Thermocapillary Convection In A Locally Heated Liquid – Gas System

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Abstract

Numerical simulation of the dynamics of a locally heated two-layer system with the deformable interface is carried out. The system is subjected to local thermal exposure due to action of the heaters of finite size arranged on outer boundaries of a working section. Influence of the lower heater size on a structure of arising convective regimes and character of interface deformations is analyzed. Feasibility of the liquid layer rupture in the heating zone are investigated for given liquid layer thickness and intensity of thermal load.

1 Introduction

Processes of heat-and-mass transfer in fluidic micro- and mini-systems exposed to local thermal load are of both fundamental and practical interests due to possible technical applications. The scientific concern is occasioned by richness of liquid behavior modes under non-isothermal processes. The utilitarian interest relates to great development of multifarious fluidic technologies in space branch, chemical and pharmaceutical industry, material engineering, thermophysics, microelectronics. One of the examples of wide application of non-isothermal fluid media in conditions of thermal impact is the thermostabilization systems with liquid cooling. Inter alia, they are used for cooling of components of on-board equipment for space vehicles of different purposes. In developing and testing the systems it becomes necessary to obtain preliminary characteristics, to determine influence of various factors on the system dynamics, to work out the control ways of arising regimes of liquid coolant flows, as well as to specify ways of suppression of undesirable effects caused by the external temperature action. In the present work the numerical modeling is performed to investigate the influence of a heat element size on the structure of the arising flows in working fluids and evolution of the liquid – gas phase boundary. Since a rupture of liquid coolant layer leads to the critical drop in effective characteristics of heat removal, then the main question is: will gap appear in the liquid layer with certain thickness under given size and temperature of heater?

2 Basic Assumptions And Problem Statement

A two-layer liquid – gas system filling the plane horizontal cuvette with solid walls is considered (Figure 1). The media contact along the phase boundary Γ_t which is the deformable thermocapillary interface defined by the equation $y = f(t, x)$. The tangential forces act lengthwise of the interface. It is assumed that the surface tension of liquid σ linearly depends on the temperature: $\sigma = \sigma_0 - \sigma_T(\theta_l - \theta_0)$. Here, σ_0, θ_0 are the reference values of

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the surface tension and temperature for the liquid, respectively, σ_T is the temperature coefficient, σ_0, $\sigma_T > 0$. At the initial instant $t = 0$ both fluids are at rest, and surface Γ_t has zero curvature. The actuation of heaters of finite size placed on the lower and upper walls of the cuvette results in onset of convective motion in the layers and the interface deformations.

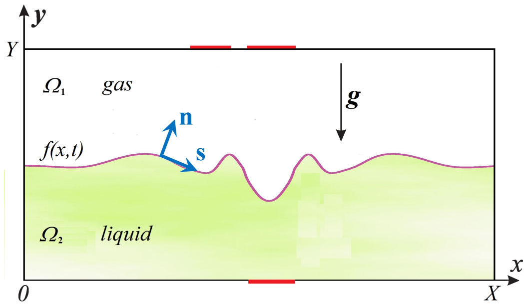

Figure 1: Configuration of the bilayered system

The flow regime pattern and character of the interface behavior depend on several factors; they are (i) thickness of the liquid layer, (ii) thermophysical properties of working media, (iii) intensity and features of external actions (gravitational, thermal etc). Here, the influence of the heater size on the system evolution is investigated. It is supposed that the heaters have length 0.5, 1 or 2 cm and can functionate in the stationary or switchable mode. In the first case the heater temperature remains constant, in another case it can be discontinuously increased or decreased.

2.1 Governing Equations

It is assumed that the bilayered system is in the field of mass forces with the gravity acceleration vector $\mathbf{g} = (0, -g)$. The Oberbeck−Boussinesq approximation of the Navier−Stokes equations is used for description of motion induced by applied external thermal exposure in j-th medium. Upon that, $j = 1$ corresponds to the upper (gas) layer, $j = 2$ refers to lower (liquid) layer (see Figure 1). In dimensionless form the governing equations are the following:

$$\begin{gathered}\partial_t\omega_j + \partial_x\big(\omega_j\partial_y\psi_j\big) - \partial_y\big(\omega_j\partial_x\psi_j\big) = \mathrm{Re}_j^{-1}\Delta\omega_j + \mathrm{Gr}_j\mathrm{Re}_j^{-2}\partial_x T_j,\\ \Delta\psi_j + \omega_j = 0, \qquad \partial_t T_j + \partial_x\big(T_j\partial_y\psi_j\big) - \partial_y\big(T_j\partial_x\psi_j\big) = \mathrm{Pr_j}^{-1}\mathrm{Re_j}^{-1}\Delta T_j.\end{gathered} \tag{1}$$

