A Variational Framework for Joint Image Registration, Denoising and Edge Detection

Jingfeng Han[1], Benjamin Berkels[2], Martin Rumpf[2], Joachim Hornegger[1], Marc Droske[3], Michael Fried[1], Jasmin Scorzin[2] and Carlo Schaller[2]

[1]Universität Erlangen-Nürnberg, [2]Universität Bonn, [3] University of California

Abstract. In this paper we propose a new symmetrical framework that solves image denoising, edge detection and non–rigid image registration simultaneously. This framework is based on the Ambrosio–Tortorelli approximation of the Mumford–Shah model. The optimization of a global functional leads to decomposing the image into a piecewise–smooth representative, which is the denoised intensity function, and a phase field, which is the approximation of the edge-set. At the same time, the method seeks to register two images based on the segmentation results. The key idea is that the edge set of one image should be transformed to match the edge set of the other. The symmetric non–rigid transformations are estimated simultaneously in two directions. One consistency functional is designed to constrain each transformation to be the inverse of the other. The optimization process is guided by a generalized gradient flow to guarantee smooth relaxation. A multi–scale implementation scheme is applied to ensure the efficiency of the algorithm. We have performed preliminary medical evaluation on T1 and T2 MRI data, where the experiments show encouraging results.

1 Introduction

Image registration, image denoising and edge detection are three important and still challenging image processing problems in the field of medical image analysis. Traditionally, solutions are developed for each of these three problems mutually independent. However, in the various applications, the solutions of these problems are depend on each other. Indeed, tackling each task would benefit significantly from prior knowledge of the solution of the other tasks. Here, we treat these different image processing problems in an uniform mathematically sound approach.

There already have been some attempts in the literature to develop methods aligning the images and detecting the features simultaneously [1, 2, 3, 4]. Due to our knowledge, most of the existing approaches are restricted to lower dimensional parametric transformations for image registration. Recently, in [5, 6] a novel approach for non–rigid registration by edge alignment has been presented. The key idea of this work is to modify the Ambrosio–Tortorelli approximation of the Mumford–Shah model, which is traditionally used for image segmentation, so that the new functional can also estimate the spatial transformation between images, but in contrast to the method proposed by Droske et al. our method is fully "symmetric" with respect to the treatment of the edge sets in both images and the transformations in both directions.

2 Method

Assume two gray images R and T are given, whose intensity values are described by the function u_R^0 and u_T^0, respectively. The goal of the joint framework is to find piecewise smooth representatives u_R and u_T (denoising), phase field edge functions v_R and v_T (edge detection) and symmetric non–rigid spatial transformations ϕ and ψ such that $u_R \circ \psi$ matches u_T and $u_T \circ \phi$ matches u_R (registration). For simplification of presentation, we denote all the unknowns with $\Phi = [u_R, u_T, v_R, v_T, \phi, \psi]$. The associated functional is defined as

$$E_G[\Phi] = E_{AT}^{u_R^0}[u_R, v_R] + E_{AT}^{u_T^0}[u_T, v_T] + E_{REG}[\Phi] \rightarrow \min, \quad (1)$$

In what follows, we define and discuss the variational formulation.

2.1 Denoising and Edge Detection

The $E_{\mathrm{AT}}^{u^0}[u, v]$ denotes the Ambrosio–Tortorelli (AT) approximation functional proposed in [7, 8]. This functional is originally designed to approximate the Mumford–Shah model [9] for image segmentation. The functional is defined as

$$E_{\mathrm{AT}}^{u^0,\epsilon}[u, v] = \int_\Omega \underbrace{\frac{\alpha}{2}(u - u^0)^2}_{E_1} + \underbrace{\frac{\beta}{2}v^2\|\nabla u\|^2}_{E_2} + \frac{\nu}{2}\Big(\underbrace{\epsilon\|\nabla v\|^2}_{E_3} + \underbrace{\frac{1}{4\epsilon}(v-1)^2}_{E_4}\Big)dx,$$

