Application of GeoGebra in Stereometry teaching

Tetiana H. Kramarenko[1][0000-0003-2125-2242], Olha S. Pylypenko[2][0000-0003-0493-8429] and Ivan O. Muzyka[3][0000-0002-9202-2973]

[1] Kryvyi Rih State Pedagogical University, 54 Gagarin Ave., Kryvyi Rih, 50086, Ukraine
kramarenko.tetyana@kdpu.edu.ua
[2] State University of Economics and Technologies,
37A Vatutin Str., Kryvyi Rih, 50096, Ukraine
banadaolga96@gmail.com
[3] Kryvyi Rih National University, 11 Vitalii Matusevych Str., Kryvyi Rih, 50027, Ukraine
musicvano@gmail.com

Abstract. The purpose of the paper is to improve methodology of teaching Mathematics via the use of cloud technology. The task of the paper is to identify the issues that require a theoretical and experimental solution. The objective of the paper is the educational process in the higher education institution, the subject of the paper is modern ICT. The result of the study is the learning tools of pedagogically considered and adequate bending of conventional and modern learning environment implemented into the educational process. The possibilities of using cloud technologies and Dynamic Mathematics system GeoGebra in the educational process through Stereometry specialized training have been revealed. The use of GeoGebra Dynamic Mathematics in Stereometry teaching will favourably influence the formation of students' STEM competencies. In order to encourage Mathematics and Computer Science teachers to implement effectively the elements of STEM education, it is suggested that cloud-based learning tools such as GeoGebra be used in the teaching process.

Keywords: GeoGebra 3D Graphing Calculator, Geometry, STEM competencies, Stereometry teaching, methodology of teaching Mathematics, cloud technologies in education.

1   Introduction

Emiliya Velikova and Magdalena Petkova [5] point out that every society needs STEM professionals, talented people who design new technologies, materials, constructions and processes. One of STEM training related fields is an integrated problem-based Mathematics training.

The results of the research study regarding the development of teachers' STEM competencies have shown that most of them are not aware of the peculiarities of students' STEM competencies formation. And some contradictions between the necessity to form the students' STEM competencies and insufficient attention to the training of future teachers of Mathematics occur.

One of the most effective tools for STEM-based Mathematics teaching is the system of dynamic mathematics, e.g. GRAN 2D tool [29] and cloud-based GeoGebra [7; 11].

Zsolt Lavicza, Theodosia Prodromou, Kristof Fenyvesi, Markus Hohenwarter, Istvan Juhos, Balazs Koren and Jose Manuel Diego-Mantecón [15] claim that GeoGebra was designed to integrate arithmetic, algebra, geometry, calculus, statistics. In recent times it is supposed to support STEM subjects as a single, integrated system available on most technology platforms. Moreover, teachers and students are offered free access to it all over the world. GeoGebra was originally created to integrate Algebra and Geometry into a single environment. GeoGebra enables to use such new technologies as Augmented [27] and Virtual Reality [16], 3D Printing [9] and mobile learning [18] in the learning Mathematics.

The use of GeoGebra Dynamic Mathematics in Stereometry teaching will enhance the formation of students' STEM competencies. It can also be helpful in the formation of key mathematical and digital competences [1], self-study competences, as well as in the development of spatial thinking.

In order to encourage Mathematics and Computer Science teachers to implement the elements of STEM education, it is suggested the cloud-based learning tools [17], such as GeoGebra, be used in the learning process.

2   Materials and methods

According to the recommendation of the European Commission [3, p. 7], competence is defined as a combination of knowledge, skills and attitudes. In the updated list of key competences of a personality eight major ones are emphasized [3]. We are keen to foster the development of students' STEM competencies: mathematical competence and competence in science, technology and engineering.

Serhiy A. Rakov explores the possibilities of mastering mathematical competencies in the learning by future teachers through the research approach using the system of dynamic mathematics DG [19]. The scientist distinguishes procedural, logical, technological, research and methodological competencies of Mathematics teacher. In particular, methodological competence implies the ability to assess the appropriateness of using mathematical methods to solve individual and social issues.

Consider in detail the opportunities which GeoGebra gives to a teacher. Markus Hohenwarter, Judith Hohenwarter, Yves Kreis and Zsolt Lavicza [10] emphasize a significantly important role of free open-source software for teaching mathematics world-wide. Open-source software do not only offer opportunities for teachers and students to use them both at home and in the classroom without any restrictions, but they also provide the tools for developing support and user communities reaching across borders. Such collaboration as well contributes to the equal access to technological resources and democratization of learning and teaching mathematics.

GeoGebra is available in multiple languages. Using GeoGebra a teacher can create interactive materials to solve the main tasks of teaching Geometry – the development of spatial imagination, practical understanding and logical thinking.

GeoGebra can be used both in standalone and online modes on mobile Internet

devices (MID) which offer new opportunities to improve learning and teaching either inside or outside of classrooms [28]. Further advantage of GeoGebra is the Resources tab containing a number of teaching materials: textbooks, articles, instructions, and illustrations. Also, a registered users can share their own educational materials on the official website via GeoGebraTube (this is a kind of file-sharing site). Its essential feature is that the created dynamic constructions can be accessed online via the use of View Sheet tool. Any user is allowed to comment on GeoGebraTube content.

