Forecasting method of multidimensional time series based on Neuro-Fuzzy Cognitive Temporal Models

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Abstract

In the article there are Neuro-Fuzzy Cognitive Temporal Models (NFCTM) described. Those provide accounting of indirect and indirect mutual impact of all the multidimensional time series (MTS) components with their temporary delays relative to each other and are oriented on forecasting of multidimensional time series. Neuro-Fuzzy Cognitive Temporal Componental Models, which provide the formation of forecasted values of the MTS components with the temporary delays demanded, are used in NFCTM concepts in order to accomplish the temporal transformation. There is the way of NFCTM coordinated training described, which consists in Neuro-Fuzzy Componental Temporary Models for each of the NFCTM component and also in coherence of these Neuro-Fuzzy Componental Temporary Models (NFCTM) between each other. There is an MTS forecasting method offered in condition of unreliability the nonlinearity of the interaction, partial inconsistency and interdependence of the MTS components, that is based on NFCTM. There are experimental studies conducted and the results of using the proposed method are presented on the example of the problem of multidimensional forecasting of the state of the urban environment in Moscow. The use of the proposed method may be in demand to provide reliable forecasting of the state of the urban environment in various regions of Russia and other countries, including into account the complex epidemiological situation.

Keywords1

multidimensional time series, Neuro-Fuzzy Cognitive Temporal Model, Neuro-Fuzzy Componental Temporal Model.

1. Introduction

Methods based on random process theory, mathematical statistics, and pattern recognition are used to predict multidimensional time series (MTS). At the same time, as a rule, they are based on approaches to forecasting of one-dimensional time series and do not fully take into account the non-linear nature of interaction between the components of the MTS, different quality, insufficient volume and incomplete information [1-3].

Currently, neural network and fuzzy methods are well established to solve these problems [4, 5], the limitations of which are the difficulty of taking into account the indirect interplay of MTS components and their partial coherency.

The multi-criteria nature of analysis and forecasting requires minimization of prediction errors for all MTS components at the same time. However, this is generally impossible to achieve for complex systems and processes in real-world conditions of uncertainty, non-linearity of interaction, partial inconsistency and substantial interdependence of TDM components.

Fuzzy cognitive maps and prediction methods based on them are aimed at solving such problems [8-10]. However, their use is also limited by the insufficient capacity of the system dynamics models

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used and the lack of consideration of the different time delays of the interdependent components of the MTS.

The article deals with Neuro-Fuzzy Cognitive Temporal Models (NFCTM) which provide direct and indirect interaction of all components of multidimensional time series (MTS) with their time delays relative to each other, and are focused on predicting multidimensional time series. The method of coordinated training of NFCTM is described, which consists in training of Neuro-Fuzzy Component Temporal Models for each NFCTM concept, as well as in matching of these Neuro-Fuzzy Component Temporal Models of NFCTM.

There are experimental studies conducted and the results of using the proposed method are presented on the example of the problem of multidimensional forecasting of the state of the urban environment in Moscow. The use of the proposed method may be in demand to provide reliable forecasting of the state of the urban environment in various regions of Russia and other countries, including into account the complex epidemiological situation.

2. Neuro-Fuzzy Cognitive Temporal Models for predicting multidimensional time series

Let's present the MTS as follows:

$$\forall t \in \{1..\tau.\} \quad S_{t} = \begin{cases} S_{1}^{(t)} = F_{1} \left(\varphi_{1,1} \left(S_{1}^{(t-1)} \dots S_{1}^{(t-L_{1}^{1})} \right) \dots \varphi_{1,N} \left(S_{N}^{(t-1)} \dots S_{N}^{(t-L_{1}^{N})} \right) \right), \\ \dots \\ S_{N}^{(t)} = F_{N} \left(\varphi_{N,1} \left(S_{1}^{(t-1)} \dots S_{1}^{(t-L_{N}^{1})} \right) \dots \varphi_{N,N} \left(S_{N}^{(t-1)} \dots S_{N}^{(t-L_{N}^{N})} \right) \right), \end{cases}$$
(1)

