Penalty Method for Reliable Allocations of Wireless Network Resources

Igor V. Konnov
Kazan Federal University
Kazan, Russia 420008
konn-igor@ya.ru

Alexey Yu. Kashuba
Kazan Federal University
Kazan, Russia 420008
leksser@rambler.ru

Erkki Laitinen
University of Oulu
Oulu, Finland FI-90014
erkki.laitinen@oulu.fi

Abstract

In the paper we consider the problem of allocation of network resources in telecommunication networks using both utility and reliability. We suggest a scalar utility maximization problem subject to capacity constraints and within the pre-defined reliability level. This problem is proposed to be solved by a penalty method. We present the results of numerical results on various test problems for the method.

1 Introduction

The current development of information technologies and telecommunications gives rise to new control problems related to efficient transmission of information and allocation of limited network resources. All these problems are determined on distributed systems where the spatial location of elements is taken into account. Due to strong variability and increasing demand of different wireless telecommunication services, fixed allocation rules usually lead to serious congestion effects and inefficient utilization of network resources despite the presence of very powerful processing and transmission devices. Hence, one has to find more flexible allocation mechanisms instead of the fixed allocation ones. These mechanisms are based on proper mathematical models; see e.g. [CW03, SWB06, WNH10]. For example, solution methods for network resource allocation based on optimization formulations of network manager problems and decomposition techniques were presented in [KKL18, KK19].

In addition, wireless networks should be reliable with respect to various attacks. The most commonly seen attack in wireless networks is eavesdropping in which attackers aim at acquiring important/private information of users, jamming and distributed DoS attacks which attempt to interfere and disrupt network operations by exhausting the resources available to legitimate systems and users. These attacks may lead to degrading the network performance and quality of service (QoS) as well as losing important data, reputations, and revenue; see e.g. [ZJT13, MZA13, ZJT13, LHW17].

In this paper we consider a problem of telecommunication network links allocation among users under reliability control of network connections. For this problem we suggest a penalty method with respect to links capacity and reliability level constraints. We present the results of numerical results on test problems for the method.

2 Problem Description

We begin our description from the basic optimal flow distribution problem in wireless telecommunication networks from [KMT98]. For a fixed time period we are given a network that contains a set $L$ of transmission links (arcs)
and accomplishes some submitted data volume transmission requirements from a set $I$ of selected pairs of origin-destination vertices. Denote by $x_i$ and $\alpha_i$ the current and maximal value of data transmission for pair demand $i \in I$, respectively, and by $c_l$ the capacity of link $l \in L$. For the sake of simplicity we suppose that each pair demand is associated with a unique data transmission path, hence each link $l$ is associated uniquely with the set $N_l$ of pairs of origin-destination vertices, whose data transmission paths contain this link. For each pair demand $i$ we denote by $u_i(x_i)$ the utility value at the data transmission volume $x_i$. Then the problem of the total utility maximization of the network is written as follows:

$$\max \rightarrow \sum_{i \in I} u_i(x_i)$$

subject to

$$\sum_{i \in N_l} x_i \leq c_l, \ l \in L,$$

$$0 \leq x_i \leq \alpha_i, \ i \in I.$$ If the functions $u_i(x_i)$ are concave, this is a convex optimization problem over a polyhedron.

In order to extend the model and take into account the reliability of connections we now suppose that the reliability depends on arc flow volumes. More precisely, let $\mu_l(f_l)$ denote the non-reliability of arc $l$ at its flow volume $f_l$. Hence, we have to add the second goal:

$$\min \rightarrow \sum_{l \in L} \mu_l(f_l)$$

where

$$f_l = \sum_{i \in N_l} x_i, \ l \in L,$$

and obtain a vector optimization problem. The scalarized goal version with some weights will take the form

$$\min \rightarrow \sigma_1 \sum_{l \in L} \mu_l(f_l) - \sigma_2 \sum_{i \in I} u_i(x_i),$$

but the choice of right weights $\sigma_1$ and $\sigma_2$ for so different goals seems too difficult here. For this reason, it is better to determine the pre-defined non-reliability level $\beta_i$ for each connection $i \in I$ and to maximize utility under all these constraints. That is, the origin-destination vertices require some desired level of their reliability to work. This problem is now formulated as follows:

$$\max \rightarrow \sum_{i \in I} u_i(x_i)$$

subject to

$$\sum_{i \in N_l} x_i = f_l, \ l \in L,$$

$$\sum_{l \in L} \mu_l(f_l) \leq \beta_i, \ i \in I,$$

$$f_l \leq c_l, \ l \in L,$$

$$0 \leq x_i \leq \alpha_i, \ i \in I.$$ If the functions $\mu_l(f_l)$ are convex, this is a convex optimization problem.

