Parametric synthesis of a dynamic object control system with nonlinear characteristics

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Abstract. The aim of the research is to solve the problem of using the method of factor experiment for the problem of parametric synthesis of a closed dynamic object control system. The method of factor experiment allows, on the basis of modeling the behavior of the system in a random external environment, to select its parameters that satisfy the requirements of the minimum of the regression functional. A quality functional that realize the requirements for a dynamic object control system and reduces the computational resource in the parametric synthesis of the system was proposed. The behavior of a closed control system was simulated considering the random external perturbations acting on the control object. The introduction of a nonlinear link with variable amplification factor widens the stability area of a closed dynamic object control system. The parametric synthesis of the closed dynamic object control system considering nonlinear characteristics on the basis of the factor experiment method was considered.

Keywords: Car, Parametric Synthesis, Quality Functional, Transient Process, Dynamic Object.

1 Introduction

Recently, the simulation methods become widespread use, in particular the method of factor experiment. This method allows, on the basis of modeling the behavior of the system in a random external environment, to select its parameters that satisfy the requirements of the minimum of the regression functional. Additive quality functional of the closed system can be used as a regression function. In this case, the behavior of a closed control system is simulated into account of the random external perturbations acting on the control object.

2 Formulation of the problem

The aim of the research is to solve the problem of applying the method of factor experiment to the problem of parametric synthesis of the dynamic object control system. Due to continuous external disturbance $M_{ed}(t)$, the position of the dynamic object is constantly changing. Changing the position of a dynamic object $M_{ed}(t)$ is a random function. In this situation, random functions are also functions $\phi_c(t)$ and $\omega_c(t)$ (Fig. 1). The accuracy of stabilizing a dynamic object is higher, the smaller the area under the dynamic process curves.

3 Literature review

The methods of classical automatic control theory do not work for high-order mathematical models. If the order of the differential equations that make up the mathematical model of the control object is higher than five, then the problem of parametric synthesis can be solved only by the methods of modern control theory, in particular, the method of state-space, the methods of the theory of analytic design of optimal regulators [1-3], minimax methods, methods of Liapunov functions, methods of simulation modeling.

Minimax methods and methods of Liapunov functions, which were applied by E.E. Alexandrov to solve the problem of parametric synthesis of the control system give satisfactory results [4-6], because they provide for the use of information in the control algorithm only about those components of the vector of the state of the control object, the measurement of which is not difficult. But in the process of parametric synthesis of a digital control system, these methods involve the transition from differential equations of the mathematical model of the control object to equations, which introduces a certain error in the synthesis process.

In addition, the high order of the differential equations describing the perturbed motion of the control object results in considerable computational difficulties associated with limited computer memory when applying the minimax method and the Liapunov function method. However, if the objects contain non-analytic nonlinearities, then the above synthesis methods are inactive. Therefore, in recent years, the simulation methods become widespread use, in particular the method of factor experiment [7, 8].

4 Research Methodology

Therefore, the quantitative accuracy of the stabilization of the dynamic object relative to the base position can be estimated by the following functional

$$I_{\phi_c} = \mathrm{M}\left[\int_0^T \phi_c^2(t)\, dt\right] \tag{1}$$

where M – is the symbol of mathematical expectation.

The quality of stabilization processes is estimated not only by the changing of position of the dynamic object, but also by the angular velocity of motion of the dynamic object relative to its base position [9, 10]. The quantitative characteristic of this movement may be functional

$$I_{\omega_c} = \mathrm{M}\left[\int_0^T \omega_c^2(t)\, dt\right] \tag{2}$$

It was an ideal case, when closed dynamic object control system solutions, both functionalities (1) and (2) would be minimized. But this is not possible and the minimums of functionals (1) and (2) correspond to different values of the parameters of the controller k_ϕ and k_ω. Therefore, it is advisable to choose the parameters k_ϕ and k_ω in condition of a minimum of additive functionality

$$I = \beta_1^2 I_{\phi_c} + \beta_2^2 I_{\omega_c} = \mathrm{M}\left[\int_0^T \left[\beta_1^2 \phi_c^2(t) + \beta_2^2 \omega_c^2(t)\right] dt\right], \tag{3}$$

where β_1 and β_2 - are the weighting factors to be selected.