where S – multidimensional time series; $S_i = (s_1^{(i)} \dots s_N^{(t)})$ – time «slice» of the MTS at the t-th instant of time; $s_j^{(i)}$ – the value of the j-th component of the MTS at the t-th instant of time; L_j^i – maximum time delay of the j-th component of the MTS relative to the i-th; $\varphi_{i,j}$ – operator for accounting for the interaction between the j-th and i-th MTS components; F_i – transformation for definition $s_i^{(t)}$, i = 1, ..., N, N – quantity of the MTS components.

Article [9] proposes a new type of NFCTM focused on MTS forecasting:

$$FCIM = \langle C, W \rangle,$$

$$C = \{C_i | i \in 1...N\}, N = |C|,$$

$$C_i: \tilde{s}_i^{(t)} = \tilde{F}_i \left(\left\{ \left(\tilde{s}_j^{\prime(t-1)} ... \tilde{s}_j^{\prime(t-L_i^j)} \right) | j \in 1...N_i \right\} \right), i = 1...N,$$

$$W = \{W_{ij} | i, j \in 1...N\},$$

$$W_{ij} = \left\{ \widetilde{w}_{ij}^{(t-l_i^j)} | l_i^j = 0...L_i^j \right\},$$

$$\tilde{s}_j^{\prime(t-l_i^j)} = \widetilde{\varphi}_{ij} \left(\widetilde{w}_{ij}^{(t-l_i^j)}, \widetilde{s}_j^{(t-l_i^j)} \right), l_i^j = 0...L_i^j,$$
(2)

where C – multiple NFCTM concepts corresponding to MTS components; \tilde{F}_i – fuzzy temporal transformation implemented by the concept C_i ; N – number of NFCTM concepts; $\tilde{s}_i^{(t)}$ – predicted fuzzy value of the concept C_i at the t-th instant of time; $\left(s_j^{\prime(t-1)} \dots s_j^{\prime(t-L_i^{l})}\right)$ – subset of the input temporal fuzzy variables of the concept C_i , associated with the corresponding output temporal fuzzy variables of the concept C_j ;

 N_i – number of NFCTM concepts directly related to the concept C_i ; l_i^j – time delay for the

corresponding input variable $s_j^{(t-l_i')}$ of the concept C_i , $l_i^j = 0...L_i^j$; W – a set of fuzzy degrees of direct impact between all pairs of NFCTM concepts; W_{ij} – a subset of fuzzy degrees of impact $\widetilde{w}_{ij}^{(t-l_i')}$ of the concept C_j on the concept C_i taking into account the time delay l_i^j ; $\widetilde{\varphi}_{ij}$ – fuzzy operator accounting for the degree of impact of the output variable of the concept C_i on the concept's input variable C_i .

3. Description of the method for predicting multidimensional time series based on Neuro-Fuzzy Cognitive Temporal Models

The method of prediction of MTS based on NFCTM consists of the stages discussed below.

Stage 1. Identification of meaningful components of the MTS for determining the composition of NFCTM concepts.

The implementation of the proposed method will be considered on the example of multidimensional forecasting of the state of the urban environment in Moscow. The state of the urban environment is characterized by the state of its facilities, systems and infrastructure and cannot be reduced to any single indicator. Basing on the results of previous studies [10-12], the following meaningful factors (components of MTS) characterizing the state of the urban environment have been determined:

- C1 ecology of the urban environment;
- C2 capacity of urban environment infrastructure;
- C3 income level of the population;
- C4 industrial consumption of fuel and energy resources;
- C5 life quality of the population;
- C6 sanitary and epidemiologic situation.

Stage 2. Determining the fuzzy degrees of impact of the components of the MTS for different time delays and forming the structure of the NFCTM.