3 Solution Method

Usually, the functions $u_i(x_i)$ and $\mu_l(f_l)$ in problem (1)–(5) are smooth, hence we can apply a number of well known smooth optimization methods; see e.g. [DR68, PD78]. However, these problems have large dimensionality and inexact data, hence their solution methods should be rather simple and provide some desired accuracy within
an acceptable time interval. Therefore, the penalty based methods are suitable here; see e.g. [FM72, GK81]. We impose penalties only on binding constraints in (2)–(3). Set

\[ X = \{ x \mid 0 \leq x_i \leq \alpha_i, \ i \in I \}, \]
\[ F = \{ f \mid 0 \leq f_l \leq c_l, \ l \in L \}, \]
\[ W = X \times F, \]

and define the penalty functions

\[ \Phi_1(x, f) = \sum_{i \in I} \left[ \sum_{j \in N_i} x_j - f_i \right]^2 \]
and

\[ \Phi_2(x, f) = \sum_{i \in I} \left[ \sum_{l \in L_i} \mu_l(f_l) - \beta_i \right]^2. \]

We take positive penalty parameters \( \tau_1 \) and \( \tau_2 \) and define the penalized problem

\[ \max_{(x, f) \in W} \Psi(x, f, \tau), \]

where \( \tau = (\tau_1, \tau_2) \),

\[ \Psi(x, f, \tau) = \sum_{i \in I} u_i(x_i) - \tau_1 \Phi_1(x, f) - \tau_2 \Phi_2(x, f). \]

Denote by \((x^*(\tau), f^*(\tau))\) a solution of problem (6)–(7). If each \( u_i \) is positive, increasing, and tending to +\( \infty \), then the corresponding points \((x^*(\tau), f^*(\tau))\) will tend to a solution of problem (1)–(5). Moreover, this is the case for some approximations of points \((x^*(\tau), f^*(\tau))\). In order to find these approximate solutions of problem (6)–(7) we propose to apply the gradient projection method; see e.g. [DR68, PD78].

Let \( g(x, f) = (g_x(x, f), g_f(x, f)) \) denote the gradient of \( \Psi(x, f, \tau) \) where \( \tau \) is fixed. At the current point \((x, f)\) we find the points

\[ \bar{x} = \pi_X[x + \lambda' g_x(x, f)], \]
\[ \bar{f} = \pi_F[f + \lambda'' g_f(x, f)], \]

where \( \lambda' > 0, \lambda'' > 0 \). The next iterate \((x^{\text{new}}, f^{\text{new}})\) can be found from \((\bar{x}, \bar{f})\) after inserting a proper line-search if necessary. That is, we set

\[ x^{\text{new}} = \eta \bar{x} + (1 - \eta)x, \]
\[ f^{\text{new}} = \eta \bar{f} + (1 - \eta)f, \text{ for } \eta \in (0, 1]. \]

The partial derivatives of \( \Psi(x, f, \tau) \) are written as follows:

\[ g_{x_i}(x, f) = u'_i(x_i) - 2\tau_1 \sum_{i \in I} \left( \sum_{j \in N_i} x_j - f_i \right) \alpha_{il}, \]

where

\[ \alpha_{il} = \begin{cases} 1, & \text{if } i \in N_l, \\ 0, & \text{otherwise,} \end{cases} \]

and

\[ g_{f_l}(x, f) = 2\tau_1 \left( \sum_{i \in N_l} x_i - f_l \right) - 2\tau_2 \sum_{i \in I} \left[ \sum_{l \in L_i} \mu_l(f_l) - \beta_i \right] \mu_l'(f_l) \gamma_{il}, \]

where

\[ \gamma_{il} = \begin{cases} 1, & \text{if } i \in N_l, \\ 0, & \text{otherwise;} \end{cases} \]
for all $i \in I$ and $l \in L$. By using these formulas the main steps in (8)–(9) decompose into single-dimensional problems:

$$\max_{0 \leq y_i \leq \alpha_i} \left\{ g_{x_i}(x, f)(y_i - x_i) - \frac{1}{2\lambda'} (y_i - x_i)^2 \right\}, \ i \in I,$$

and

$$\max_{0 \leq v_l \leq \alpha_l} \left\{ g_{f_l}(x, f)(v_l - f_l) - \frac{1}{2\lambda''} (v_l - f_l)^2 \right\}, \ l \in L,$$

respectively. Their solutions can be found by explicit formulas.

4 Numerical Experiments

As part of the work, a numerical study of the considered method was carried out on test examples. The method was implemented in C++ with a PC with the following facilities: Intel(R) Core(TM) i7-4500, CPU 1.80 GHz, RAM 6 Gb.

