Simulation Modeling of External Perturbations Affecting Wheeled Vehicles of Special Purpose

Oleg Nikonov[0000-0002-8878-4318], Igor Kyrychenko [0000-0002-2128-3500],

Vladyslav Shuliakov[0000-0002-6654-6749]

Kharkiv National Automobile and Highway University,
Str. Yaroslava Mudrogo 25, Kharkiv, 61002, Ukraine
nikonov.oj@gmail.com, igk160450@gmail.com, jason07@ukr.net

Abstract. External perturbations affecting wheeled vehicles of special purpose on the basis of 3D terrain model are estimated. The mathematical model of the system of sprinkling for special purpose wheeled vehicles was developed on the basis of the use of continuous "color" and "fractional" noise as external perturbations, affecting the system, which allows to determine the influence of the impact of the rough terrain on wheeled vehicles of special purpose with integrated information and control telematics systems. A comparative analysis of the algorithms for the formation of external perturbations on the basis of "color" and "fractional" noise was carried out. 3D models of terrain were developed on the basis of the actual normal Markov field, which gives an opportunity to raise a qualitative level mathematical models of special purpose wheeled vehicles with integrated information and control telematics systems. It was generated an algorithm of a two-dimensional fractional field. The proposed approach allows you to simulate 3D terrain with given characteristics and analyze the work of special purpose wheeled vehicles systems on different operating modes.

Keywords. Wheeled Vehicles, External Perturbations, Simulation Modeling

1 Introduction

Solving various problems related to the automated design of systems and units of special purpose wheeled vehicles (WV) the dynamic process should be simulated in the process of its movement in rough terrain.

In real conditions, the WV of special purpose does not move along the microprofile of the road, but on the surface; therefore, the replacement of the movement of a three-dimensional object by the surface with the movement of a two-dimensional object along the road profile is an excessive simplification of the model, which may lead to significant differences between the simulation model and reality.

Secondly, the use of ICTS, GIS systems, satellite communications requires data processing and visualization in 3D format. Therefore, when simulating the terrain by a probable method, it is necessary to use 3D terrain models with given characteristics.

2 Formulation of the problem

The aim of the research is to develop the mathematical model of the system of sprinkling for special purpose wheeled vehicles on the basis of the use of continuous "color" and "fractional" noise as external perturbations, affecting the system, which allows to determine the influence of specific conditions on the system. A comparative analysis of the algorithms for the formation of external perturbations on the basis of "color" and "fractional" noise should be carried out. It is necessary to develop 3D models of terrain on the basis of the actual normal Markov field, what can give an opportunity to raise a qualitative level mathematical models of special purpose wheeled vehicles with integrated information and control telematics systems.

The algorithm of a two-dimensional fractional field should be generated. To consider an approach which allows to simulate 3D terrain with given characteristics and analyze the work of special purpose wheeled vehicles systems on different operating modes.

3 Literature Review

One of the important issues that arises in these problems is the simulation of random surfaces, in which the process of movement of vehicles takes place [1]. At present, for the aforementioned tasks, it is accepted to use complex functional mathematical models (CFMM) of special purpose WV in cooperation with the terrain and the driver [2, 3]. In such CFMM the terrain can be represented by a probable method or determinative one. Usually a probable representation method is used, because the determinative method in the case of a detailed description of a rather large area of the terrain is cumbersome and excessive [4-6]. The purpose of the work is to estimate external perturbations affecting wheeled vehicles of special purpose on the basis of 3D terrain model.

Let's consider in more detail the modeling of the oscillations of the body of wheeled vehicles of special purpose. The experience in the operation of automatic devices and systems shows that automation tools operating on vehicles and other machines are effective when their elements are properly tuned [7, 8]. An efficient approach to vehicle design is to simulate dynamic processes in real time [9-11]. It should be noted that the simulation as itself must be carried out taking into account the real properties of the environment, which is especially important for wheeled vehicles of special purpose.