But, many experiments [11] proved that the obtained values of the varied parameters, which give a minimum of functionals (1) and (2) are almost not different (within 5%). Considering this fact, as well as the fact that the output signal from the control unit is limited and does not allow the dynamic object to pass excessive speed, it is advisable to use functional (1) or its modifications for this particular system.

In general, a functional that evaluates the quality of the stochastic system looks like

$$I = \mathrm{M}\left[\int_0^T f\left[e(t), x(t), y(t), t\right] dt\right], \tag{4}$$

where f – is the function of error, input and output signals, as well as time. Using different combinations of system and time variables, different quality estimates can be obtained.

To reduce the contribution of a significant initial error and to account for a future error, it is more appropriate to use the form functionality

$$I = \mathrm{M}\left[\int_{t_1}^T t\, \left|\Delta\phi_c(t)\right| dt\right], \tag{5}$$

and when $M_{ed}(t) = 0$ functional (5) is transformed into $I = \int_{t_1}^{T} t \left|\Delta\phi_c(t)\right| dt$.

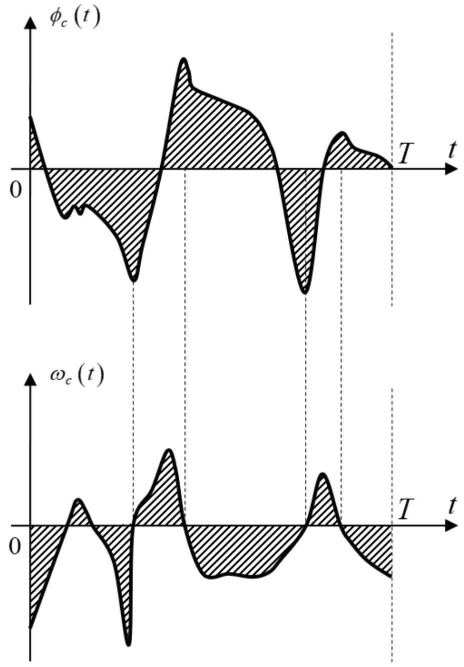

Fig. 1. Dynamic processes in a closed dynamic object control system

Choosing as variables parameters the coefficients k_ϕ and k_ω^*, and as the optimization parameters, the functional (5), we use the factor experiment theory [7, 8] to find the values of the variable parameters of the controller that give the least functional (5) for the dynamic object control system.

The points a, b correspond to the minimum of the objective function (5) without and taking into account $W_{H_1}\left(A_{H_1}\right)$ respectively. Parameters for (5): $t_1 = 0.25$ s, $T = 5$ s. For the case without taking into account $W_{H_1}\left(A_{H_1}\right)$ – $I = 0.005958$, $k_\phi^* = 199.5$, $k_\omega^* = 16.3$. For the case considered $W_{H_1}\left(A_{H_1}\right)$ – $I = 8.186339$, $k_\phi^* = 323.2$, $k_\omega^* = 21.0$.

In Fig. 2 the transients processes of a closed dynamic object control system with the obtained values of the variable parameters of the controller k_ϕ^* and k_ω^* for the case without taking into account $W_{H_1}\left(A_{H_1}\right)$ are presented. As you can see, transients processes are smooth without significant fluctuations. The amount of overshoot was 0.5 %.