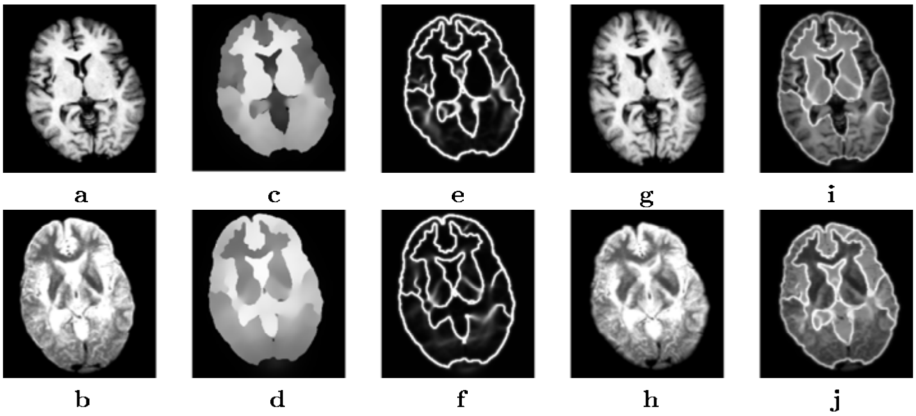
with parameters $\alpha, \beta, \nu \geq 0$. In the Ambrosio–Tortorelli approximation, the edge set is depicted by a phase field function v with $v(x) \approx 0$ if x is an edge point and $v(x) \approx 1$ otherwise. The term E_1 favors u to be as similar to u^0 as possible. The term E_2 allows u to be singular (large $\|\nabla u\|^2$) where $v \approx 0$ and favors u to be smooth (small $\|\nabla u\|^2$) where $v \approx 1$. The term E_3 constrains v to be smooth. The last term E_4 prevents the degeneration of v, i.e. without E_4 the functional would be minimized by $v \equiv 0, u \equiv u_0$. For the details of the Ambrosio–Tortorelli approximation, we refer to [7].

2.2 Edge Alignment

The main goal of the registration functional E_{REG} is to find the transformations that match the edge sets of image R and image T to each other. In order to explicitly enforce the bijectivity and invertibility of spatial mapping, we estimate the two transformations in two directions simultaneously: $\phi : \Omega \rightarrow \Omega$ is the transformation from image T to image R and $\psi : \Omega \rightarrow \Omega$ is the one from R to T. The functional E_{REG} is a linear combination of an external functional E_{ext}, an internal functional E_{int} and a consistent functional E_{con}:

$$E_{\mathrm{REG}}[\Phi] = \mu E_{\mathrm{ext}}[\Phi] + \lambda E_{\mathrm{int}}[\phi, \psi] + \kappa E_{\mathrm{con}}[\phi, \psi], \quad (2)$$

Fig. 1. Results of registration of T1/T2 slices with parameters: $\alpha = 2550, \beta = 1, \nu = 1, \mu = 0.1, \lambda = 20, \kappa = 1, \epsilon = 0.5h$. (**a**, **b**): The original images u^0_{T1} and u^0_{T2}. (**c**, **d**): Piecewise smooth functions u_{T1} and u_{T2}. (**e**, **f**): Phase field functions v_{T1} and u_{T2}. (**g**, **h**): The registered T1 and T2 slices. (**i**): Blending of transformed T1 slice and phase field function of T2 slice. (**j**): Blending of transformed T2 slice and phase field function of T1 slice.



where μ, λ and κ are just scaling parameters. The three functionals $E_{\text{ext}}, E_{\text{int}}, E_{\text{con}}$ are defined as follows:

$$E_{\text{ext}}[\Phi] = \int_\Omega \frac{1}{2}(v_T \circ \phi)^2 \|\nabla u_R\|^2 + \frac{1}{2}(v_R \circ \psi)^2 \|\nabla u_T\|^2 \, dx \,, \tag{3}$$

$$E_{\text{int}}[\phi, \psi] = \int_\Omega \frac{1}{2} \|D\phi - \mathbb{1}\|^2 + \frac{1}{2} \|D\psi - \mathbb{1}\|^2 \, dx \,, \tag{4}$$