As a rule, in simulation modeling, stochastic modeling based on the use of the "white" noise process is used as a perturbation factor associated with the profile properties [7]. The cross-country profile on which wheeled vehicles of special purpose move contains obstacles such as pits, ditches, logs, reinforced concrete structures, etc. To describe profiles of this kind in transport problems of simulation modeling, it is necessary to use stochastic processes, whose properties are close to the indicated perturbations, and thus different from the properties of white noise. The known process of "fractional" noise, it seems, can be used in these tasks.

4 Research Methodology

We consider an integral quadratic function based on the valid normal Markov two-dimensional field (NMD field) $H(x,y)$ [6]

$$J[\mathrm{H}] = \int_0^a \int_0^b h^2(x,y)\, dxdy \tag{1}$$

where $h = h(x,y)$ – the realization of a Gaussian two-dimensional field $H(x,y)$ in a rectangular region $\{ x \in [0,a], y \in [0,b] \}$ on surface. The determining property of a stationary NMD field is its correlator

$$K_{\mathrm{XY}}(x,y;x',y') = \mathrm{E}_{\mathrm{H}}\left[h(x,y)\ h(x',y') \right] = pq\sigma_{\mathrm{H}},$$

$$p = \exp\left(-\nu\ |x-x'|\right),\ \ q = \exp\left(-\mu\ |y-y'|\right),$$

where $\mathrm{E}_{\mathrm{H}}[.]$ – the operator of mathematical expectation;
$\sigma_{\mathrm{H}} = \mathrm{E}_{\mathrm{H}}\left[h^2(x,y) \right]$ – field intensity;
ν and μ – decrements of the field fading of the axes X and Y respectively.
The generalization of known constructions - transition probabilities density for the normal Markov process of Ornshtein-Ulenbeck (OU-process) – can be the following transition probability distribution density for the NMD field

$$\begin{aligned} f_{\mathrm{H}}\left(h(x,y)\ \middle|\ h(x',y'), h(x,y'), h(x',y') \right) &= \frac{1}{\sqrt{2\pi\ \left(1-p^2\right)\ \left(1-q^2\right)\ \sigma_{\mathrm{H}}}} \times \\ &\times \exp\left\{ -\frac{\left[h(x,y) - ph(x',y) - qh(x,y') + pqh(x',y') \right]^2}{2\ \left(1-p^2\right)\ \left(1-q^2\right)\ \sigma_{\mathrm{H}}} \right\} \end{aligned} \tag{2}$$

Steering $x' \to -\infty$ and $y' \to -\infty$, we obtain limited transitions probability distribution density, which serve as a transition for partial OU processes

$$f_{\mathrm{H}}\left(h(x,y)\ \middle|\ h(x',y) \right) = \frac{1}{\sqrt{2\pi\ \left(1-p^2\right)\ \sigma_{\mathrm{H}}}} \exp\left\{ -\frac{\left[h(x,y) - ph(x',y) \right]^2}{2\ \left(1-p^2\right)\ \sigma_{\mathrm{H}}} \right\} \tag{3}$$

$$f_{\mathrm{H}}\left(h(x,y)\ \middle|\ h(x,y')\right)=\frac{1}{\sqrt{2\pi\ \left(1-q^{2}\right)\ \sigma_{\mathrm{H}}}}\exp\left\{-\frac{\left[h(x,y)-qh(x,y')\right]^{2}}{2\ \left(1-q^{2}\right)\ \sigma_{\mathrm{H}}}\right\} \tag{4}$$

The peak density distribution of probabilities of an equilibrium type for a random variable - the realization of the NMD field at the coordinate point (x,y) has this form

$$f_{\mathrm{H}}(x,y)=\frac{1}{\sqrt{2\pi\sigma_{\mathrm{H}}}}\exp\left\{-\frac{h^{2}(x,y)}{2\sigma_{\mathrm{H}}}\right\} \tag{5}$$