A user is provided with a possibility to create their own GeoGebraBook, a folder where selected materials can be added and subfolders can be created. A user can publish a game, a collection of visuals, lesson workbook that may include text messages, GeoGebra models, web pages, questions, videos, images and manuals public to anyone. GeoGebraTube content can be shared via social networks Facebook, Twitter, emailed and uploaded to Google Drive. Cloud-based tools affect favourably teacher-student interactions in an informal learning environment.

Ljubica Diković statistically confirms the fact that the use of GeoGebra applications in learning calculus had a positive effect on the development of students' reasoning and visualization skills [4].

The issues of using GeoGebra are highlighted in the research studies of Olena V. Semenikhina [24], Maryna H. Drushliak [23], Natalya V. Rashevska [20] and others. Application of GeoGebra 5.0 to the solution of solid geometry problems has been analyzed in the article [23]. Examples of solid geometry problems with detailed solution and learning clues have been presented. Among them there are problems on auxiliary section, polyhedron net, locus problems, space transformations etc. The article [24] discusses the relevance of the study of the dynamic Mathematics software for future teachers of Mathematics in Ukraine.

Modelling implies simplifying thus providing the model with its own characteristics, independent from the original reality. Consequently, geometry, in its interpretation and processing, acts like a new reality both abstract in its logic and concrete in its representation modes.

In the article [21] the authors begin with classical theories of modelling representation processes in order to apply them to 3D dynamic geometry software, GeoGebra3D, computing development seeking to bring geometric models closer to the reality of a school setting. The authors introduce three key notions, developed from Mathematics teaching, in order to support the exploration of two interaction situations between mathematics and their teaching. Finally, they finish on a few general considerations for the teaching of mathematics.

In teaching Stereometry, the assignment worksheets on the construction of sections through the trace method, internal projection and combination method by Volodymyr A. Sydoruk [25] should be taken into consideration. Each of the assignments is provided with the possibility of step-by-step procedure of the construction building. Thus, students are given an opportunity to develop both mathematical competence and the ability to learn independently. Some assignments include constructions created on a 3D canvas.

N. F. Fariha and H. P. Lestari [6] describe in detail how to realize the Dandelin Sphere more realistically, using GeoGebra Classic 5, which supports the creation of 3D

images.

At a profound methodological level, a number of assignments on combinations of geometrical solid shapes was created by Mykhailo Y. Rykovskyi [22]. These models are designed as constructions of plane geometric shapes through the use of parallel projection properties. They are intended to be used to visualize the properties of geometrical shapes. However, students might face a challenge with reproducing models as the algorithm of the construction and details of the construction are not available for them.

The models are designed as constructions of plane geometric shapes with the use of the properties of parallel projection. In particular, a book / collection of visuals covering the topic "Prism" is recommended. There is a number of visuals that represents combinations of stereometric shapes: sphere and pyramid, sphere and prism, sphere and cylinder etc. Consider the visual "There is a cone inscribed in the sphere. And there is the right triangular pyramid inscribed into the cone. A pyramid is inscribed in the sphere". Even the name itself suggests that integrity is inherent to visuals. It can be used in the process of teaching Stereometry in school or teaching Methodology of Mathematics on various topics. While using visual aids, users are advised to study the properties of shapes inscribed into the sphere, to look into the relationship between the sphere radius, radius of cone base, the height of the cone and element of the cone.

These visuals are equipped with the Checkbox tool which allows a user to do the revision and answer to the questions: Which cone is called inscribed into the sphere? What element of the cone is the center of the sphere? Which pyramid is called inscribed in the cone? Which pyramid is called inscribed into the sphere? How to find the axial section of the cone? Which circle of the sphere is called a great circle? Which properties of the diameter of the sphere perpendicular to its great circle?

3 Setting a task

Mobile applications such as Geometry, Graphing Calculator, 3D Graphing Calculator have been relatively recently launched and now they are available on the official GeoGebra website. These applications are still undergoing improvements. However, the methodology of their use, and especially in teaching Stereometry, is not yet well developed. Therefore, this paper aims at elucidating the features of creating visuals with GeoGebra and their use in teaching how to solve stereometric problems.

4 Results

4.1 GeoGebraBook "Models for Stereometric problems" as training manual on the methodology of teaching mathematics

The methodical materials of the paper authors related to the use of the GRAN software are presented in the manual "Innovative information and communication technologies of teaching Mathematics". The issue of GeoGebra use has been considered in an updated version of the manual [12]. On the GeoGebra site, users can find the materials

which contain relevant visuals at GeoGebraBook "Models for stereometric tasks" [14]. One can find here problem situations to be solved by students independently, self-assessment tests.

