A novel contribution to an hybrid INS/GPS system using a fuzzy approach*

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***Abstract*—In this paper, we present a technique based on fuzzy logic to improve the performance of an integrated inertial navigation system with GPS. The fuzzy technique proposed is mainly used to predict position and velocity measurements during the absence of GPS signals. As long as GPS measurements are available, the Q-SUKF [1] filter for INS / GPS integration works efficiently and provides an accurate estimate of the states of navigation. Nevertheless, during the disturbance of GPS signals, the fuzzy technique will be used with the Q-SUKF filter to correct the performance degradation of the algorithm. Finally, an experimental part on the use of the fuzzy technique proposed with the Q-SUKF has been validated. The aftereffects of our experiment have demonstrated the adequacy and the critical effect of the fuzzy method used. It decreases the error's estimation of the position and velocity during GPS blackout periods.**

***Index Terms*—Inertial Navigation System, Global Positioning System, Takagi-Sugeno Fuzzy Model, Fuzzy C-Means, Quaternion Scaled Unscented Kalman Filter.**

I. Introduction

The last decades have seen an increase in demand for inertial navigation systems (INS) low-weight, low cost and low-power consumption, in many applications such as personal, automotive and air navigation. The technological progress of micro-electro-mechanical systems (MEMS) showed a promised signs for the development of these systems. Compared to higher quality systems, a low-cost inertial navigation system may cause a strong drift on the accuracy of the estimate of position, velocity and attitude on short time intervals. This is mainly due to the great uncertainty of the outputs of MEMS sensors and their sensitivity to changes in the environment. If the accuracy of low-cost INS can be improved, its cost can be reduced in existing applications and new applications may emerge. Like most uncertainties exist in the behavior of the sensor errors, the calibration would significantly improve the accuracy. However, intensive calibration would increase considerably the cost. Another way to improve the accuracy would be to hybridize the inertial navigation systems with other complementary external sensors. Choosing the appropriate method of hybridization is the key for the development of Hybridised inertial navigation problem. Currently, three approaches have been identified in research on filtering methods for hybrid inertial navigation system. The first is the use of the statistical approach techniques such as the extended Kalman filter (EKF) and filters based on sampling such as the unscented Kalman filter (UKF) [2], [3]. The second is to use of the probabilistic approach techniques such as the particle filters [4], and finally we have the methods based on Artificial Intelligence (AI), such as the Artificial Neural Networks (ANN) [5] or adaptive information systems for neural networks and fuzzy methods (ANFIS) [6]. The Kalman filtering provides a powerful tool for creating synergy between different sensors to hybridize. It can take advantage of the benefits and characteristics of different sensors to provide an hybrid inertial navigation system that performs better than unaided inertial navigation. It gives the optimal estimate by minimizing the mean squared error (MMSE). The extended Kalman filter (EKF) is probably the most common and popular approach to deal with non-linear system [7], [8], [9], [10], [11]. However, it has some disadvantages. A new evaluation method exists called 'Unscented Kalman Filter' (UKF) [12], [13], [14]. IT is nowadays considered a superior alternative to EKF in treating the errors of inertial navigation systems aided by one or more additional external sensors. In [1], [15], we have proposed a new type of UKF filter called Quaternion-Scaled Unscented Kalman Filter (Q-SUKF) which combines the scaled unscented Kalman filter (SUKF) and the use of quaternions as attitude representation parameters [1], [15]. This new hybridization digital filter makes it possible to obtain a recalibrated state of the inertial navigation from information provided by external sensors. The external measurements used are the position and / or velocity resulting from satellite navigation (GPS, Galileo). The Low-cost inertial navigation systems provide accurate and reliable navigation parameter solutions when integrated with the GPS aided sensor in the Q-SUKF filter. However, the performance of the integrated system will deteriorate considerably during periods of GPS failure. During the last decade, the Takagi-Sugeno (TS) fuzzy systems have been used to model successfully the non-linear systems and have proved a good representation of dynamic systems [16], [17], [18], [19]. In these approaches, the non linear behavior of a system is represented by a composition of If-Then rules, concatenating a set of local linear sub models. In this article, a Takagi-Sugeno fuzzy model is used to estimate the position and velocity measurements to the integrated INS/GPS system during the various GPS blackouts. The fuzzy model requires an offline learning phase extracted from a large number of input-output coupled data when GPS signals are available. This phase aims to identify the parameters of the fuzzy model used with the filter Q-SUKF. The Input-Output data cover different dynamics and types of movement (straight and rotation). During the learning stage, the inputs of the fuzzy model are position, velocity predicted by the Q-SUKF filter. The outputs of the fuzzy model are the positions and velocity measured by the GPS. At the end of the learning stage, the best estimates of the parameters of the fuzzy model are achieved. These measurements allow to maintain the update mode phase of Q-SUKF fiter. In this paper, we describe a new hybridization filter, called Adaptive Fuzzy Logic Quaternion Scaled Unscented Kalman Filter, denoted by (A) (FL) Q-SUKF. It is based on the application of the fuzzy model with the Q-SUKF filter. Next, an experimental part on the (A) (FL) Q-SUKF algorithm has been validated.