To consider the equation of movement for the amplitude of the NMD field [6]. We place on the surface Cartesian coordinate system with the beginning at the point (0,0). The dynamics of a random field $H(x,y)$ in a rectangle $\left\{\ x\in[0,a], y\in[0,b]\ \right\}$ with a vertex in $(0,0)$ it can be described with the help of a generalized Langevin equation for the OU-process

$$\left(\frac{\partial}{\partial x}+\nu\right)\left(\frac{\partial}{\partial y}+\mu\right)h(x,y)=\sqrt{\sigma_{H}}\ u(x,y) \tag{6}$$

where $u(x,y)$ – is a random field with properties of Gaussian two-dimensional noise with unit intensity. As the limited conditions to (6), we use two normal stochastic processes, which are described by the Langevin equations

$$\begin{aligned}&\left(\frac{\partial}{\partial x}+\nu\right)h(x,0)=\sqrt{\sigma_{H}}\ u(x,0);\\&\left(\frac{\partial}{\partial y}+\mu\right)h(0,y)=\sqrt{\sigma_{H}}\ u(0,y),\end{aligned} \tag{7}$$

which are realized along axes X and Y accordingly, and the initial condition to them will be the peak values of the random variable

$$h(0,0)=\sqrt{\sigma_{H}}\ u(0,0) \tag{8}$$

The solution of equation (6) with conditions (7) and (8) is as follows

$$h(x,0) = h(0,0)\exp(-\nu x) + \sqrt{\sigma_H}\int_0^x \exp\left[-\nu(x-x')\right] u(x',0)\, dx';$$

$$h(0,y) = h(0,0)\exp(-\mu y) + \sqrt{\sigma_H}\int_0^y \exp\left[-\mu(y-y')\right] u(0,y')\, dy';$$

$$\begin{aligned} h(x,y) &= h(0,0)\exp(-\nu x - \mu y) + \sqrt{2\nu\sigma_H}\exp(-\mu y)\times \\ &\times \int_0^x \exp\left[-\nu(x-x')\right] u(x',0)\, dx' + \sqrt{2\mu\sigma_H}\exp(-\nu x)\times \\ &\times \int_0^y \exp\left[-\mu(y-y')\right] u(0,y')\, dy' + \sqrt{4\nu\mu\sigma_H}\times \\ &\times \int_0^x\int_0^y \exp\left[-\nu(x-x') - \mu(y-y')\right] u(x',y')\, dx'\, dy' \end{aligned} \quad (9)$$

From the obtained solution it is obvious that the Markov property is performed along the axes X and Y.

Using the known rules of discretization OU process in the nodes of a rectangular lattice and renormalizing the generating field $u(x,y)$, it is possible to construct a numerical algorithm for generation of a NMD field based on the solution (9). This hierarchical algorithm for generating the reference in the nodes of a random normal stationary Markov field in a rectangular surface can be given by the following four steps:

1. Generation of the countdown at the vertex

$$h_{0,0} = \sqrt{\sigma_H}\; u_{0,0}.$$

2. Generation of the countdown of the process along the X-limit rectangle ($j > 0$)

$$h_{j+1,0} = p h_{j,0} + \sqrt{\left(1-p^2\right)\sigma_H}\; u_{j+1,0}.$$

3. Generation of the countdown of the process along the Y-limit rectangle ($k > 0$)

$$h_{0,k+1} = q h_{0,k} + \sqrt{\left(1-q^2\right)\sigma_H}\; u_{0,k+1}.$$

4. Sequential filling of the countdown internal nodes of the rectangle ($j > 0$, $k > 0$)

$$h_{j+1,k+1} = p h_{j,k+1} + q h_{j+1,k} - pq h_{j,k} + \sqrt{\left(1-p^2\right)\left(1-q^2\right)\sigma_H}\; u_{j+1,k+1}.$$

In the presented algorithm

$$p = \exp(-\nu\Delta_x); \quad q = \exp(-\mu\Delta_y),$$

where ν and μ – partial decrements;

Δ_x and Δ_y – steps of nodes on the axes X and Y respectively.