The study of the effectiveness of the use of GeoGebra mobile applications in the process of teaching Stereometry was conducted by us within the preparation of future teachers of Mathematics during Mathematics teaching methodology classes, advanced training courses at Kryvyi Rih State Pedagogical University, in teaching students of Kryvyi Rih State College of Economics and Technologies, and students of classes with mathematical specialization of Kryvyi Rih schools.

Particular interest in using GeoGebra was shown during the advanced training courses by Kryvyi Rih teachers of Mathematics who have been working at school for a considerable time. The study was also conducted on the effectiveness of the use of GeoGebra applications by students in order to enhance students' motivation during extra-curricular vocational activities at the university. And it was studied as well during the course of Sereometry lessons conducted by masters majoring in Mathematics education completing professional practice in secondary education institutions.

When using applications, it is useful for a student to be able to reproduce the construction in the browser window, open other temporarily hidden drawing elements in case of necessity, and receive text notifications. However, a considerable number of constructions should be created later or simultaneously with the viewing via the mobile application.

Consider the peculiarities of the implementation of specific constructions and provide recommendations concerning their use.

One of the main issues that school students and students of higher education establishments face is the recognition of a 2D drawing representing the image of a 3D shapes. In order to develop the spatial imagination of students at schools and universities it is necessary to provide them with the assignments on constructing sections of polyhedron by planes.

4.2 Construction of sections of polyhedron with plane via the applications GeoGebra Geometry and 3D graphics

Consider examples of problems for constructing a section of a polyhedron with a plane passing through three given points that are not on the same line; through a straight line and a point that does not belong to it etc. It is necessary to construct a linear angle of a dihedral angle and measure them; the angle between the straight line and the plane.

Look into what four approaches to constructing a section of a polyhedron in a plane which should be distinguished by a teacher in order to be able to provide their students with the most appropriate one for their educational trajectory.

If a student has not fully mastered GeoGebra tools, it is advisable to provide them with a sheet of paper guiding the algorithm to follow in order to build a construction. Moreover, a student should be supplied with information regarding the icons that correspond to the tools in question. It is appropriate to place QR codes on printable worksheets for demonstrations.

1. The construction on a 2D canvas is rather cumbersome and requires logically justified steps for the construction. At first, it is necessary to follow step-by-step construction procedures to build several drawings, created via "the trace method" on a 2D canvas. To intensify the material dissemination, students may be at once provided with a constructed polyhedron. However, in the course of the study, there were cases when the students constructed the polyhedron wrongly. That is, after its construction on the plane, they were not able to change it dynamically. For example, to build a prism image on a 2D canvas made of an *n*-sided polygonal base (*n*>3), it is necessary to follow the following steps. Firstly one should build an n-sided polygon via the use of the appropriate tool. Then it is preferable to build a vector along one of the edges of the prism. Further, one should apply a parallel translation of the n-sided polygon to the constructed vector. The given vertexes are connected. Such a construction is represented taking into account the properties of the parallel projection.

2. Next it is necessary to compare the previous drawing with the one presented according to the same problem situation but on the 3D canvas. In this case, all straight lines for the construction of the section can be presented step by step. The advantages of such a construction include the possibility to change the constructed polyhedron dynamically and the location of points which the secant plane passes through. Via the GeoGebra 3D Geometry [8] app, one can simulate external actions aimed at geometrical solid shapes which are necessary for a student to be able to apply internal thinking regarding them and as a result develop spatial thinking. In our opinion, this approach is the most appropriate for the development of students' spatial imagination, logical thinking. In Fig. 1 the construction of the section of the pyramid with a secant plane which passes through the point on the lateral edge and a straight line drawn in the plane of the base is presented.
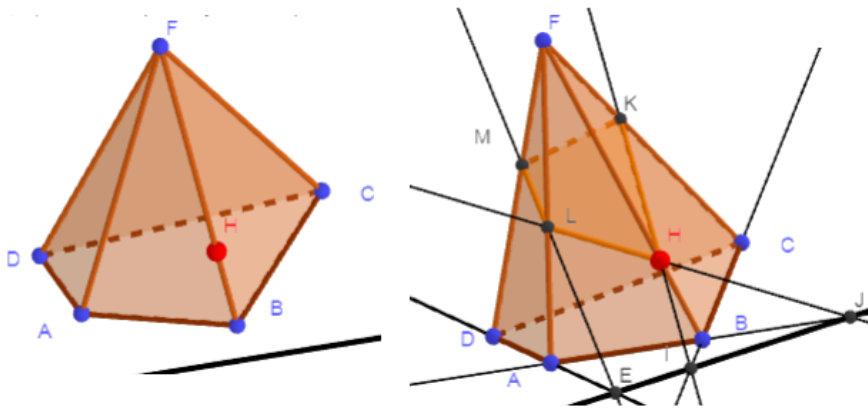

Fig. 1. Construction of a section of the pyramid with a plane.

3. The third option for constructing a section on the 3D canvas. Firstly, a polyhedron, and a secant plane should be built, and then the tool "Intersection of two surfaces" should be applied. A student, by "turning" the polyhedron, will have an opportunity

for the better perception and understanding of the construction. The approach in question is appropriate when augmented reality tools are used enabling to align the drawings with the image of a real object [13]. It is advisable to encourage students to find examples of such sections among the surrounding objects, in architecture etc.