Functions ψ_j, ω_j, T_j (stream function, vorticity and temperature in j-th layer, respectively) are the required ones. Similarity criteria $\mathrm{Gr}_j = \beta_j T_* g h_1^3/\nu_j^2$ (Grashof number), $\mathrm{Re_j} = u_* h_1/\nu_j$ (Reynolds number), $\mathrm{Pr_j} = \nu_j/\chi_j$ (Prandtl number) are introduced by usual way for each medium. They are assumed to be given and defined by the thermophysical parameters (coefficients of thermal expansion β_j, kinematic viscosity ν_j, thermal diffusivity χ_j) as well as the characteristic values (temperature drop T_*, velocity of viscous stresses relaxation u_* in the gas phase, height of the upper layer h_1 in the initial (unperturbed) state). Symbols ∂_t, ∂_x, ∂_y denote the partial-differential operators with respect to corresponding independent variables.

2.2 Boundary Conditions

Boundary conditions on the common interface Γ_t are the result of relations on a strong discontinuity, conservation laws for mass, momentum and energy, and some additional assumptions [And12]. To state the conditions in "stream function−vorticity" variables, the unit vectors of tangent and normal lines to Γ_t are introduced:

$$\mathbf{s} = \left(\frac{1}{\sqrt{1+\partial_x^2 f}}, \frac{\partial_x f}{\sqrt{1+\partial_x^2 f}}\right), \qquad \mathbf{n} = \left(-\frac{\partial_x f}{\sqrt{1+\partial_x^2 f}}, \frac{1}{\sqrt{1+\partial_x^2 f}}\right).$$

Here, it is taken into account that $\mathbf{n}$ is the vector of external normal to Γ_t for lower fluid (Figure 1). Further, the normal and tangent components of the velocity are determined for points, which lie on the interface: $v_n = -\partial_s\psi$, $v_s = \partial_n\psi$; here, symbols ∂_s, ∂_n denote the derivatives in the tangential and normal direction. Thus, velocity of any point on the interface is presented in the form $\mathbf{v} = v_n\mathbf{n} + v_s\mathbf{s}$. Besides, the standard continuity conditions for the total velocity vector, temperature and heat fluxes are assumed to be fulfilled on the phase boundary. The continuity condition for the tangential velocities together with the volume conservation requirement for each of the media ensure the fulfilment of the following relations for the stream functions on Γ_t:

$$\psi_1 = \psi_2, \qquad \partial_n \psi_2 - \partial_n \psi_1 = 0.$$

The kinematic condition can be written in the form:

$$\partial_t f + \sqrt{1 + \partial_x^2 f}\, \partial_s \psi_2 = 0.$$

Solving the equation allows one to determine a profile of the phase boundary at every instant.

The analogues of the tangential and normal components of the dynamic condition are the matching conditions for the vorticity functions on Γ_t:

$$\omega_2 - \bar{\rho}\bar{\nu}\omega_1 = F_1(t, x), \qquad \partial_n \omega_2 - \bar{\rho}\bar{\nu}\partial_n \omega_1 = F_2(t, x). \tag{2}$$

Here, function F_1 has regard to the thermocapillary force action, and F_2 takes into account a contribution of pressure-jump and effects of the problem nonstationarity. Detailed derivation of relations (2) and exact expressions for F_1 and F_2 functions are presented in [Bek19].