II. Validity of the Q-SUKF

The Q-SUKF is an iterative procedure of calculation intended to rectify the errors of the INS through outer estimations given by the GPS. For whatever length of time that the GPS estimations are accessible, the Q-SUKF works proficiently and gives a close valuation of the navigation parameters states. Nevertheless, during GPS blackouts, the general execution of INS/GPS framework is altogether corrupted by the quick collection of blunders which influence the inertial unit components estimation. To fix this trouble, a fuzzy logic model expressed by (A)(FL) is suggested. The fuzzy logic is a collection of scientific hypotheses which manages the portrayal and control the defective information (inaccurate, ambiguous or deficient). It doesn't try to remove them; oppositely, it will look to keep maintain them utmost. Then, its aim is to make flexible the representative structure and information's treatment, inspiring thus from the human mental procedures. The viable utilizations of fuzzy logic are various. Models include: automatism, robotics, expert systems, decision support, etc. In this paper, the fuzzy logic is characterized as a reasoning which uses the general role of "expert system" in handling the information. When GPS signals are accessible, this model is educed offline from a countless amount of a paired input-output data during a period called learning stage. The inputs of the fuzzy model are, position and velocity, calculated by the Q-SUKF. The outputs of the fuzzy model are the positions and velocities estimated by the GPS, as demonstrated in fig.1. At the final stage of the learning phase, the best assessment of the parameters of the Fuzzy model are accomplished. When a GPS blackout happens, the fuzzy model (A) (FL) reproduce instead an evaluated measures of position and velocity which should be the GPS estimations if they were accessible. Thus, the Q-SUKF keeps on utilizing the conditions of the estimation update phase, as demonstrated in fig.2. The Q-SUKF filer is refered to (A) (FL) Q-SUKF when it is utilized with the proposed fuzzy model.

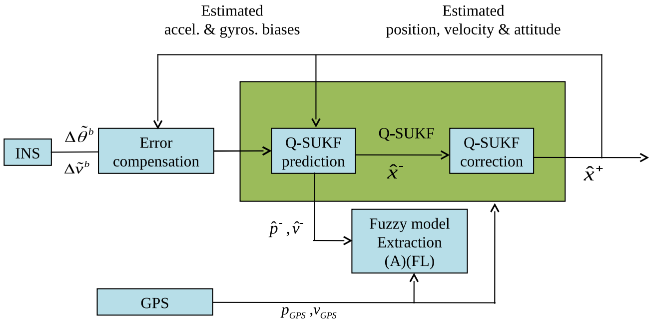


Fig. 1: Information accumulations for the extraction of fuzzy model (A) (FL) during the learning stage