4. The use of both 2D and 3D canvases simultaneously. Such a combination is advisable if there is some kind of difficulty in constructing a polygon that is for a base of the pyramid or specifying the position of the vertex of the pyramid, etc. For instance, if a trapezoid, in particular, a rectangular or equilateral one, is for a base of the pyramid or prism. Or, for example, the vertex of a pyramid is projected into one of the top of the base or to the side of the base (see Fig. 2).
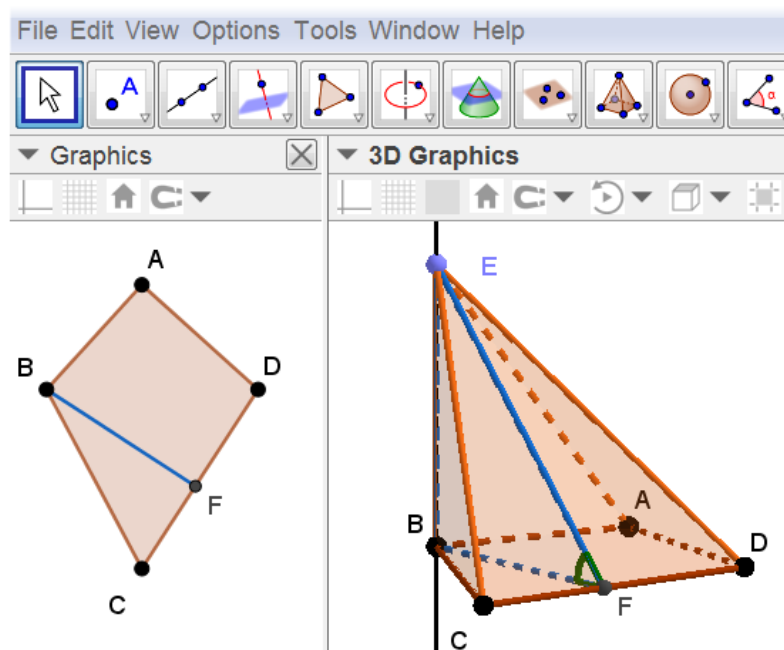

Fig. 2. Pyramid, two faces of which are perpendicular to the plane of the base.

It is worth pointing out that such constructions can only be built in the classic version of GeoGebra, and the construction preview may also be available via MID after uploading the file to the GeoGebra cloud repository.

4.3 The peculiarities of the definition and angle construction between planes and linear angle of the dihedral angle

Via the assistant of GeoGebra (3D canvas) it is possible to build the base of the pyramid (OXY) on the canvas, and in the process the constructions can be synchronously displayed on the 3D canvas (see Fig. 2, Fig. 3).
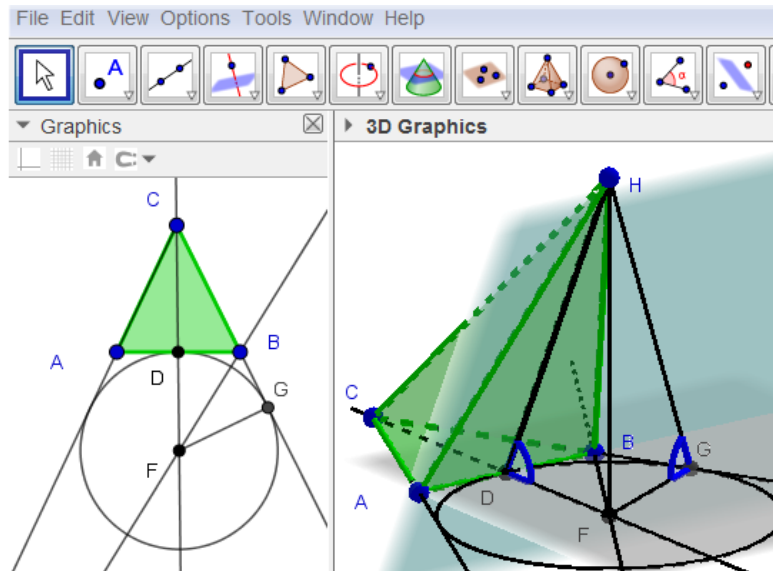
Fig. 3. Image of the pyramid and linear angles.

Attention should be drawn to the convenience and ease of the construction with the help of the linear angle of the dihedral angle at the base. According to the definition, to determine the linear angle of the dihedral angle at the base a plane perpendicular to the edge of the dihedral angle should be drawn. Then the angle between the rays formed as a result of the intersection of this plane with the edges of the dihedral angle will determine its linear angle (see Fig. 2) [2].

It should be stated that the measure of the angle between the planes varies from 0° to 90°. While the measure of the dihedral angle can vary from 0° to 360° [22]. Here is an example of the problem from the textbook with major in Mathematics (No. 836, [2, p. 162]).

At the base of the pyramid there is an isosceles triangle with angle β at the vertex and radius R of the circle described. The plane of each face of the pyramid forms an angle α with the plane of the base. Find the area of the side surface.