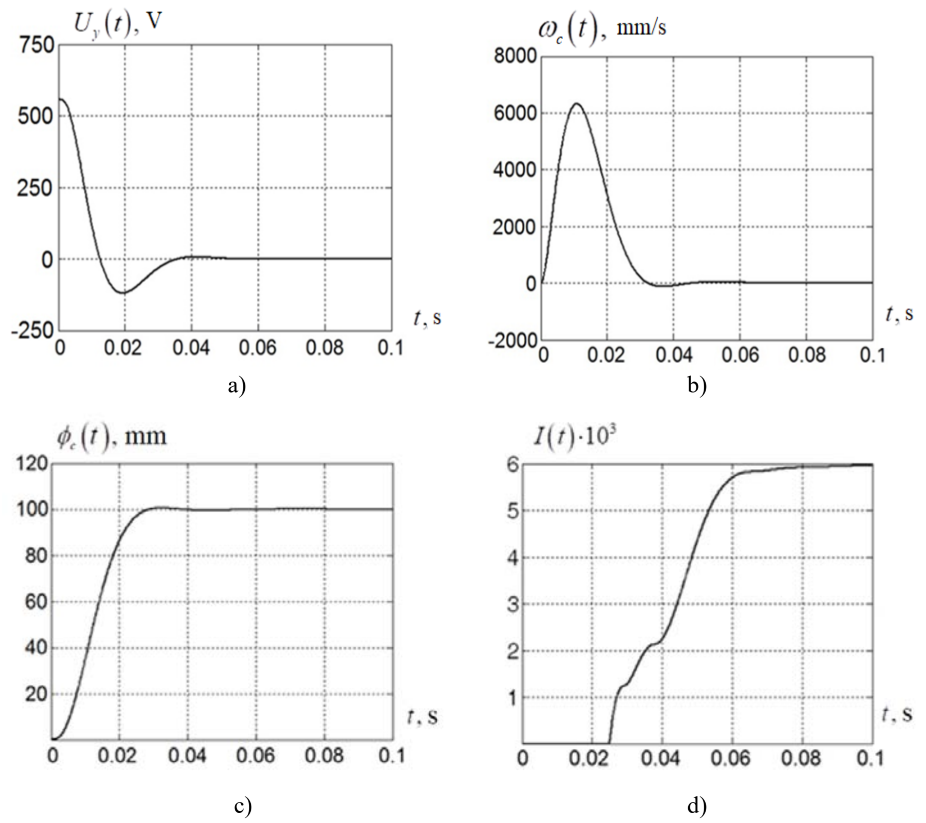

Fig. 2. Transients processes of a closed dynamic object control system when $k_\phi^* = 199.5$, $k_\omega^* = 16.3$: control voltage $U_y(t)$ (a), dynamic object speed $\omega_c(t)$ (b) and for the value of moving a dynamic object 100mm $\phi_c(t)$ (c) and the objective function $I(t)$ (d)

In Fig. 3 the transients processes of a closed dynamic object control system with the obtained values of the variable parameters of the controller k_ϕ^* and k_ω^* in the case of the given $W_{H_1}\left(A_{H_1}\right)$ are presented. As you can see, transients processes are smooth without significant fluctuations. The amount of overshoot was 0.1 %.

A structural diagram of a dynamic object control system, in addition to a nonlinear link with a transfer function $W_{H_1}\left(A_{H_1}\right)$ and characteristic, of the limited (saturation) zone type also contains a nonlinear link with a transfer function $W_{H_2}\left(A_{H_2}\right)$ and characteristic with variable amplification factor.

In this case, this nonlinear link is introduced into the structural diagram in order to improve the quality of transients processes and increase the reliability of the system as a whole by extending the stability area.