To note that for selected Δ_x, Δ_y (i.e., for given dimensions of the rectangle a, b and the corresponding number of steps $N_x = a/\Delta_x$, $N_y = b/\Delta_y$) the intensity in this algorithm needs to be renormalized so that the energy of the NMD field per unit area coincides with the given in any number of steps.

Thus, the aforementioned algorithm for generating random countdown in a rectangle on a surface is stationary [8, 9].

In conducting many experiments, the following results were obtained. The simulation was carried out in a rectangular area $\{ x \in [0,10], y \in [0,10] \}$. It is shown three realizations of a random NMD field (fig. 1), which differ only in the value of the field intensity (case a, b, c). The picture shows the dynamics of the formation of fluctuations of NMD-fields on both coordinates. In fig. 2 it is shown the original output profile $s(x,y) = [\sin(x)\cdot\cos(y)] / 2$. When the additive overlay of a random field (fig. 1b) on a regular profile (fig. 2) is obtained the surface $p(x,y) = s(x,y) + h(x,y)$, shown in fig. 3.

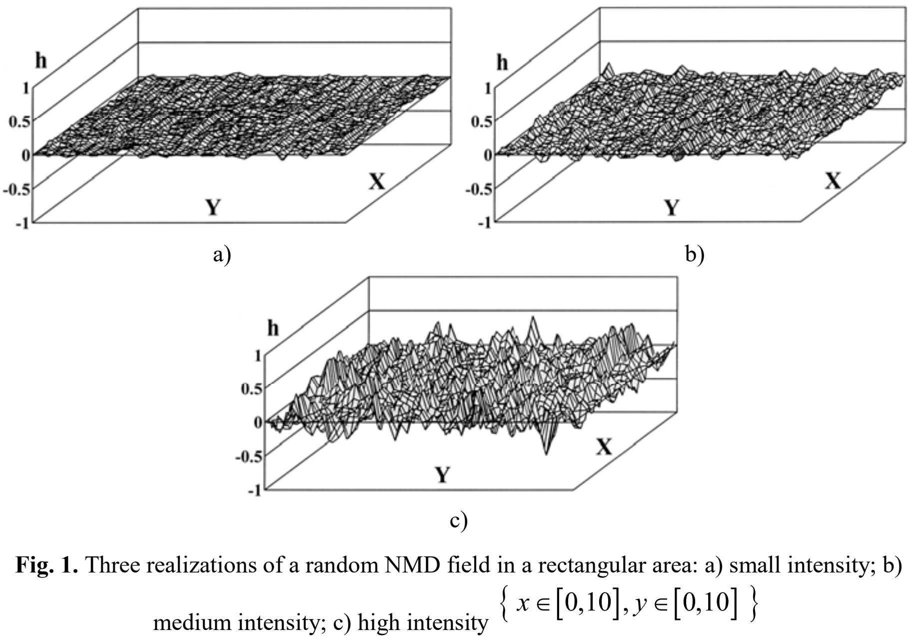

Fig. 1. Three realizations of a random NMD field in a rectangular area: a) small intensity; b) medium intensity; c) high intensity $\{ x \in [0,10], y \in [0,10] \}$

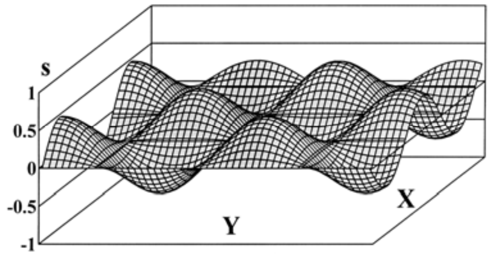

Fig. 2. The original output profile $s(x,y)=\left[\sin(x)\cdot\cos(y)\right]/2$

In the study of spatial images on a given rectangle $\left[(0,a)\times(0,b)\right]$ one of the main questions of the theory of evaluation is the information on the distribution of random values η of the functional J[H].