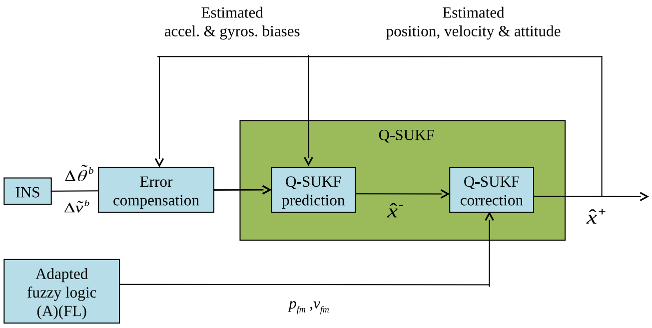


Fig. 2: Operating mechanism of the fuzzy model (A) (FL) with the Q-SUKF during GPS blackouts.

III. Projection of the Fuzzy Inference System

The projected fuzzy model uses a fuzzy inference system of Takagi-Sugeno type (FIS-TS) which has specific features since it symbolizes the nonlinear systems as an interjection between local linear models. The FIS-TS fuzzy model proposed can be written in a general structure as:

$$R_i : \text{If } \mathbf{x}_k \text{ is } A_i(\mathbf{x}_k) \text{Then } \tilde{\mathbf{y}}_i = a_i^T \mathbf{x}_k + b_i, i = 1, \ldots, r \quad (1)$$

Where $A_i(\mathbf{x}_k)$ is a gaussian membership function of the input variable vector at observation k, x_k, in the fuzzy set A_i. a_i and b_i are the components of the consequent parameters vector $\Theta_i = \left[a_i^T \, b_i\right]^T$ of the i-th fuzzy rule which describes the local linear model. R_i is the i-th fuzzy rule, r is the total number of rules and $\tilde{\mathbf{y}}_i$ is the estimated output of the local linear model. The proposition " $\mathbf{x}_k$ is $A_i(\mathbf{x}_k)$ " can be defined for the different components in form of conjunction:

$$\begin{aligned} R_i \text{ :If } \mathbf{x}_{1,k} \text{ is } A_{i,1}(\mathbf{x}_{1,k}) \,\&\, \ldots \,\&\, \mathbf{x}_{n,k} \text{ is } A_{i,n}(\mathbf{x}_{n,k}) \quad (2) \\ \text{Then } \tilde{\mathbf{y}}_i = a_i^T \mathbf{x}_k + b_i, \qquad i = 1, \ldots, r \end{aligned}$$

Where n is the dimension of the vector $\mathbf{x}_k$. The components of $\mathbf{x}_k$ are the three elements of the position $\mathbf{p}_k^n = [\Phi_k \; \lambda_k \; h_k]$, and the velocity $\mathbf{v}_k^n$. The choice of these parameters as input of the fuzzy model is convenient because they are the main factors to affect the prospected outputs of the fuzzy model (position and velocity vectors). In addition, these states are all determined in the navigation frame and easily obtained from the prediction phase of the Q-SUKF filter. Two classes have been assigned iteratively to each component of the entry vector where a Gaussian function has been implemented to represent the membership degree to each class. Based on the number of entries equal to 6 and the class number equal to 2, the number of rules is therefore equal to $2^6 = 64$. The estimated outputs of the fuzzy model are the position and velocity vectors, $\mathbf{p}^n$ and $\mathbf{v}^n$, expressed in the navigation frame and which can be calculated from the equation (1) of the FIS as follows:

$$\tilde{\mathbf{y}}_k = \frac{\sum_{i=1}^{r} \beta_i(\mathbf{x}_k)(a_i^T \mathbf{x}_k + b_i)}{\sum_{i=1}^{r} \beta_i(\mathbf{x}_k)} \tag{3}$$

where $\beta_i(\mathbf{x}_k)$ denotes the degree of fulfillment of the i-th rule:

$$\begin{aligned} \beta_i(\mathbf{x}_k) &= \prod_{j=1}^{n} A_{i,j}(\mathbf{x}_{j,k}) \\ &= \prod_{j=1}^{n} \exp\left(\frac{-1}{2} \frac{(\mathbf{x}_{j,k} - V_{i,j})^2}{\sigma_{i,j}^2} \right) \end{aligned} \tag{4}$$

$V_{i,j}$, $\sigma_{i,j}^2$ represent the center and the variance of the Gaussian fuzzy membership functions respectively. To identify the FIS of the fuzzy model, the antecedent parameters $V_{i,j}$, $\sigma_{i,j}^2$, and the consequent parameters, Θ_i, must be determined.