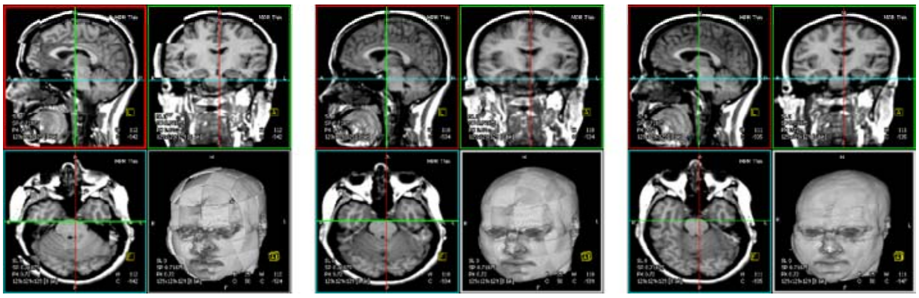
$$E_{\text{con}}[\phi, \psi] = \int_\Omega \frac{1}{2}\|\phi \circ \psi(x) - x\|^2 + \frac{1}{2}\|\psi \circ \phi(x) - x\|^2 dx \,. \tag{5}$$

Here, $\mathbb{1}$ is the identity matrix. The external functional E_{ext} favors transformations that align zero–regions of the phase field of one image to regions of high gradient in the other image. The internal functional E_{int} imposes a common smoothness prior on the transformations. The consistency functional E_{con} constrains the transformations to be inverse to each other, since it is minimized when $\phi = \psi^{-1}$ and $\psi = \phi^{-1}$.

2.3 Variational Formulation

The definition of the global functional $E_G[\Phi]$ is mathematically symmetrical with respect to the two groups of unknown $[u_R, v_R, \phi]$ and $[u_T, v_T, \psi]$. Thus, we restrict here to variations with respect to $[u_R, v_R, \phi]$. The other formulas can be deduced in a complementary way.

Fig. 2. Results of 3D registration. We denote the original MRI volume as R and the artificially deformed volume as T. After symmetric registration, the resampled volume are denoted as $R^{'}$ and $T^{'}$ respectively. (**a**) The check board volume of R and T. (**b**) The check board volume of R and $T^{'}$.(**c**) The check board volume of T and $R^{'}$. The parameter setting: $\alpha = 2550, \beta = 1, \nu = 1, \mu = 0.1, \lambda = 20, \kappa = 1, \epsilon = 0.5h$.



a **b** **c**

For testfunctions $\vartheta \in C_0^\infty(\Omega), \zeta \in C_0^\infty(\Omega, \mathbb{R}^d)$, we obtain

$$\begin{aligned}
\langle \partial_{u_R} E_G, \vartheta \rangle &= \int_\Omega \alpha(u_R - u_R^0)\vartheta \ + \beta v_R^2 \nabla u_R \cdot \nabla \vartheta \ + \mu(v_T \circ \phi)^2 \nabla u_R \cdot \nabla \vartheta dx \ , \\
\langle \partial_{v_R} E_G, \vartheta \rangle &= \int_\Omega \beta \left\| \nabla u_R \right\|^2 v_R \vartheta \ + \frac{\nu}{4\epsilon}(v_R - 1)\vartheta dx \\
&\quad + \int_\Omega \nu\epsilon \nabla v_R \cdot \nabla \vartheta \ + \mu \|\nabla u_T \circ \psi^{-1}\|^2 v_R \vartheta \left|\det D\psi\right|^{-1} dx \ , \\
\langle \partial_\phi E_{\mathrm{G}}, \zeta \rangle &= \int_\Omega \mu \left\| \nabla u_R \right\|^2 (v_T \circ \phi) \nabla (v_T \circ \phi) \cdot \zeta \ + \lambda D\phi : D\zeta \\
&\quad + \kappa([\phi \circ \psi](x) - x) \cdot [\zeta \circ \psi](x) \ + ([\psi \circ \phi](x) - x) D\psi(h(x)) \cdot \zeta(x) dx \ .
\end{aligned}$$

where $A : B = \sum_{ij} A_{ij} B_{ij}$.