To determine the degree of mutual impact $\widetilde{w}_{ij}^{(t-l_i')}$ taking into account time delays l_i^j for NFCTM concepts, various methods of data analysis can be used, based on the establishment of interdependencies between all the components of the MTS. For example, for the example under consideration (due to the different quality of the urban environment, the expert nature of their assessment, the non-linear relationship between them and the non-stochastic uncertainty), a fuzzy extension of the multiple linear regression method has been chosen [13].

In table 1 shows the formed matrix of fuzzy relations W of impact of concept sources on concept receivers of NFCTM for solved task of multidimensional forecasting of urban environment state. For clarity, only modal values of fuzzy degrees of impact are shown.

ormed matrix of fuzzy impact relationships between NFC five concepts							
W	l_i^j	C_1	C_2	C_3	C_4	C_5	C_6
	1	0	0,75	0	0,52	0	0
C_1	2	0	0,84	0	0	0	0
	3	0	0,40	0	0,40	0	0
	1	0	0	0,79	1,0	0	0
C_2	2	0	0	0	0	0	0
	3	0	0	0	0	0,52	0,57
	1	0,55	0	0,68	0,50	0,40	0,43
C_3	2	0	1,0	0	0,46	0	0
	3	0,61	0	0	0,88	0,99	0
C_4	1	0	0,48	0,67	0,79	0	0
	2	0	0,41	0	0,43	0	0
	3	0.41	0.40	0	0.54	0.49	0

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Formed matrix of fuzzy impact relationships between NFCTM concepts

	1	0	0,68	0,62	0,42	0,45	1,00	
C_5	2	0	0,40	0	0	0,48	0	
	3	1,0	1,00	1,00	0,47	1,00	0,54	
	1	0	0	0	0	0,53	0,59	
C_6	2	0	0	0	0	0,51	0	
	3	0	0	0	0	0	0	

Formation of NFCTM structure consists in definition of structural relationships between NFCTM concepts weighted by fuzzy values $\tilde{w}_{ij}^{(t-l_i')}$. The formed structure of the NFCTM for multidimensional forecasting of the state of the urban environment of Moscow is shown in figure 1.



Figure 1. Neuro-Fuzzy Cognitive Temporal Model for multidimensional forecasting of the state of the urban environment in Moscow

As Neuro-Fuzzy Component Temporal Models FS_i , that implement fuzzy temporal transformations \tilde{F}_i , modified ANFIS models (Adaptive Neuro-Fuzzy Inference System), providing generation of predicted fuzzy values of MTS components with required time delays [9].

The input variables of the model FS_i concept C_i are related to the output variables of those concepts that have a direct impact on the concept C_i . At the same time input variables C_i are «weighted» by fuzzy degrees of impact $\widetilde{w}_{ij}^{(l-l_i)}$:

$$\tilde{s}_{j}^{(t-l_{i}^{j})} = \left(\tilde{w}_{ij}^{(t-l_{i}^{j})} \mathrm{T} \; \tilde{s}_{j}^{(t-l_{i}^{j})}\right), \; l_{i}^{j} = 0, ..., L_{i}^{j},$$
(3)

where T – operation of the t-norm (min-operation).

The output variables of the model FS_i of the concept C_i are intended to generate the predicted values of the i-th MTS component, corresponding to reasonable time delays.

To build models FS_i , both expert information about the components of the MVR and experimental data can be used. Next, we will consider a mixed version, when the model's rule base is formed by an expert, and its training is carried out on the basis of a training sample. Let's consider this particular case as an example of building the structure (and later parametric configuration) of a Neuro-Fuzzy Component Temporal Model FS_1 . The input variables of the model $FS_1 - S_1' = \left\{ \tilde{s}_3''(r^{-1}), \tilde{s}_3'(r^{-3}), \tilde{s}_4'(r^{-3}), \tilde{s}_1'(r^{-3}), \tilde{s}_1'(r^$