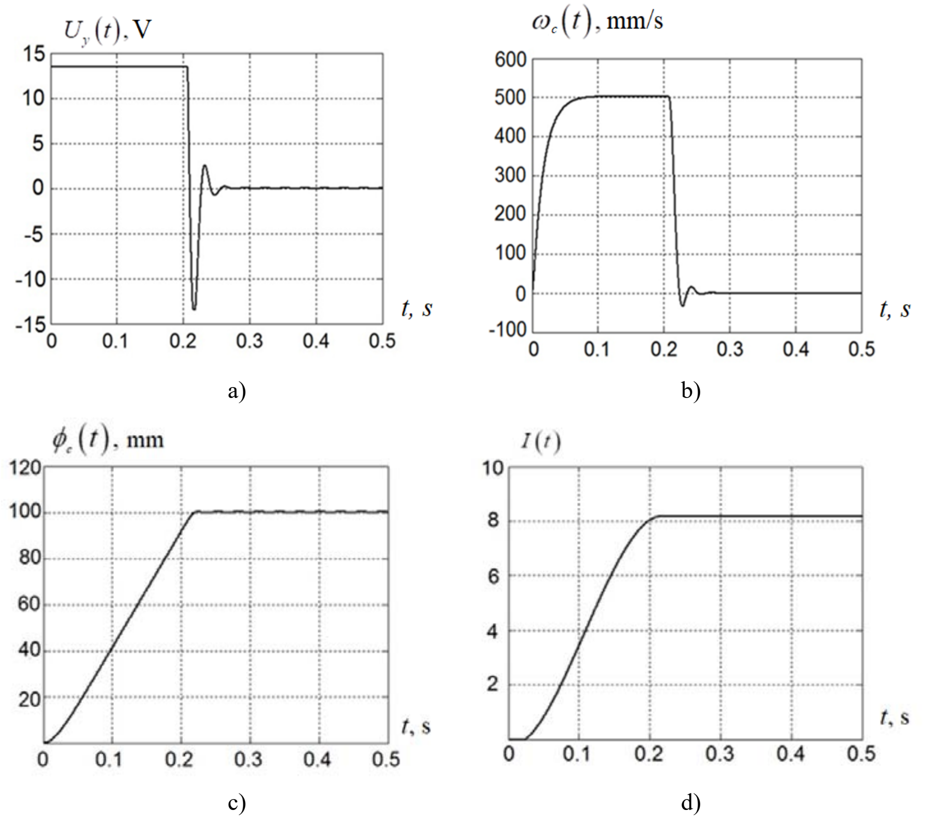

Fig. 3. Transients processes of a closed dynamic object control system when $k_{\phi}^{*} = 323.2$, $k_{\omega}^{*} = 21.0$: control voltage $U_y(t)$ (a), dynamic object speed $\omega_c(t)$ (b) and for the value of moving a dynamic object 100mm $\phi_c(t)$ (c) and the objective function $I(t)$ (d)

From the structural diagram we find the transfer function of the closed circuit with the nonlinear link with the transfer function $W_{H_2}\left(A_{H_2}\right)$

$$W'_{3HH}(s) = \frac{W_{3H}(s) \cdot W_{H_2}\left(A_{H_2}\right)}{1 + W_{3H}(s) \cdot W_{H_2}\left(A_{H_2}\right) \cdot k_{\omega} \cdot \dfrac{k_{gs}}{T_{\omega 1}^2 s^2 + T_{\omega 2} s + 1} \cdot s}. \tag{6}$$

We find the transfer function of the whole open system

$$W_{4HH}(s) = \frac{k_{\phi} \cdot W_{3H}(s) \cdot W_{H_2}(A_{H_2})}{1 + W_{3H}(s) \cdot W_{H_2}(A_{H_2}) \cdot k_{\omega} \cdot \dfrac{k_{gs}}{T_{\omega 1}^2 s^2 + T_{\omega 2} s + 1} \cdot s}$$

and a closed system

$$W'_{4HH} = \frac{W_{4HH}(s)}{1 + W_{4HH}(s)}. \tag{7}$$

We construct the stability domain of a closed dynamic object control system in the plane of variable parameters of the controller k_{ϕ} and k_{ω}, taking into account a nonlinear link such as a zone of limitation (saturation) and a nonlinear link with a variable amplification factor.