A. Determination of Antecedent Parameters

Abonyi in [20] has proposed the Fuzzy C-Means classification algorithm (FCM) to identify the antecedent parameters of Takagi-Sugeno fuzzy model. The FCM algorithm aims to divide the data points into homogeneous classes or groups. Thus, the points in the same class are as similar as possible while points in different classes are as dissimilar as possible. The FCM algorithm, which issued from the works of [21] and improved later by [22], constitutes an important reference among the different methods of fuzzy coalescence [23] based on the minimization of the objective function, of the form:

$$J_{FCM}(X; U, V) = \sum_{i=1}^{c} \sum_{k=1}^{N} \mu_{ik}.D_{ik}^2 \tag{5}$$

Where X is the data matrix, N is the number of observations, μ_{ik} is the Fuzzy partition of fuzzy subset i, $U = [\mu_{ik}]$ is the fuzzy partition matrix of dimension $c \times N$, $V = [V_1, V_2, \ldots, V_c]$ is a matrix of cluster centroid vectors which must be determined, with $V_i \in \mathcal{R}^n$, $1 \leq i \leq c$, in our case, the number of cluster c is equal to the number of rules r and D_{ik} is the euclidean distance between the observation $\mathbf{x}_k$ and the Cluster centroid vector V_i. In the equation (5), the dissimilarity measure expressed by the term $J_{FCM}(X; U, V)$ is the sum of the squares of the distances between each observation $\mathbf{x}_k$ and the corresponding center V_i. The effect of this distance is weighted by the degree of activation of the class, μ_{ik} corresponding to $\mathbf{x}_k$. The value of the objective function can be seen as a measure of the total variance of $\mathbf{x}_k$ with respect to the centers V_i. The minimization of $J_{FCM}(X; U, V)$ is a non-linear optimization problem that can be solved by different methods; the most used is the Fuzzy C-Means (FCM) algorithm [22]. It can be achieved by finding the cluster centroid vectors and the standard deviation of the membership Gaussian functions iteratively [24]:

$$V_i^l = \frac{\sum_{k=1}^{N} \mu_{ik}^{l-1} \mathbf{x}_k}{\sum_{k=1}^{N} \mu_{ik}^{l-1}}, \sigma_{i,j}^{2(l)} = \frac{\sum_{k=1}^{N} \mu_{ik}^{l-1} (\mathbf{x}_{j,k} - V_{i,j}^l)^2}{\sum_{k=1}^{N} \mu_{ik}^{l-1}} \tag{6}$$
$$1 \leq i \leq c,\ 1 \leq j \leq n$$

Where the membership degree μ_{ik} is calculated as follows:

$$\mu_{ik} = \frac{1}{\sum_{j=1}^{c} (D_{ik}/D_{jk})^{2/(m-1)}} \quad 1 \leq i \leq c,\ 1 \leq j \leq n \tag{7}$$

$$\text{avec}\, \mu_{ik} \in < 0, 1 > \text{ et } \sum_{i=1}^{c} \mu_{ik} = 1$$

$m \in [1, \infty)$ is the fuzziness parameter of the partition. The parameter m influences the form of the classes in the data space of the system. When m approaches 1, the shape of the membership function of each class is close to be Boolean function ($m \in \{0, 1\}$). The partition can range from a hard partition ($m = 1$) to a completely fuzzy partition ($m \to \infty$) when there is no changes significantly of the fuzzy partition matrix between two successive iterations. Although the choice of m depends on the data [25], usually this parameter is initialized to a value between 1.5 and 2.5. The iterative process stops when the partition becomes stable, *i.e.*, when it no longer changes significantly, between two successive iterations. This is generally expressed by checking the expression (8) where the left term indicate a matrix norm and the coefficient ϵ defines the convergence threshold:

$$\left\| U^{(l)} - U^{(l-1)} \right\| < \epsilon \tag{8}$$

The expression $U^{(l)}$ represents the fuzzy partition matrix of the l-iteration.