Here is an example of one fuzzy production rule of the model FS_1 for the concept C_1 of NFCTM:

$$If \left(\tilde{s}_{1}^{\prime(t-1)} is \tilde{L}\right) AND\left(\tilde{s}_{3}^{\prime(t-3)} is \tilde{L}\right) AND\left(\tilde{s}_{4}^{\prime(t-3)} is \tilde{M}\right) AND\left(\tilde{s}_{5}^{\prime(t-3)} is \tilde{M}\right) AND\left(\tilde{s}_{1}^{\prime(t-3)} is \tilde{H}\right),$$
(4)
Then $\left(\tilde{s}_{1}^{(t)} is \tilde{M}\right) AND\left(\tilde{s}_{1}^{(t-1)} is \tilde{M}\right) AND\left(\tilde{s}_{1}^{(t-2)} is \tilde{L}\right),$

where $\tilde{L}, \tilde{M}, \tilde{H}$ – fuzzy sets of prerequisites and conclusions of model rules FS_1 .

Figure 2 shows an example of a Neuro-Fuzzy Component Temporal Model FS₁.



Figure 2. Neuro-Fuzzy Component Temporal Model

The model FS_1 consists of the following layers of elements.

Layer 1. Layer elements are used to determine the degrees of truth for input variable values relative to the corresponding fuzzy statements of the prerequisites of all model rules. For *p*-th rule (p = 1, ..., P) models:

$$\mu_{\tilde{L}}\left(\tilde{s}_{1}^{\prime(t-1)}\right) = \tilde{s}_{1}^{\prime(t-1)} \wedge \tilde{L}, \quad \mu_{\tilde{L}}\left(\tilde{s}_{3}^{\prime(t-3)}\right) = \tilde{s}_{3}^{\prime(t-3)} \wedge \tilde{L}, \quad \mu_{\tilde{M}}\left(\tilde{s}_{4}^{\prime(t-3)}\right) = \tilde{s}_{4}^{\prime(t-3)} \wedge \tilde{M},$$

$$\mu_{\tilde{M}}\left(\tilde{s}_{5}^{\prime(t-3)}\right) = \tilde{s}_{5}^{\prime(t-3)} \wedge \tilde{M}, \quad \mu_{\tilde{H}}\left(\tilde{s}_{1}^{\prime(t-3)}\right) = \tilde{s}_{1}^{\prime(t-3)} \wedge \tilde{M}.$$
(5)

Layer 2. Layer elements aggregate the truth degrees of rule prerequisites. For p-th rule:

$$\alpha_{p} = \min\left(\mu_{\tilde{L}}\left(\tilde{s}_{1}^{\prime(t-1)}\right), \mu_{\tilde{L}}\left(\tilde{s}_{3}^{\prime(t-3)}\right), \mu_{\tilde{M}}\left(\tilde{s}_{4}^{\prime(t-3)}\right), \mu_{\tilde{M}}\left(\tilde{s}_{5}^{\prime(t-3)}\right), \mu_{\tilde{H}}\left(\tilde{s}_{1}^{\prime(t-3)}\right)\right).$$
(6)

Layer 3. Layer elements activate rule conclusions according to the degrees of truth of their prerequisites based on the implication operation (here, Mamdani implication). For the considered rule:

$$\mu_{\tilde{M}}\left(\tilde{s}_{1}^{(t)}\right) = \min\left(\alpha_{p}, \tilde{M}\right).$$
(7)