We took the linear functions of arc non-reliability:

$$\mu_l(f_l) = \mu_{0,l} f_l, \ \mu_{0,l} > 0, \ l \in L,$$

and logarithmic functions of connection utilities

$$u_i(x_i) = u_{2,i} \log(u_{0,i} + u_{1,i} x_i), \ u_{j,i} > 0, \ j = 0, 1, 2, \ i \in I.$$

The coefficients $\mu_{0,l}$, $u_{0,i}$, $u_{1,i}$, and $u_{2,i}$ were determined on the basis of trigonometric functions:

$$\mu_{0,l} = \frac{|\cos(l)|}{|\cos(l)|} + 1,$$

$$u_{0,i} = 2 \frac{\sin(2i)}{|\cos(l)|} + 1,$$

$$u_{1,i} = \frac{|\sin(i + 1)|}{|\cos(l)|} + 1,$$

$$u_{2,i} = 3 \frac{\sin(2i)}{|\cos(l)|} + 1.$$

The maximal flow $c_l$ along arc $l$ was selected in the segment $[1, 10]$ depending on the arc number as follows:

$$c_l = 10 \frac{|\cos(l)|}{|\cos(l)|} + 1.$$

The maximal flow $\alpha_i$ for connection $i$ was selected in the segment $[1, 7]$ depending on the connection number as follows:

$$\alpha_i = 7 \frac{|\sin(i - 1)|}{|\cos(l)|} + 1.$$

The upper non-reliability bound $\beta_i$ was selected in the segment $[1, 5]$ depending on the connection number as follows:

$$\beta_i = 3 \frac{|\cos(i - 1)|}{|\cos(l)|} + 1.$$

The parameters $\tau_1$, $\tau_2$, $\lambda'$, and $\lambda''$ were fixed as follows:

$$\tau_1 = 0.9, \tau_2 = 0.9, \lambda' = 0.009, \lambda'' = 0.009.$$

The distribution of the available arcs across the connections was chosen either uniformly or according to the normal distribution law. We took two versions of the gradient projection method. The first does not involve any line-search, the second involves an Armijo type line-search.

Let us introduce the additional notations:

$\varepsilon$ is the accuracy of a solution of the problem;

$T_{\varepsilon}$ is the total solution time (in seconds) of the penalty method containing the gradient projection method without line-search;

$T_{\varepsilon,ls}$ is the total solution time (in seconds) of the penalty method containing the gradient projection method with line-search;

$I_{\varepsilon}$ is the number of iterations of the penalty method containing the gradient projection method without line-search;

$I_{\varepsilon,ls}$ is the number of iterations of the penalty method containing the gradient projection method with line-search.
Table 1: Calculations for $|I| = 620$, $|L| = 310$ and different $\varepsilon$

<table>
<thead>
<tr>
<th>$\varepsilon$</th>
<th>$T_\varepsilon$</th>
<th>$T_{e,ls}$</th>
<th>$T_{e,ls}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$10^{-1}$</td>
<td>0.031</td>
<td>280</td>
<td>0.063</td>
</tr>
<tr>
<td>$10^{-2}$</td>
<td>0.094</td>
<td>914</td>
<td>0.219</td>
</tr>
<tr>
<td>$10^{-3}$</td>
<td>0.156</td>
<td>1310</td>
<td>0.313</td>
</tr>
<tr>
<td>$10^{-4}$</td>
<td>0.219</td>
<td>1729</td>
<td>0.422</td>
</tr>
</tbody>
</table>

The penalty method was stopped if the norm difference between two sequential iterates appeared less than the accuracy. In Table 1, the numerical results are given for the case where $|I| = 620$, $|L| = 310$ and for different values of the accuracy $\varepsilon$.

Note that the working time was less than one second. We can also observe that similar results were obtained for fixed $\varepsilon$ and $|L|$ and different $|I|$ (see 2), and for fixed $\varepsilon$ and $|I|$ and different $|L|$ (see 3).

Table 2: Calculations for $\varepsilon = 10^{-2}$, $|L| = 310$ and different $|I|$

| $|I|$ | $T_\varepsilon$ | $T_{e,ls}$ | $T_{e,ls}$ |
|------|--------------|-------------|-------------|
| 310  | 0.032        | 626         | 0.079       |
| 620  | 0.094        | 914         | 0.219       |
| 930  | 0.157        | 869         | 0.328       |
| 1240 | 0.203        | 809         | 0.406       |

Table 3: Calculations for $\varepsilon = 10^{-2}$, $|I| = 310$ and different $|L|$

| $|L|$ | $T_\varepsilon$ | $T_{e,ls}$ | $T_{e,ls}$ |
|------|--------------|-------------|-------------|
| 310  | 0.032        | 626         | 0.079       |
| 620  | 0.047        | 559         | 0.109       |
| 930  | 0.109        | 538         | 0.203       |
| 1240 | 0.109        | 519         | 0.235       |

From the experiments we conclude that the version of the penalty method containing the gradient projection method without line-search appeared somewhat better than that with the line-search. In general, the method attained a solution with low accuracy quickly enough, but the additional analysis and selection of parameters for more effective implementation of the method are necessary.

4.0.1 Acknowledgments

The results of the first author in this work were obtained within the state assignment of the Ministry of Science and Education of Russia, project No. 1.460.2016/1.4. In this work, the first author was also supported by Russian Foundation for Basic Research, project No. 19-01-00431. The work of the second author was performed within the Russian Government Program of Competitive Growth of Kazan Federal University. The first and third authors were also supported by grant No. 331833 from Academy of Finland.

References