B. Determination of Consequent Parameters

After the learning of the antecedent parameters using equations (5) and (6), the equation (3) can be rewritten as follows:

$$\begin{aligned} \tilde{\mathbf{y}}_k &= \sum_{i=1}^{c} \bar{\beta}_i(\mathbf{x}_k).\mathbf{x}_k^e \Theta_i \\ &= \left[\bar{\beta}_1(\mathbf{x}_k).\mathbf{x}_k^e \ \ldots \ \bar{\beta}_c(\mathbf{x}_k).\mathbf{x}_k^e \right] \left[\Theta_1 \ \ldots \ \Theta_c \right]^T \\ &= \left[\bar{\beta}_1(\mathbf{x}_k).\mathbf{x}_k^e \ \ldots \ \bar{\beta}_c(\mathbf{x}_k).\mathbf{x}_k^e \right] \Theta \end{aligned} \tag{9}$$

Where $\mathbf{x}_k^e = \begin{bmatrix}\mathbf{x}_k \; 1\end{bmatrix}$ is the input vector to the fuzzy model augmented by the unit element. Θ is the M-dimensional consequent parameters vector where M=$c \times (n+1)$. $\bar{\beta}_i$ is defined by the following formula:

$$\bar{\beta}_i(\mathbf{x}_k) = \frac{\beta_i(\mathbf{x}_k)}{\sum_{i=1}^{c} \beta_i(\mathbf{x}_k)} \tag{10}$$

The linear equation (9) of the consequent parameters vector can be written as follows:

$$Z\Theta = Y \tag{11}$$

Given M the number of linear consequent parameters and N the number of learning input-output data, the dimensions of the matrices Z, Θ, Y are $N \times M$, $M \times 1$ and $N \times 1$ respectively. As N is always greater than M, the system of linear equations (9) is an underdetermined system, therefore generally there is no exact or unique solution which can be reached. To get there, the least squares estimation (LSE) method is exploited to minimize the squared distance between the vector Y and the linear combination $Z\Theta$. It is a classical problem that forms the basic in many applications such as linear regression, adaptive filtering and signal processing. The famous formula for solving systems of underdetermined equations uses the pseudo-inverse matrix of Θ as follows [26]:

$$\Theta^* = (Z^T Z)^{-1} Z^T Y \tag{12}$$

Where Z^T is the transpose of Z. $(Z^T Z)^{-1} Z^T$ is the pseudo-inverse matrix of Z if $Z^T Z$ is non-singular. Despite that the equation (12) is expressed in few words, it is very costly in terms of computation time when it comes to the calculation of the inverse of a matrix $Z^T Z$ and, in addition, it becomes poorly defined if this matrix is singular. To avoid the large computation time or the problem of singularity, sequential formulas are used to calculate the least square estimation of Θ. This sequential method is more efficient, especially when M is small. If the i^{th} row of the matrix Z in equation (11) is denoted by z_i^T and the i^{th} element of the vector Y is denoted by y_i^T, then Θ may be calculated iteratively using the following sequential formulas [26], [27]:

$$\begin{aligned} \mathbf{S}_{i+1} &= \mathbf{S}_i + \frac{\mathbf{S}_i z_{i+1} z_{i+1}^T \mathbf{S}_i}{1 + z_{i+1}^T \mathbf{S}_i z_{i+1}}; i = 0, \dots, N-1 \\ \Theta_{i+1} &= \Theta_i + \mathbf{S}_{i+1} z_{i+1} (y_{i+1} - z_{i+1}^T \Theta_i) \end{aligned} \tag{13}$$

Where S_i is often called the covariance matrix and the estimated least squares Θ^* is equal to Θ_N. The initial condition of the equation (13) is $\Theta_0 = 0$ and $\mathbf{S}_0$=ηI, where η is an arbitrary positive number which is large and I is the $M \times M$ dimensional identity matrix.