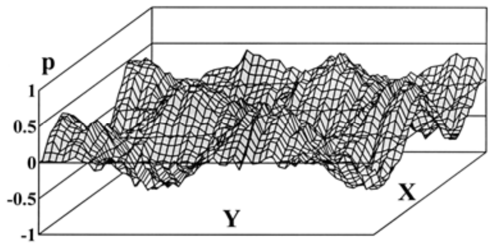

Fig. 3. Additive overlay of a random field (fig 1b) on outgoing original output profile (fig 2)

Because of the positive definiteness and additivity of the integral on a rectangle [(0,*a*)×(0,*b*)] it is convenient to describe the properties of a functional *J*[*H*] using a function in the form of the next mathematical expectation

$$Q_{XY}(\lambda)=\mathrm{E}_{\mathrm{H}}\left[\exp\left(-\lambda J[\mathrm{H}]\right)\right]=\mathrm{E}_{\mathrm{H}}\left[\exp\left(-\lambda\int_0^a\int_0^b h^2(x,y)\,dxdy\right)\right] \tag{10}$$

where λ – random parameter.

The desired function can be represented as a completely convergent result [6]

$$Q_{XY}(\lambda)=\prod_{n=1}^{\infty}\prod_{m=1}^{\infty}\frac{1}{\sqrt{1+\dfrac{\lambda}{\lambda_{X,n}\lambda_{Y,m}}}} \tag{11}$$

where $\{\lambda_X\}$ and $\{\lambda_Y\}$ – sets of proper values of partial correlators K_X and K_Y, associated with solutions of integral equations corresponding to the limits of a rectangle

$$\phi_X(x)=\lambda_X\left(K_X,\phi_X(x)\right)=\lambda_X\sigma_{\mathrm{H}}\int_0^a\exp\left(-\nu\,|x-x'|\right)\phi_X(x')\,dx' \tag{12}$$

$$\phi_Y(y) = \lambda_Y\left(K_Y, \phi_Y(y)\right) = \lambda_Y \sigma_{\text{H}} \int_0^b \exp\left(-\mu\ |y - y'|\right)\ \phi_Y(y')\ dy' \qquad (13)$$

In the considered case, the NMD field for partial functions $Q_X(\lambda)$ and $Q_Y(\lambda)$ is well known [6], that

$$Q_X(\lambda) = \sqrt{\frac{4\nu r_X \exp(\nu a)}{(r_X + \nu)^2 \exp(r_X a) - (r_X - \nu)^2 \exp(-r_X a)}} \qquad (14)$$

$$Q_Y(\lambda) = \sqrt{\frac{4\mu r_Y \exp(\mu b)}{(r_Y + \mu)^2 \exp(r_Y b) - (r_Y - \mu)^2 \exp(-r_Y b)}} \qquad (15)$$

where $r_X = r_X(\lambda) = \sqrt{\nu^2 + 2\lambda\nu\sigma_{\text{H}}}$; $r_Y = r_Y(\lambda) = \sqrt{\mu^2 + 2\lambda\mu\sigma_{\text{H}}}$.

The simple zeros of the denominators of the given partial functions $Q_X(\lambda)$ and $Q_Y(\lambda)$ the essence of the negatively defined sets of numbers $\left\{\lambda_{X,n}\right\}_{n=1}^{\infty}$ and $\left\{\lambda_{Y,m}\right\}_{m=1}^{\infty}$. Thus, on the basis of these sets, it is necessary to construct a formula for the desired function. In other words, it is necessary to reduce the result (11) and to obtain a constructive analytical expression for $Q_{XY}(\lambda)$, without the necessity to find the specific values of the sets $\{\lambda_X\}$ and $\{\lambda_Y\}$.