It is recommended to use 2D and 3D canvases simultaneously to represent the drawing corresponding to the set problem [14].

The reason why mistakes are often made is that only one case is under consideration. Whereas the concept of "angle between the plane of the lateral face and the plane of the base" is substituted by "dihedral angle at the base". If the problem situation said that "dihedral angles at the base were congruent", then we would consider the pyramid the vertex of which is projected into the center of the circle inscribed into the triangle of the base (Pyramid of Type 1).

Since it concerns only congruent angles between planes, the problem will have two solutions. It is also necessary to take under consideration the case when one of the dihedral angles at the edge of the base is obtuse (Pyramid of Type 2). In this case, the vertex of the pyramid is projected into the center of the inscribed circle lying

outside (see Fig. 3).

Using the problem mentioned above and similar ones we have conducted a research on the basis of the two 11th grade classes specialized in Mathematics, and in particular with the teachers of Mathematics Svitlana Shahmatova and Ruslan Kaluhin [12, p. 439-445]. 16 students of the experimental group (EG students) and 18 students of the control group (CG) have participated in the research. The similar research has also been conducted at the advanced training courses among the teachers of Mathematics in Kryvyi Rih State Pedagogical University (group 1 – 22 teachers, group 2 – 26 teachers). Some teachers among the participants of the courses did not teach students in high school that is why they do not deal with solving these very problems in their teaching.

The suggested questionnaire consisted of several questions and corresponded mainly to the Pyramid of Type 2. The participants were supposed to deal with the questions during the problem solving. Alongside the questionnaire we present the results of the survey.

1. On reading the problem situation, have you distinguished two types of the Pyramid?

A positive answer: EG – 1 student (6%), CG – no students (0%), group 1 – 5 teachers (23%), group 2 – 6 teachers (23%).

2. The participants received a clue about the way it is possible to construct the Pyramid with the obtuse dihedral angle at the base (Pyramid of type 2). The question was the following: Is it possible to construct the vertex of the pyramid projection on the plane base and in fact the Pyramid itself?

A positive answer: EG – 6 students (28%), CG – 5 students (28%), group 1 – 12 teachers (55%), group 2 – 13 teachers (50%).

3. First, a dynamic model, constructed with GeoGebra, was demonstrated to the participants. The algorithm of the Pyramid construction via the use of the tool GeoGebra Construction Steps was displayed. The task was to construct the Pyramid of Type 2 and create a problem solving algorithm individually. Next, a paper Pyramid model was demonstrated to the participants. The algorithm of the construction was discussed. The task was to construct the Pyramid of Type 2 and create a problem solving algorithm.

A positive answer: EG – 12 students (75%), CG – 8 students (44%), group 1 – 19 teachers (86%), group 2 – 17 teachers (65%).

Fisher's angular transformation [26] was used for the research results processing. For this purpose, the task performance proportions in the groups during the first and second stages were compared.

φ(EG, CG) = 0,61 < 1,64; φ(group1, group2) = 0,31 < 1,64: the received results do not differ considerably both in the groups of students and teachers.

After stage 3, the proportions of successful task performance in groups were compared one more time. φ(EG, CG) = 1,85 > 1,64; φ(group1, group2) = 1,73 > 1,64: the results differ significantly this time.

It has been defined that the use of dynamic models created with GeoGebra was more effective in comparison with the static paper ones.

On discussing the use of dynamic models created with GeoGebra, participants pointed out the following advantages: variability; dynamic visuals which enhance the development of spatial imagination; multiple reproducibility of the construction, which will contribute to the development of algorithmic thinking.

4.4 Tasks on combinations of polyhedron and solids of revolution

Consider the way it is possible to inscribe a sphere into the right rectangular pyramid via the use of 3D Geometry. In order to construct the base of the pyramid, it is necessary to use the Right Polygon tool, by pointing two points on the 3D canvas – adjacent vertexes of the base, and indicating that the right polygon has 4 vertexes. Then one should construct the diagonals of the square (the Segment tool) and define the center (Intersection point). Then through the center of the square, which is also the center of the circle inscribed in the square, one draws a straight line perpendicular to the plane of the square. On this straight line, one chooses an arbitrary point (Point on the object) and constructs a polyhedron (Pyramid). The perpendicular to the plane of the square straight line is the geometric location of points, equidistant from the sides of the base of the right pyramid.
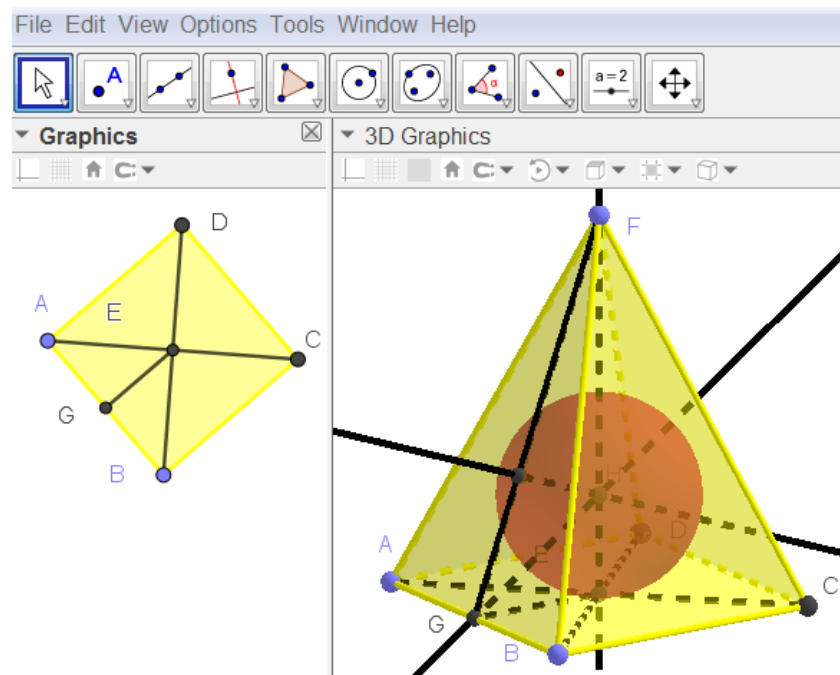