IV. Simulations and Results

A. Simulations

To test the effectiveness of the (A) (FL)Q-SUKF filter and its impact on the accuracy of the navigation parameters calculation (specially the position and velocity), a simulated data of inertial measurement unit, GPS and magnetometer were used. The experiment was conducted using a car driving (reference trajectory) for 30 mimutes. This reference trajectory was generated by the function progencar of INS toolbox version 3.0 created by GPSoft. This trajectory covers different dynamic (static and kinematic) and scenarios of motion (rotation and rectilinear). The data of the inertial navigation system (angle and velocity increments) were simulated from the parameters of the profile of the automobile using certain functions of the INS toolbox. These angle and velocity increments have been corrupted with various sources of errors such as biases, scale factors and noises in order to generate outputs close to real data of an inertial navigation system. The characteristics of the errors models of the inertial sensors used in the experiment are presented in TABLE I, where the two parameters T and σ describe the first-order markov process $\mathbf{x}$ represented by:

$$\dot{\mathbf{x}} = -\frac{1}{T}\mathbf{x} + \omega \tag{14}$$

Where T is the correlation time of the process $\mathbf{x}$ and ω is a wiener process with variance $2\sigma^2/T$.

TABLE I: Characteristics of error models of the inertial sensors used in the experience

Parameter	Model	Accelerometer	Gyroscope
Noise	Random Walk	$0.6\ \mathrm{m/s/\sqrt{h}}$	$3.5\ \mathrm{deg/\sqrt{h}}$
Bias	1^{st} Order Gauss Markov	$\sigma_a = 0.1\ \mathrm{m/s^2}$ T=1hour	$\sigma_g = 100\ \mathrm{deg/h}$ T=1hour
Scale Factor	1^{st} Order Gauss Markov	$\sigma_{sa} = 1000$ PPM T= 4 hours	$\sigma_{sg} = 1000$ PPM T= 4hours

The GPS data (position and velocity) were generated by adding to the positions and velocities data of the reference trajectory a gaussian white noise. The initial standard deviation of the position expressed in Cartesian coordinates in the navigation frame is equal to $2cm$ in the horizontal plane and is equal to $4cm$ in the vertical plane. The initial standard deviation of the velocity expressed in the navigation frame is equal to $0.25m/s$ for the horizontal components and is equal to $0.4m/s$ for the vertical component. Two simulations of GPS outages with a duration of $30s$ have been considered along the path as shown in Fig. 3.

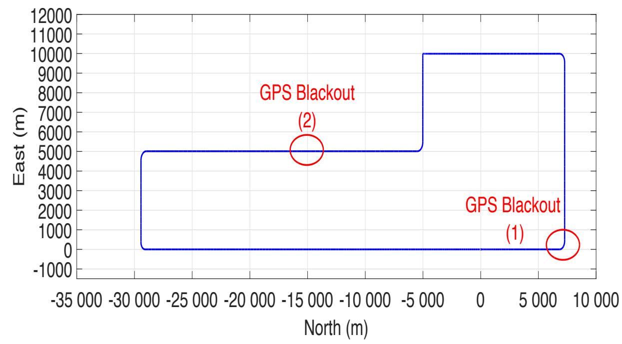
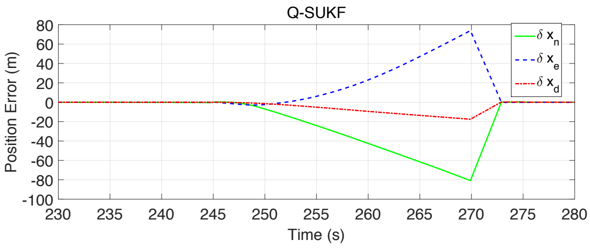


Fig. 3: Simulated trajectory with GPS outages indicated.