Layer 4. The layer element performs the max-disjunction operation, accumulating the activated conclusions of all the model rules:

$$\tilde{s}_{1}^{(t)} = \max\left(\mu_{\tilde{L}}\left(\tilde{s}_{1}^{(t)}\right), ..., \mu_{\tilde{M}}\left(\tilde{s}_{1}^{(t)}\right), ..., \mu_{\tilde{H}}\left(\tilde{s}_{1}^{(t)}\right)\right).$$

$$(8)$$

Layer 5. Layer elements are designed to normalize and output model output variable values with required time delays:

$$\tilde{s}_{1(norm)}^{(t)} = Z^{0} \begin{pmatrix} \tilde{s}_{1}^{(t)} \end{pmatrix}, \quad \tilde{s}_{1(norm)}^{(t-1)} = Z^{-1} \begin{pmatrix} \tilde{s}_{1}^{(t)} \end{pmatrix}, \quad \tilde{s}_{1(norm)}^{(t-2)} = Z^{-1} \begin{pmatrix} \tilde{s}_{1}^{(t-1)} \end{pmatrix}.$$
(9)

Next, we use the notation $s_i^{(t)}$ for normalized values $s_{i,norm}^{(t)}$.

Value of the output fuzzy variable $\tilde{s}_i^{(t)}$ of the model FS_i if necessary, is defuzzified using the «center of gravity» method [14]:

$$s_{i}^{(t)} = def\left(\tilde{s}_{i}^{(t)}\right) = \frac{\sum_{m=1}^{M} \left(s_{i,m}^{(t)} \cdot \mu_{\bar{s}_{i}^{(t)}}\left(s_{i,m}^{(t)}\right)\right)}{\sum_{m=1}^{M} \mu_{\bar{s}_{i}^{(t)}}\left(s_{i,m}^{(t)}\right)}, \quad M = \left|Supp\left(\tilde{s}_{i}^{(t)}\right)\right|,$$

$$\tilde{s}_{i}^{(t)} = \left\{ \left(\mu_{\bar{s}_{i}^{(t)}}\left(s_{i,m}^{(t)}\right) / s_{i,m}^{(t)}\right) | m = 1, ..., M\right\},$$
(10)

where $s_i^{(t)}$ – defuzzified value of the output variable $\tilde{s}_i^{(t)}$ of the model FS_i in timepoint *t*; $def(\tilde{s}_i^{(t)})$ – defuzzification operator using the «center of gravity» method; $S_{i,m}^{(t)}$ – *m*-th the discretized value of a variable $\tilde{s}_i^{(t)}$, m = 1, ..., M; $\mu_{\tilde{s}_i^{(t)}}(s_{i,m}^{(t)})$ – degree of the identity of the variable $\tilde{s}_i^{(t)}$ for the value $s_{i,m}^{(t)}$; $Supp(\tilde{s}_i^{(t)})$ – variable carrier $\tilde{s}_i^{(t)}$.

Set of values $\{s_i^{(t)} | i = 1, ..., N\}$ at the output of the corresponding models $\{FS_i | i = 1, ..., N\}$ comprehensively characterizes the predicted state of the urban environment at a given time *t*.

Stage 3. The coordinated training of NFCTM

For coordinated training of NFCTM, a method is proposed comprising the following two procedures:

firstly, training Neuro-Fuzzy Component Temporal Models for each NFCTM concept;

secondly, matching of Neuro-Fuzzy Component Temperature models with each other.

Training procedure for Neuro-Fuzzy Component Temporal Models FS_i is preceded by the formation of training samples:

$$\left(\left\{\left(\tilde{s}_{j}^{\prime(t-1)}(k),...,\tilde{s}_{j}^{\prime(t-L_{i}^{j})}(k)\right)|j\in 1,...,N_{i}\right\},\tilde{s}_{i}^{(t)}(k)\right),\ k=1,...,K,$$
(11)

where $\left\{ \left(\tilde{s}_{j}^{\prime(t-1)}(k), ..., \tilde{s}_{j}^{\prime(t-L_{i})}(k) \right) | j \in 1, ..., N_{i} \right\}, \tilde{s}_{i}^{(t)}(k)$ – input and output variable values in k-th example; K

- number of examples in the training sample.

For the models FS_i implementing Mamdani's inference algorithm [14], modal values and blur degrees of fuzzy sets of prerequisites and rule conclusions are configurable parameters.