The outer boundaries of the cuvette ($x = 0$, $x = X$, $y = 0$, $y = Y$) are the solid impermeable walls (Figure 1). Conditions for the stream functions ψ_j on these boundaries correspond to the no-slip conditions for viscous fluid: $\psi_j = 0$, $\partial_n \psi_j = 0$. Relations for the temperature functions consider the presence of thermal elements on the substrate ($y = 0$) and upper wall ($y = Y$):

$$T_1\,|_{y=Y,\, x\notin Q_p^{up}} = 0, \quad T_1\,|_{y=Y,\, x\in Q_p^{up}} = q_p^{up}(t), \qquad T_2\,|_{y=0,\, x\notin Q_l^{s}} = 0, \quad T_2\,|_{y=0,\, x\in Q_l^{s}} = q_l^{s}(t),$$

where Q_l^s is the area of the substrate occupied by l-th heater with the temperature q_l^s, Q_p^{up} is the part of the upper boundary on that p-th thermal element with the temperature q_p^{up} is arranged.

The numerical algorithm based on the Ovcharova method [Ovch14] is used to solve the stated adjoint problem. The outline of the algorithm and some details of the numerical method are given in [Bek20].

3 Results Of Numerical Simulation And Discussion

A series of calculations was performed for the ethanol−nitrogen system filling the vessel with length $X = 0.2$ m and height $Y = 0.01$ m. The condition of terrestrial gravity with $g = 9.81$ m/s^2 were considered. In the unperturbed state the thickness of each layer was taken to be equal to $5 \cdot 10^{-3}$ m. The dynamics of system were numerically investigated under different operating modes of the heaters of various size. The case was considered when the thermal elements were simultaneously arranged on the bottom and upper walls of the cuvette. Upon that, one heater with the length of 0.01 or $5 \cdot 10^{-3}$ m was placed on the lower boundary, and it was operated in commutated mode. Two heaters with the length of 0.02 and 0.015 m were arranged on the upper wall; along with this, one of them was located above the lower thermal element. The top heaters had the same constant temperature. Considering that the primary influence on the amplitude of the interface deformations was governed by action of the lower heater [Bek19], the parameters of this thermal element, namely, temperature and size were changed.

The specific feature of the switchable (commutated) operating mode is the abrupt change in heater temperature (both increase and drop) to some limit values. Similar regime simulates periodical heating (under actuation) or cooling (after switching off) of a heater on the working area of real fluidic path. For practical systems of thermal control the working range of the temperature is $0 - 25\,^{\mathrm{O}}$C. The modeling was carried out precisely for such temperature drops of lower heater; the temperature q^s was varied by $2.5\,^{\mathrm{O}}$C under each switching. The character of changes for thermal and hydrodynamical fields and the behavior of the liquid−gas interface were analyzed. The comparison of characteristics with those investigated in [Bek19] for case of heater with larger size was performed.

When the lower heater of small size is switched on the solitary thermal plume (upper right picture in Figure 2) evolves in zone of thermal impact due to the convective mechanism and dual-vortex flows occurs in each layer (lower right picture in Figure 2). Along with this, the thermocapillary deflection caused by change in the surface tension of Γ_t appears. The formation of similar flow pattern with the typical temperature plume in a locally heated liquid with a free surface is experimentally confirmed [Kon16]. In contrast to the case when the lower thermal element has larger size (the thermal and hydrodynamical structures of flow for this case are presented on the left pictures in Figure 2), in the system with the small heater the two-vortex regime in the liquid layer is directly set in above zone of heating. In such a case, the transient process from the two-vortex flow to the quadruple-vortex one and back is absent (the description of mechanisms generating such alteration is given in [Bek19]). The differences are explained by the structure of the thermal field. In the case of large lower heater the double-type thermal plume appears in the domain of thermal exposure (upper left picture in Figure 2), therefore, four-vortex flow in the liquid layer is able to develop. Such a dependence of the plume form on the heater size was established in experiments [Kon16].