$$\begin{aligned}
&T_y\ A_3\ T_{\omega 1}^2\ s^7 + \left(T_y\ A_3\ T_{\omega 2} + T_y\ A_2\ T_{\omega 1}^2 + A_3\ T_{\omega 1}^2\right) s^6 + \\
&+\left(A_2\ T_{\omega 1}^2 + T_y\ A_2\ T_{\omega 2} + T_y\ A_3 + A_3\ T_{\omega 2} + T_y\ A_1\ T_{\omega 1}^2\right) s^5 + \\
&+\left(T_y\ T_{\omega 1}^2 + A_2\ T_{\omega 2} + T_y\ A_2 + A_3 + T_y\ A_1\ T_{\omega 2} + A_1\ T_{\omega 1}^2\right) s^4 + \\
&+\left(T_{\omega 1}^2 + A_1\ T_{\omega 2} + T_y\ A_1 + T_y\ T_{\omega 2} + A_2\right) s^3 + \\
&+\left(T_{\omega 2} + A_1 + T_y + k_{\phi}\ k\ \ W_{H_1}(A_{H_1})\ \ W_{H_2}(A_{H_2})\ k_y\ k'_2\ T_{\omega 1}^2\right) s^2 + \\
&\left(1 + k\ \ W_{H_1}(A_{H_1})\ \ W_{H_2}(A_{H_2})\ k_y\ k'_2\left(k_{\phi 2}\ T_{\omega 2} + k_{\omega}\ k_{gs}\right)\right) s + \\
&+k_{\phi}\ k\ \ W_{H_1}(A_{H_1})\ \ W_{H_2}(A_{H_2})\ k_y\ k'_2 = 0.
\end{aligned} \tag{8}$$

From the transfer function of the closed system (7) we write the characteristic equation (8). In characteristic equation (8) we make a substitution $s = j\omega$, select the real and imaginary parts and equal them to zero. Obtained algebraic equations we solve referring to the parameters k_{ϕ} and k_{ω}:

$$k_\phi = \frac{\left(T_y\ T_{\omega 1}^2 + A_2\ T_{\omega 2} + T_y\ A_2 + A_3 + T_y\ A_1\ T_{\omega 2} + A_1\ T_{\omega 1}^2\right)\ \omega^4}{k\ W_{H_1}\left(A_{H_1}\right)\ W_{H_2}\left(A_{H_2}\right)\ k_y\ k_2'\left(\omega^2 T_{\omega 1}^2 - 1\right)} - \tag{9}$$

$$- \frac{\left(T_y\ A_3\ T_{\omega 2} + T_y\ A_2\ T_{\omega 1}^2 + A_3\ T_{\omega 1}^2\right)\ \omega^6 + \left(T_{\omega 2} + A_1 + T_y\right)\ \omega^2}{k\ W_{H_1}\left(A_{H_1}\right)\ W_{H_2}\left(A_{H_2}\right)\ k_y\ k_2'\left(\omega^2 T_{\omega 1}^2 - 1\right)};$$

$$k_\omega = \frac{\left(T_{\omega 1}^2 + A_1\ T_{\omega 2} + T_y\ A_1 + T_y\ T_{\omega 2} + A_2\right)\ \omega^2 + T_y\ A_3\ T_{\omega 1}^2\ \omega^6}{k\ W_{H_1}\left(A_{H_1}\right)\ W_{H_2}\left(A_{H_2}\right)\ k_y\ k_2'\ k_{gs}} -$$

$$- \frac{\left(A_2\ T_{\omega 1}^2 + T_y\ A_2\ T_{\omega 2} + T_y\ A_3 + A_3\ T_{\omega 2} + T_y\ A_1\ T_{\omega 1}^2\right)\ \omega^4}{k\ W_{H_1}\left(A_{H_1}\right)\ W_{H_2}\left(A_{H_2}\right)\ k_y\ k_2'\ k_{gs}} - \tag{10}$$

$$- \frac{k_\phi\ k\ W_{H_1}\left(A_{H_1}\right)\ W_{H_2}\left(A_{H_2}\right)\ k_y\ k_2'\ T_{\omega 2} + 1}{k\ W_{H_1}\left(A_{H_1}\right)\ W_{H_2}\left(A_{H_2}\right)\ k_y\ k_2'\ k_{gs}}.$$

In Fig. 4 and 5 are shown the limits of stability area of the closed dynamic object control systems in the plane of variable parameters of the controller k_ϕ and k_ω taking into account a nonlinear variable with amplification factor, constructed using relations (9) and (10), where a, b, c – are the points of minimum of the objective function (5) without and with considering $W_{H_1}\left(A_{H_1}\right)$, $W_{H_2}\left(A_{H_2}\right)$ respectively.