At first, a finite element approximation in space is applied [10]. Then, we minimize the corresponding discrete functional by finding a zero crossing of the variation. Because of the high dimensionality of the minimization problem (six unknown functions, two of them vector valued), we employ an EM type algorithm, i.e. we iteratively solve for zero crossings of the variations given before. Since the variations with respect to the images and the phase fields are linear in the given variable, we can solve these equations directly with a CG method. The nonlinear equations for the transformation are solved with a time discrete, regularized gradient flow, which is closely related to iterative Tikhonov regularization, see [11].

3 Results

The first experiment was performed on a pair of T1/T2 MRI slices (See Fig.1**a**,1**b**), which have the same resolution (257×257) and come from the same patient. The experiment results in Fig.1 show that the proposed method successfully removes the noise (Fig.1**c**,1**d**) and detects the edge features (Fig.1**e**,1**f**) of

T1/T2 slices. Moreover, the method computes the transformations such that the two transformed slices (Fig.**1g**,**1h**) optimally align to the original images according to the edge features, see (Fig.**1i**,**1j**). The second experiment was designed to demonstrate the effect of the proposed method in 3D. We deformed one MRI volume ($129 \times 129 \times 129$) with Gaussian radial basis function (GRBF) and seek to recover the artificially introduced transformation via symmetric registration method. See the registration results in Fig.2.

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References

1. Zöllei L, Yezzi A, Kapur T. A Variational Framework for Joint Segmentation and Registration. In: MMBIA'01: Proceedings of the IEEE Workshop on Mathematical Methods in Biomedical Image Analysis. Washington, DC, USA: IEEE Computer Society; 2001. p. 44–51.
2. Chen Y, Thiruvenkadam S, Huang F, Gopinath KS, Brigg RW. Simultaneous segmentation and registration for functional MR images. In: Proceedings. 16th International Conference on Pattern Recognition. vol. 1; 2002. p. 747–750.
3. Young Y, Levy D. Registration-based morphing of active contours for segmentation of CT scans. Mathematical Biosciences and Engineering 2005;2(1):79–96.
4. Pohl KM, Fisher J, Levitt JJ, Shenton ME, Kikinis R, Grimson WEL, et al. A Unifying Approach to Registration, Segmentation, and Intensity Correction. In: MICCAI; 2005. p. 310–318.
5. Droske M, Ring W. A Mumford-Shah Level-Set Approach for Geometric Image Registration. SIAM Appl Math 2005; to appear.
6. Droske M, Ring W, Rumpf M. Mumford-Shah Based Registration. Computing and Visualization in Science manuscript 2005; submitted.
7. Ambrosio L, Tortorelli VM. On the approximation of free discontinuity problems. Boll Un Mat Ital B 1992;6(7):105–123.
8. Ambrosio L, Tortorelli VM. Approximation of functionals depending on jumps by elliptic functionals via Γ-convergence. Comm Pure Appl Math 1990;43:999–1036.
9. Mumford D, Shah J. Boundary detection by minimizing functional. In: Proceedings. IEEE conference on Computer Vision and Pattern Recognition. San Francisco, USA; 1985.
10. Bourdin B, Chambolle A. Implementation of an adaptive Finite-Element approximation of the Mumford-Shah functional. Numer Math 2000;85(4):609–646.
11. Clarenz U, Henn S, Rumpf K M Witsch. Relations between optimization and gradient flow methods with applications to image registration. In: Proceedings of the 18th GAMM Seminar Leipzig on Multigrid and Related Methods for Optimisation Problems; 2002. p. 11–30.