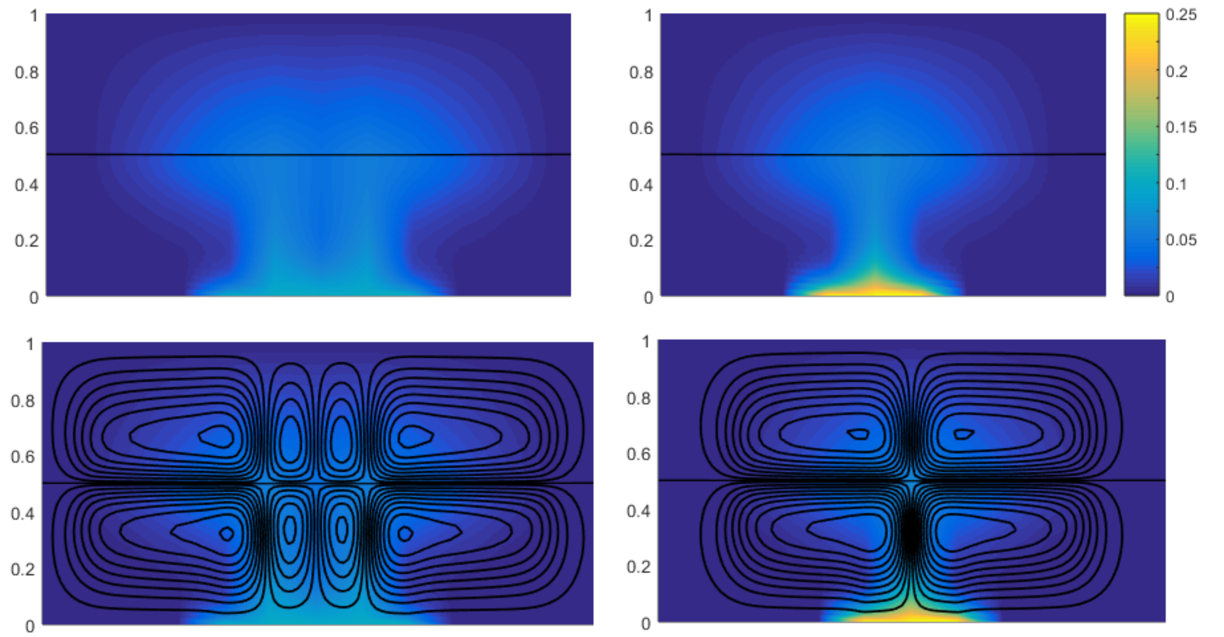

Figure 2: Comparison of the thermal (from above) and topological (from below) pattern of the flows in the heating zone in the systems with large (from the left) and small (from the right) heater

At each successive jump-like increasing of the heater temperature the interface undergoes oscillations accompanied by the changes in amplitude and shape of the flexure. It should be noted that the system responds on the variations of thermal exposure intensity with certain time lag. The delay time corresponds to period in which heat from the thermal element on the substrate comes to the liquid – gas surface. The convection speed depends on both the rate of thermal attack and the liquid layer thickness, and also on the intensity of the gravity field etc. [Bek20]. With time the boundary layer is formed near the interface. It deforms the convective cells and leads to the formation of the regime with drifting from the heating zone vortices (Figure 3).

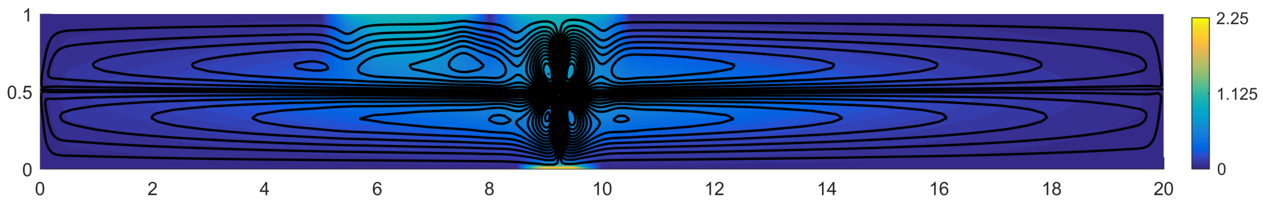

Figure 3: Field of velocity and temperature in the ethanol – nitrogen system at the lower heater temperature $q^s = 22.5\,^{\mathrm{O}}\mathrm{C}$ (155 seconds after switching on the heater)

Action of the upper thermal elements has a little effect on the system dynamics. Heat from the sources arranged on the top walls transfers to the interface due to the thermal conductivity and scarcely affects both the interface deformation and the flow topology. In the upper layer the vortices are only slightly deformed (Figure 3).

With increase of the heater temperature the amplitude of interface deformation in the zone of thermal attack grows (Figure 4). If the temperature of lower heater is not increased with time, then steady thermocapillary deflection with concave profile is formed above the thermal element.