Fig. 4. The sphere inscribed in the pyramid.

To determine the position of the center inscribed sphere in the pyramid, one constructs a geometric location of points that are equidistant from the edges of the dihedral angle

at the base of the pyramid. Since there is no construction of the bisector plane in the GeoGebra tools, it is necessary to construct a linear angle of the dihedral angle at the base and then bisector of the very angle. The plane passing through the vertex of the pyramid perpendicular to the edge of the base is built (Plane through the point perpendicular to the straight line; Intersection point). Instead of a plane, it is possible to draw a straight line from the vertex of the pyramid perpendicular to the edge of the base (straight, perpendicular to straight). Next, one should find the intersection point of the constructed plane / perpendicular with the edge of the base (Intersection point of the straight line and the plane / Intersection point of two straight lines). Then one builds the bisector of the obtained linear angle.

The point of its intersection with the perpendicular to the base of the pyramid, drawn from the top of the pyramid, will determine the center of the inscribed sphere (Point of intersection). Finally, one constructs the inscribed sphere (Sphere outside the center and radius) (see Fig. 4), specifying in sequence the center of the sphere and the point of intersection of the diagonals of the square.

For better understanding and mastering of the algorithm the construction of the sphere inscribed around the pyramid the students setting of the canvas are adjusted to be able to show the step-by-step procedure of the construction.

5   Conclusions and prospects for further research

1. The use of dynamic geometry software GeoGebra as a modeling tool makes it possible for scientific experimental logic to engage students in discovering new mathematical facts. Teachers who are concerned by the issues of instrumented learning help to create activities which imply that the main students' motivation is in the understanding of mathematical concepts.
2. In the process of Stereometry teaching, it necessary to divide the students into subgroups according to the type of spatial thinking. This will contribute to the implementation of the individual approach to the development of the spatial imagination, taking into account individual peculiarities, gradually complicating the task, supplementing the teaching material with visual aids, focusing on the practical application of knowledge. Actions with the models created with GeoGebra serve as an intermediate link between external actions with geometrical solid shapes and internal mental processes. Thinking must precede the external ones to engage and develop a person's imagination.
3. In order to increase the efficiency of the perception and assimilation of stereometric material, to overcome the difficulties in transcoding the symbols of a spatial body and to create an adequate spatial image, it is necessary to supplement the theoretical material with multimedia demonstration models created by the means of ICT, to encourage students and to engage them in the process of creating such models for classes independently. After all, the acquisition of mathematical competence depends not on memory, but on the activity in which the person is involved, on the system of mental operations that they perform in the process of knowledge acquisition.

4. In general, the result of the research study is the improvement of teaching methods of Mathematics via the use of cloud technologies, the implementation in the educational process of methodological materials of pedagogically balanced and appropriate blending of traditional and innovative learning tools, which contribute to the intensification of teachers' readiness for the use of STEM training in teaching.

In the future, it is necessary to explore the conditions for the effective use of other GeoGebra mobile applications in STEM learning.

References

1. Astafieva, M.M., Zhyltsov, O.B., Proshkin, V.V., Lytvyn, O.S.: E-learning as a mean of forming students' mathematical competence in a research-oriented educational process. In: Kiv, A.E., Shyshkina, M.P. (eds.) Proceedings of the 7th Workshop on Cloud Technologies in Education (CTE 2019), Kryvyi Rih, Ukraine, December 20, 2019, CEUR-WS.org, online (2020, in press)

2. Bevz, H.P., Bevz, V.H., Vladimirova, N.H., Vladimirov, V.M.: Geometry 11 grade. Geneza, Kyiv (2011)

3. Council Recommendation of 22 May 2018 on key competences for lifelong learning (Text with EEA relevance). The Council of the European Union, Official Journal of the European Union. – C 189 – P. C1-13. – 4.6.2018. https://eur-lex.europa.eu/legal-content/EN/TXT/PDF/?uri=CELEX:32018H0604(01)&from=EN (2018). Accessed 25 Oct 2019