The main analytical result is the following representation for the function $Q_{XY}(\lambda)$ of the functional (1)

$$Q_{XY}(\lambda) = \exp\left\{-\frac{1}{(2\pi i)^2} \iint_C \left(\ln Q_X(x) \frac{d}{dx}\right)\left(\ln Q_Y(y) \frac{d}{dy}\right) \ln\left(1 + \frac{\lambda}{xy} \frac{\langle\eta\rangle_{XY}}{\langle\eta\rangle_X \langle\eta\rangle_Y}\right) dxdy\right\} \qquad (16)$$

where $\langle\eta\rangle_{XY} = ab\sigma_{\text{H}}$ – the average value of the functional (1);

$\langle\eta\rangle_X = a\sigma_{\text{H}}$ and $\langle\eta\rangle_Y = b\sigma_{\text{H}}$ – partial mathematical expectations;

C – the contour of integration in the complex surface passing through the imaginary axis bypassing the point $(0,0)$ on the left is a fairly small radius of a semicircle.

The density of the probability distribution $f_J(\eta)$ of random values η of a functional $J[\text{H}]$ is determined, therefore, on the basis of the inverse Laplace transformation of the function $Q_{XY}(\lambda)$. Such a transformation for functions (14), (15) or (16) can only be performed numerically, so this procedure will be implemented the better, the more accurate the analytical information about the function $Q_{XY}(\lambda)$.

To reason the results of the simulation, families of histograms corresponding to the distribution of the integral quadratic functional $J[\text{H}]$ for a particular field were obtained. The volume of the statistical sample in the simulation was 10^4, the relative sample dispersion did not exceed 3%. The obtained histograms were the same with the given accuracy. A number of similar experiments showed the preservation of the type of histograms in numerical simulation for any region.

Also the algorithm was developed for generation of two-dimensional fractional field based on the description of fractional noise. In fig. 4 it is shown four realization of random fractional fields for different values of the number of strokes *K*.

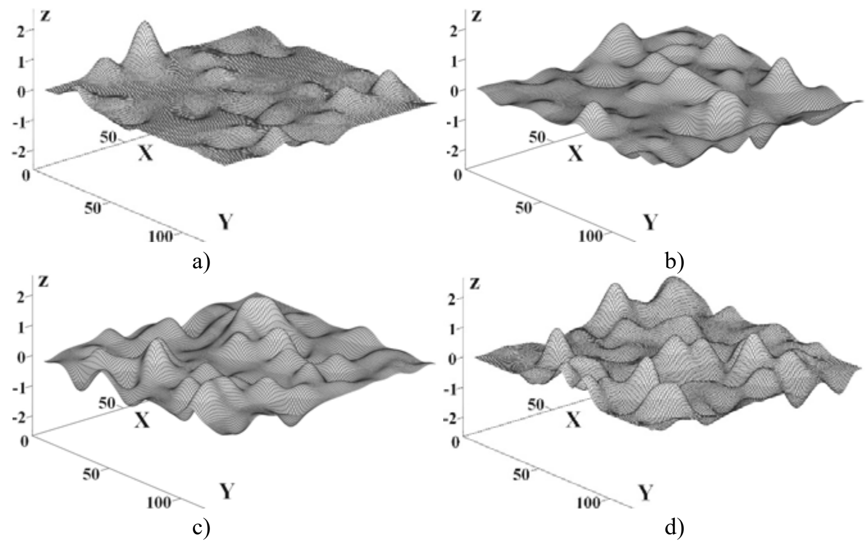

Fig. 4. Realization of a random fractional field in a rectangular region at: a) $K = 50$; b) $K = 100$; c) $K = 150$; d) $K = 200$

The picture shows well the dynamics of the formation of fluctuations on both coordinates, which agrees with the results of 2D fractional noise modeling. The calculation of the fluctuations of the body of special purpose wheeled vehicles and its main systems and aggregates, taking into account the movement of the 3D field, is carried out according to the known calculation formulas.