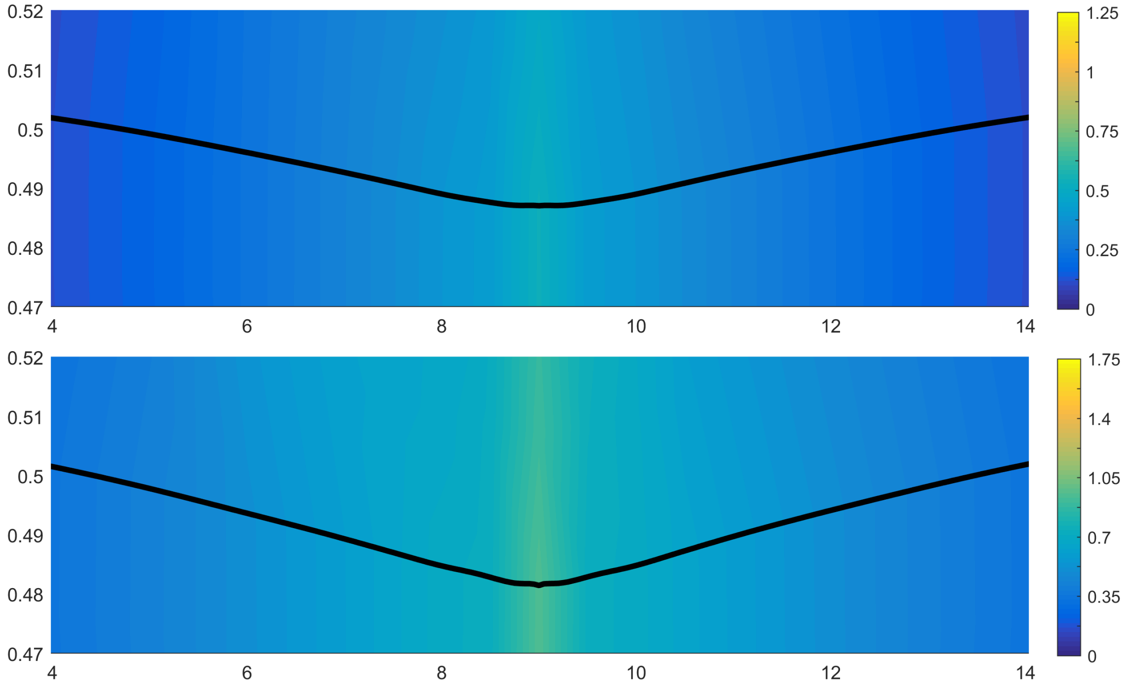

Figure 4: Position of Γ_t and temperature distribution near the interface in the thermal exposure zone for the ethanol−nitrogen system: at $q^s = 12.5^{\mathrm{O}}$C (50 seconds after switching on the heaters, from above), at $q^s = 17.5^{\mathrm{O}}$C (137 seconds after switching on the heaters, from below)

4 Conclusions

It was established that for the considered range of working temperature of the heaters (from 0 to 25 $^{\mathrm{O}}$C), if the thickness of the lower layer was $5 \cdot 10^{-3}$ m then the rupture of liquid layer did not occur despite long duration of heating and unfavourable (switchable) operation mode. In this range of working temperature the gap of liquid layer can appear only if the initial thickness of the lower layer is significantly less than $5 \cdot 10^{-3}$ m. Furthermore, it was found that the size of the lower heater slightly influences the amplitude of the interface deformation. And finally, it was shown that the arrangement of additional upper thermal sources (even directly above the lower heater) does not result in significant alteration of the flow regimes and changes in the interface behavior. Thus, one of the ways to avoid critical deformations of the phase boundary is to arrange the heater on the upper wall of the working section. Also, upper thermal elements can be used to prevent the appearance of thermal hysteresis where the parameters of an electronic device do not revert to the initial values after its heating and subsequent cooling. Switching to these additional upper heaters allows one to ensure continuity of the operation of onboard equipment without losing effective power. The lifetime of thermal elements can be prolonged by placing a duplicate heater and periodic switching of the electric circuit from one heater to another while the system relaxes.

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