4. Diković, L.: Applications GeoGebra into Teaching Some Topics of Mathematics at the College Level. Computer Science and Information Systems **6**(2), 191–203(2009). doi:10.2298/csis0902191D

5. E. Velikova, M. Petkova: Analysing Students' Creativity in Integrating GeoGebra Applets in Solving Geometrical Problems. Baltic J. Modern Computing **7**(3), 419–429 (2019). doi:10.22364/bjmc.2019.7.3.08

6. Fariha, N.F., Lestari, H.P.: Construction of dandelin sphere on definition of conics using geogebra classic 5. Journal of Physics: Conference Series **1320**, 012085 (2019). doi:10.1088/1742-6596/1320/1/012085

7. Flehantov, L., Ovsiienko, Yu.: The Simultaneous Use of Excel and GeoGebra to Training the Basics of Mathematical Modeling. In: Ermolayev, V., Mallet, F., Yakovyna, V., Kharchenko, V., Kobets, V., Korniłowicz, A., Kravtsov, H., Nikitchenko, M., Semerikov, S., Spivakovsky, A. (eds.) Proceedings of the 15th International Conference on ICT in Education, Research and Industrial Applications. Integration, Harmonization and Knowledge Transfer (ICTERI, 2019), Kherson, Ukraine, June 12-15 2019, vol. II: Workshops. CEUR Workshop Proceedings **2393**, 864–879. http://ceur-ws.org/Vol-2393/paper_288.pdf (2019). Accessed 30 Jun 2019

8. GeoGebra Team German: Learn GeoGebra 3D Calculator. https://www.geogebra.org/m/aWhYSpvy. Accessed 25 Oct 2019

9. Hevko, I., Potapchuk, O., Sitkar, T., Lutsyk, I., Koliasa, P.: Formation of practical skills modeling and printing of three-dimensional objects in the process of professional training of IT specialists. In: Semerikov, S., Chukharev, S., Sakhno, S., Striuk, A., Osadchyi, V., Solovieva, V., Vakaliuk, T., Nechypurenko, P., Bondarenko, O., Danylchuk, H. (eds.) The International Conference on Sustainable Futures: Environmental, Technological, Social and Economic Matters (ICSF 2020). Kryvyi Rih, Ukraine, May 20-22, 2020. E3S Web of

Conferences **166**, 10016 (2020). doi:10.1051/e3sconf/202016610016

10. Hohenwarter, M., Hohenwarter, J., Kreis, Y., Lavicza, Z.: Teaching and learning calculus with free dynamic mathematics software GeoGebra. In: ICME 11 – 11th International Congress on Mathematical Education. Monterrey, Nuevo Leon, Mexico (2008)

11. Hohenwarter, M.: GeoGebra Quickstart. Check-as-you-go GeoGebra Tutorials. https://www.geogebra.org/m/Ebm5wBW5. Accessed 29 Nov 2019

12. Kramarenko, T.H., Korolsky, V.V., Semerikov, S.O., Shokaliuk, S.V.: Innovatsiyni informatsiyno-komunikatsiyni tekhnolohii navchannia matematyky (Innovative information and communication technologies of Mathematics teaching), 2nd edn. Kryvyi Rih Pedagogical University, Kryvyi Rih (2019)

13. Kramarenko, T.H., Pylypenko, O.S., Zaselskiy, V.I.: Prospects of using the augmented reality application in STEM based Mathematics teaching. Educational Dimension **53**(1), 199–218 (2019). doi:10.31812/educdim.v53i1.3843

14. Kramarenko, T.H.: Vybrani pytannya elementarnoyi matematyky z GeoGebra (Selected questions of Elementary Mathematics from GeoGebra), https://www.geogebra.org/m/gqpk8yfu. Accessed 28 Nov 2019

15. Lavicza, Z., Prodromou, T., Fenyvesi, K., Hohenwarter, M., Juhos, I., Koren, B., Diego-Mantecon, J.: Integrating STEM related technologies into mathematics education at a large scale. International Journal for Technology in Mathematics Education **27**(1), 3–12 (2020)

16. Lavrentieva, O.O., Arkhypov, I.O., Kuchma, O.I., Uchitel, A.D.: Use of simulators together with virtual and augmented reality in the system of welders' vocational training: past, present, and future. In: Kiv, A.E., Shyshkina, M.P. (eds.) Proceedings of the 2nd International Workshop on Augmented Reality in Education (AREdu 2019), Kryvyi Rih, Ukraine, March 22, 2019. CEUR Workshop Proceedings **2547**, 201–216. http://ceur-ws.org/Vol-2547/paper15.pdf (2020). Accessed 10 Feb 2020