5 Conclusions

The mathematical model of the system of sprinkling for special purpose wheeled vehicles was developed on the basis of the use of continuous "color" and "fractional" noise as external perturbations, affecting the system, which allows to determine the influence of the impact of the rough terrain on wheeled vehicles of special purpose with integrated information and control telematics systems. It is proved that for the simulation of the terrain with the help of CFMM it is most appropriate to use the probable representation method of the terrain, because the determinative method in the case of a detailed description of a rather large area of the terrain is cumbersome and excessive. A comparative analysis of the algorithms for the formation of external perturbations on the basis of "color" and "fractional" noise was carried out. On the basis of many experiments it has been proved that in the problems of modeling the movement of special purpose wheeled vehicles, it is possible to use the external perturbations based on the use of "fractional" noise and "color" noise, depending on the type of terrain. 3D models of terrain were developed on the basis of the actual normal Markov field, which gives an opportunity to raise a qualitative level mathematical models of special purpose wheeled vehicles with integrated information and control telematics systems. The proposed approach allows you to simulate virtually any 3D terrain with given characteristics and analyze the operation of wheeled vehicles control systems at different operating modes, as well as to use calculated 3D terrain models for geoinformation programs of information and control telematics systems. It was generated an algorithm of a two-dimensional fractional field. The proposed approach allows you to simulate 3D terrain with given characteristics and analyze the work of special purpose wheeled vehicles systems on different operating modes.

References

1. Xu, G., Ma, Z, Lu, F., Hou, P.: Kinematic Analysis of Hydraulic Excavator Working Device Based on DH Method. In: International Conference on Applied Mechanics, Mechanical and Materials Engineering, pp. 8 (2016). doi: 10.12783/dtmse/ammme2016/6857
2. Xue-wen, Chen, Yue, Zhou: Modelling and Analysis of Automobile Vibration System Based on Fuzzy Theory under Different Road Excitation Information. In: Complexity, Jinzhou, 9 pages (2018) https://doi.org/10.1155/2018/2381568
3. Shahgholi, G., Chiyaneh, H., Gundoshmian T.: Modeling of Soil Compaction Beneath the Tractor Tire using Multilayer Perceptron Neural Networks. Journal of Agricultural Machinery, vol.8, pp. 105-118 (2018)
4. Liu Y., Tong S.: Adaptive fuzzy control for a class of unknown nonlinear dynamical systems. In: Fuzzy Sets and Systems, vol. 263, pp. 49-70 (2015)
5. Gao, Y.: Adaptive fuzzy optimal control using direct heuristic dynamic programming for chaotic discrete-time system. Journal of Vibration and Control, vol. 22, pp. 595-603 (2016)
6. Mazmanishvili, A., Rafalovich, O.: Construction example of regional phase maps of error probability for chosen satellite communication network configuration. In: Engineering Simulation, vol. 15, pp. 715-719 (1998) (in Russian)
7. Aleksandrov, E.: Dynamics of transport and wheeled and tracked vehicles. Kharkiv, KHDATU (2001) (in Russian)
8. Aleksandrov, E.: To the task of optimizing the parameters of the vehicle suspension system. Mechanics and engineering. Kharkiv, KHDPU, pp. 35-41 (2000) (in Russian)
9. Uspensky, B., Avramov, K., Liubarskyi, B., Andrieiev, Y., Nikonov, O.: Nonlinear torsional vibrations of electromechanical coupling of diesel engine gear system and electric generator. Journal of Sound and Vibration, vol. 460, 114877 (2019) doi:10.1016/j.jsv.2019.114877
10. Gurko, A., Kyrychenko, I., Yaryzhko, A.: Trajectories Planning and Simulation of a Backhoe Manipulator Movement. CMIS, pp. 771-785 (2019)
11. Uspensky, B., Avramov, K., Nikonov, O.: Nonlinear modes of piecewise linear systems forced vibrations close to superharmonic resonances. Proceedings of the Institution of Mechanical Engineers, Part C: Journal of Mechanical Engineering Science, vol. 233, Issue 23-24, pp. 7489-7497 (2019) doi: 10.1177/0954406219869967