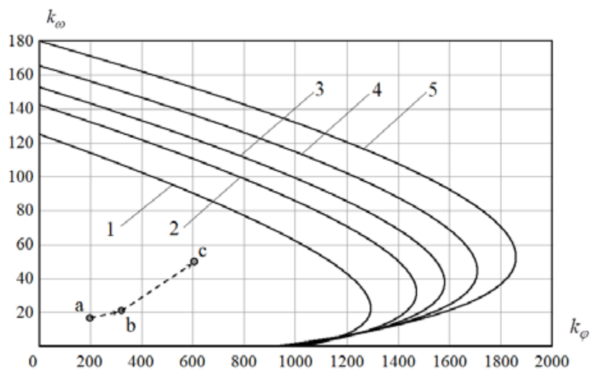

Fig. 4. Stability area of a closed dynamic object control system considering the nonlinear link with variable amplification factor at $A_{H_2}/b_{H_2} = 2$: 1 – $k_{H_2} = k_{H_3}$; 2 – $k_{H_2} = 0.8\ k_{H_3}$; 3 – $k_{H_2} = 0.7\ k_{H_3}$; 4 – $k_{H_2} = 0.6\ k_{H_3}$; 5 – $k_{H_2} = 0.5\ k_{H_3}$

For the case with $W_{H_1}\left(A_{H_1}\right)$ and $W_{H_2}\left(A_{H_2}\right)$ based on the method of factor experiment we obtain: $I = 8.190428$, $k_{\phi}^{*} = 607.1$, $k_{\omega}^{*} = 49.9$. Parameters for (5): $t_1 = 0.25$ s, $T = 5$ s.

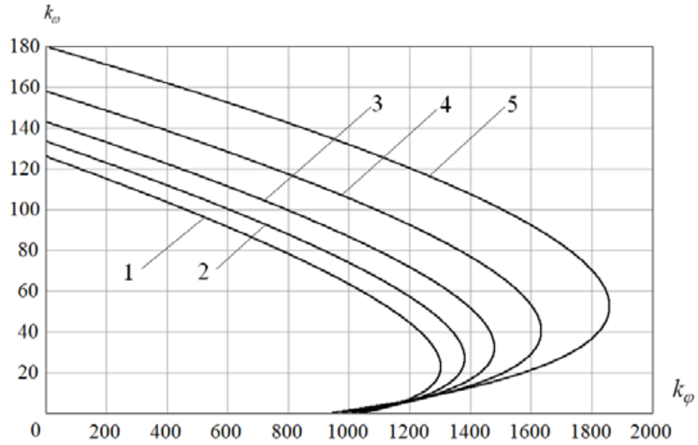

Fig. 5. Stability area of a closed dynamic object control system considering the nonlinear link with variable amplification factor at $k_{H_2} = 0.5\,k_{H_3}$: 1 – $A_{H_2}/b_{H_2} = 100$; 2 – $A_{H_2}/b_{H_2} = 10$; 3 – $A_{H_2}/b_{H_2} = 5$; 4 – $A_{H_2}/b_{H_2} = 3$; 5 – $A_{H_2}/b_{H_2} = 2$

As you can see from Fig. 4, the introduction of a nonlinear link [12 - 14] with variable amplification factor widens the stability area of a closed dynamic object control system (with $k_{H_2} = k_{H_3}$ (curve 1), the stability area coincides completely with the stability of the linear system).

The optimal point c in this case moves to the area of high coefficients k_{ϕ} and k_{ω}. The above helps to improve the reliability and accuracy of a closed dynamic object control system.

As you can see from Fig. 5, when $A_{H_2}/b_{H_2} = 100$ the equation $W_{H_2}\left(A_{H_2}\right) \to k_{H_3}$, and the stability area approaches to the linear.