17. Markova, O.M.: The tools of cloud technology for learning of fundamentals of mathematical informatics for students of technical universities. In: Semerikov, S.O., Shyshkina, M.P. (eds.) Proceedings of the 5th Workshop on Cloud Technologies in Education (CTE 2017), Kryvyi Rih, Ukraine, April 28, 2017. CEUR Workshop Proceedings **2168**, 27–33. http://ceur-ws.org/Vol-2168/paper5.pdf (2018). Accessed 21 Mar 2019

18. Modlo, Ye.O., Semerikov, S.O., Shajda, R.P., Tolmachev, S.T., Markova, O.M., Nechypurenko, P.P., Selivanova, T.V.: Methods of using mobile Internet devices in the formation of the general professional component of bachelor in electromechanics competency in modeling of technical objects. In: Kiv, A.E., Shyshkina, M.P. (eds.) Proceedings of the 7th Workshop on Cloud Technologies in Education (CTE 2019), Kryvyi Rih, Ukraine, December 20, 2019, CEUR-WS.org, online (2020, in press)

19. Rakov, S., Gorokh, V., Osenkov, K.: Mathematics, computer mathematical systems, creativity, art. In: Braman, J., Vincenti, G., Trajkovski, G. (eds.) Handbook of Research on Computational Arts and Creative Informatics, pp. 253–279. IGI Global, Hershey (2009). doi:10.4018/978-1-60566-352-4.ch015

20. Rashevska, N.V., Semerikov, S.O., Slovak, K.I.: Intehratsiia systemy dynamichnoi heometrii GeoGebra v systemu dystantsiinoho navchannia Moodle (Integration of dynamic geometry system GeoGebra in the distance learning system Moodle). In: Persha vseukrainska naukovo-praktychna konferentsiia "MoodleMoot Ukraine 2013. Teoriia i praktyka vykorystannia systemy upravlinnia navchanniam Moodle", Kyiv, KNUBA, 30–31 travnia 2013 r.), p. 63. KNUBA, Kyiv (2013)

21. Richard, P.R., Blossier, M.: Instrumented modelling and preliminary conceptions in three-dimensional dynamic geometry with geogebra-3D. In: Bastiaens, T., Marks, G. (eds.) Proceedings of E-Learn 2012 – World Conference on E-Learning in Corporate,

Government, Healthcare, and Higher Education 1, pp. 322–330. Association for the Advancement of Computing in Education, Montréal, Quebec. https://www.learntechlib.org/primary/p/41611 (2012). Accessed 25 Oct 2019

22. Rykovskyi, M.Y.: Mykhailo Yosypovych Rykovskyi – Resources – GeoGebra. https://www.geogebra.org/u/mirinf. Accessed 28 Nov 2019

23. Semenikhina, O.V., Drushliak, M.H.: GeoGebra 5.0 Tools and Their Use in Solving Solid Geometry Problems. Information Technologies and Learning Tools **44**(6), 124–133 (2014). doi:10.33407/itlt.v44i6.1138

24. Semenikhina, O.V.: Metodolohichni pidkhody do formuvannia profesiynoi hotovnosti vchytelia matematyky do vykorystannia zasobiv kompiuternoi vizualizatsii matematychnykh znan (Methodological Approaches to the Formation of Professional Readiness of Mathematics Teacher for the Use of Mathematical Knowledge Computer Visualization Tools). Pedahohichni nauky: teoriya, istoriya, innovatsiyni tekhnolohiyi 2(66), 129–138 (2017)

25. Sidoruk, V.A.: Pobudova pereriziv mnohohrannykiv (Construction of polyhedra sections). https://www.geogebra.org/m/Jd4va4rs. Accessed 28 Nov 2019

26. Stevens, W.L.: Tables of the Angular Transformation. Biometrika **40**(1/2), 70–73 (1953). doi:10.2307/2333098

27. Striuk, A.M., Rassovytska, M.V., Shokaliuk, S.V.: Using Blippar Augmented Reality Browser in the Practical Training of Mechanical Engineers. In: Ermolayev, V., Suárez-Figueroa, M.C., Yakovyna, V., Kharchenko, V., Kobets, V., Kravtsov, H., Peschanenko, V., Prytula, Ya., Nikitchenko, M., Spivakovsky A. (eds.) Proceedings of the 14th International Conference on ICT in Education, Research and Industrial Applications. Integration, Harmonization and Knowledge Transfer (ICTERI, 2018), Kyiv, Ukraine, 14-17 May 2018, vol. II: Workshops. CEUR Workshop Proceedings **2104**, 412–419. http://ceur-ws.org/Vol-2104/paper_223.pdf (2018). Accessed 30 Nov 2018

28. Tomaschko, M., Kocadere, S.A., Hohenwarter, M.: Opportunities for Participation, Productivity, and Personalization Through GeoGebra Mathematics Apps. In: Khan, A.A., Umair, S. (eds.) Handbook of Research on Mobile Devices and Smart Gadgets in K-12 Education, pp. 45–56. IGI Global, Hershey (2018). doi:10.4018/978-1-5225-2706-0.ch004

29. Zhaldak, M.I., Vitiuk, O.V.: Kompiuter na urokakh heometriyi (Computer on Geometry lessons). Dinit, Kyiv (2004)