B. Results

To test the implementation of the proposed methodology, an initial attitude error of 60 degrees is given on each axis. The diagonal terms of the initial covariance matrix represent variances or mean squared errors. The off-diagonal terms are set to be zeros. The parameters used in the Q-SUKF are given by scaling parameters $\alpha = 0.05$ and $\beta = 2$, and by weight of 0^{th} point $\omega_0 = 0.5$. The Fig. 4, Fig. 5, Fig. 6 and Fig. 7 demonstrate the position and velocity errors during the two periods of GPS blackouts 1, 2 respectively, before and after the use of the proposed technique of the fuzzy model to Q-SUKF.



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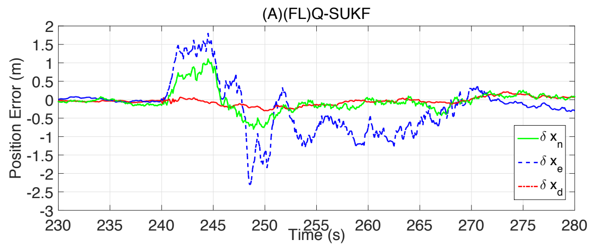
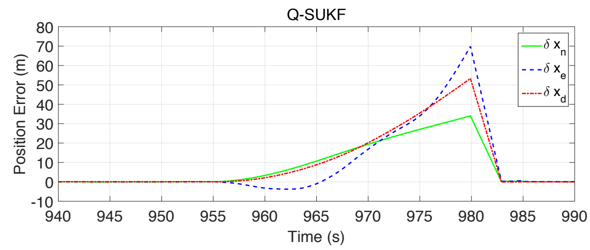


(b)

Fig. 4: Position error estimated during GPS blackout1



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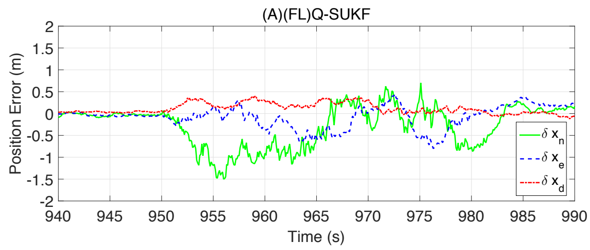
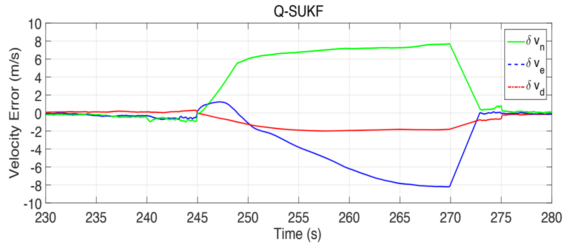


(b)

Fig. 5: Position error estimated during GPS blackout2



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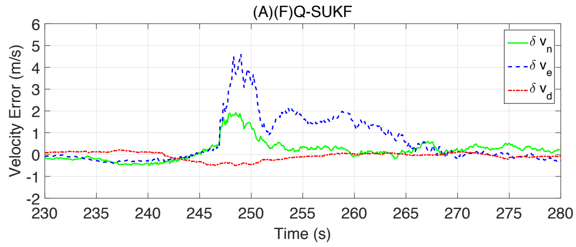
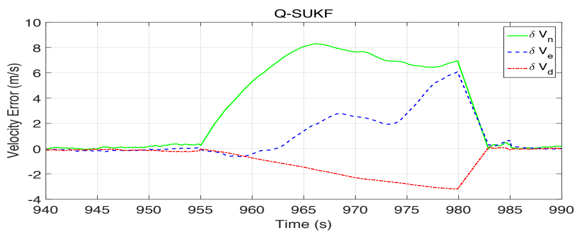