In Fig. 6 the transients processes of a closed dynamic object control system with the obtained values of the variable parameters of the controller k_{ϕ}^{*} and k_{ω}^{*} considering $W_{H_1}\left(A_{H_1}\right)$ and $W_{H_2}\left(A_{H_2}\right)$ are presented. The amount of overshoot was 0.1%.

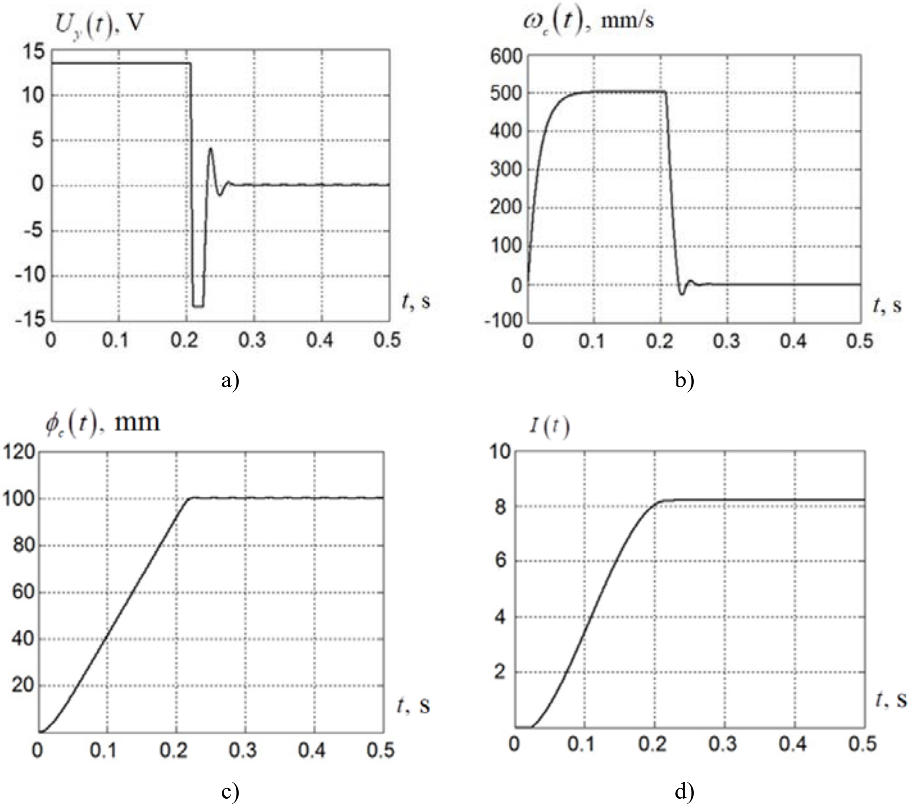

Fig. 6. Transients processes of a closed dynamic object control system when $k_{\phi}^{*} = 607.1$, $k_{\omega}^{*} = 49.9$: control voltage $U_y(t)$ (a), dynamic object speed $\omega_c(t)$ (b) and for the value of moving a dynamic object 100mm $\phi_c(t)$ (c) and the objective function $I(t)$ (d)

For a more positive result when using a variable structures need to use a more complicated law of the changing the structure of the system, but it can reduce the reliability of the system as a whole, due to the additional elements and connections.

5 Conclusions

The aim of the research to solve the problem of using the method of factor experiment for the problem of parametric synthesis of a closed dynamic object control system was considered. Quality functional that realize the requirements of a dynamic object control system and reduce the calculating resource of parametric system synthesis was proposed. Parametric synthesis of a closed dynamic object control system with nonlinear characteristics on the basis of the factor experiment method was made by using three practical examples.

It was proved that the method of factor experiment allows, on the basis of modeling the behavior of the system in a random external environment, to select its parameters that satisfy the requirements of the minimum of the regression functional. The behavior of a closed control system was simulated considering the random external perturbations acting on the control object. It was defined that the introduction of a nonlinear link with variable amplification factor widens the stability area of a closed dynamic object control system.

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