(b)

Fig. 6: Velocity error estimated during GPS blackout1



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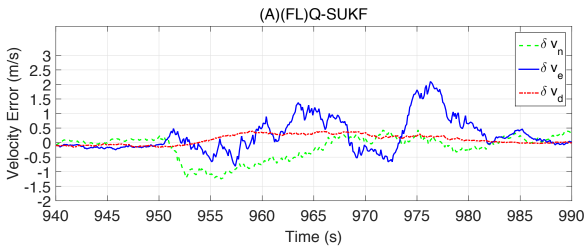

(b)

Fig. 7: Velocity error estimated during GPS blackout2

We notice in these figures that the maximum errors of the position and velocity components have been reduced considerably after the application of the proposed technique of the fuzzy model to the Q-SUKF filter during the two periods of GPS blackouts. The TABLE II summarizes the percentage of the reduction of these errors.

The fuzzy model applied to the Q-SUKF conduct to an important enhancement of 75.32 % and 43.90 % at least in reducing the errors estimation of position and velocity respectively. Finally, in spite of the fact that these first outcomes cannot be extrapolated, they might anticipate to give the green light for future research presenting GPS blackouts in various situations of real scenarios for the purpose of a generalization.

TABLE II: Reduction of the maximum error of the position and the velocity of Q-SUKF filter after using the fuzzy model.

Maximum Error	**Outage GPS1**	**Outage GPS2**
$\delta\mathbf{x}_n(m)$ & $\delta\mathbf{v}_n(m/s)$ of Q-SUKF	81,20 & 7.70	34.20 & 8.30
$\delta\mathbf{x}_n(m)$ & $\delta\mathbf{v}_n(m/s)$ of (A)(FL)Q-SUKF	1.10 & 1.93	1.50 & 1.25
Reduction % of $\delta\mathbf{x}_n$ & $\delta\mathbf{v}_n$	98.64 &75.32	75.32 &84.93
$\delta\mathbf{x}_e(m)$ & $\delta\mathbf{v}_e(m/s)$ of Q-SUKF	74.40 & 8.20	70,80 & 6.10
$\delta\mathbf{x}_e(m)$ & $\delta\mathbf{v}_e(m/s)$ of (A)(FL)Q-SUKF	2.30 & 4.60	0.80 & 2.10
Reduction % of $\delta\mathbf{x}_e$ & $\delta\mathbf{v}_e$	96.90 &43.90	98.87 &65,57
$\delta\mathbf{x}_d(m)$ & $\delta\mathbf{v}_d(m/s)$ of Q-SUKF	17.60 & 2.00	5.70 & 3.20
$\delta\mathbf{x}_d(m)$ & $\delta\mathbf{v}_d(m/s)$ of (A)(FL)Q-SUKF	0.30 & 0.50	0.40& 0.40
Reduction % of $\delta\mathbf{x}_d$ & $\delta\mathbf{v}_d$	98.29 &74.75	99.25 &87.50

V. CONCLUSION

This paper displays a novel hybridization filter of the inertial navigation system with GPS. This new filter, denoted (A) (FL) Q-SUKF, depends on the use of Q-SUKF with a fuzzy approach. For whatever length of time that the GPS estimations are accessible, the fusion INS/GPS gives great outcomes. At the point when the estimations of GPS are not reliable or inaccessible, the fuzzy model permits the Q-SUKF to keep on rectifying the errors of the estimation of the parameters of navigation (position and velocilty) of the vehicule by giving simulated GPS position and velocity measures. The consequences of the experimental validation have demonstrated the adequacy and the critical effect of the fuzzy strategy utilized with the Q-SUKF in the decrease of errors estimation of the position and velocity in the tried situations. The (A) (FL) Q-SUKF gives us progressively precise computation of the rotation matrix which is integrated in the computing of the position and velocity, and therefore leads to a better estimation of the parameters of navigation.The results obtained on synthetic data have shown the contribution of the fuzzy logic and have approved the methodology proposed.

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