Training procedure for all NFCTM models FS_i includes the following steps.

Step 1. For each example of the training selection based on the values of input variables $\left\{ \left(\tilde{s}_{j}^{\prime(t-1)}(k),...,\tilde{s}_{j}^{\prime(t-L_{i}^{j})}(k) \right) | j \in 1,...,N_{i} \right\}$ the model FS_{i} calculates the value of the output variable $\tilde{s}_{i(cur)}^{(t)}(k)$.

Step 2. For all examples of teaching sample, the error function is calculated, depending on the parameters of the model to be configured:

$$E_{i} = \sqrt{\frac{1}{K} \sum_{k=1}^{K} \left(\tilde{s}_{i}^{(t)}(k) - \tilde{s}_{i\,(cur)}^{(t)}(k) \right)^{2}}.$$
(12)

Step 3. In accordance with the learning algorithm (e.g., an error reverse propagation algorithm or a genetic algorithm), adjustments are made to the parameters to be adjusted.

Steps 1-3 are iteratively repeated, and model training is considered complete when for each of them the total error does not exceed the set threshold.

Procedure for matching all Neuro-Fuzzy Component Temporal Models FS_i , i = 1, ..., N is performed after their individual training and consists in such change of modal values and degrees of blur of fuzzy degrees of impact $\left\{\widetilde{w}_{ij}^{(t-l_i^j)} \mid l_i^j = 0, ..., L_i^j\right\}$ between concepts of NFCTM, which provides maximum increase of prediction accuracy of each component of MTS without deterioration of prediction accuracy of at least one of other components of MTS. This procedure is preceded by a teaching sample consisting of data for all TDM components:

$$\left\{ \left(\left\{ \left(\tilde{s}_{j}^{\prime(t-1)}(q), ..., \tilde{s}_{j}^{\prime\left(t-L_{i}^{j}\right)}(q) \right) \mid j \in 1, ..., N_{i} \right\}, \tilde{s}_{i}^{(t)}(q) \right) \mid i = 1, ..., N \right\}, \ q = 1, ..., Q,$$

$$(13)$$

where Q – Number of examples in this additional teaching sample.

Procedure for matching all NFCTM FS_i , i = 1, ..., N consists in the following steps.

Step 1. For each example from a matching training sample based on the values of input variables $\left\{\left\{\left(\tilde{s}_{j}^{\prime(t-1)}(q),...,\tilde{s}_{j}^{\prime(t-L_{i})}(q)\right)| j \in 1,...,N_{i}\right\} | i = 1,...,N\right\}$ all the models $FS_{i}, i = 1,...,N$ calculate the values of input variables $FS_{i}, i = 1,...,N$ calculate the values $FS_{i}, i = 1,...,N$ calculate the values of $FS_{i}, i = 1,...,N$ calculate the values $FS_{i}, i = 1,...,N$ calculate the values $FS_{i}, i = 1,...,N$ calculate the values $FS_{i}, i = 1,...,N$ calculate $FS_{i}, i = 1,...,N$ calculate the values $FS_{i}, i = 1,...,N$ calculate $FS_{i}, i = 1,...,N$

output variables $\tilde{s}_{i(cur)}^{(t)}(q), i = 1, ..., N$.

Step 2. For all sample examples for all models FS_i , i = 1, ..., N error functions that depend on configurable fuzzy impact parameters $\left\{\widetilde{w}_{ij}^{(t-l_i^j)} \mid l_i^j = 0, ..., L_i^j\right\}$ between NFCTM concepts:

$$E_{i} = \sqrt{\frac{1}{K} \sum_{q=1}^{Q} \left(\tilde{s}_{i}^{(t)}(q) - \tilde{s}_{i\,(cur)}^{(t)}(q)\right)^{2}}, \ i = 1, ..., N.$$
(14)

Step 3. According to the genetic algorithm used (e.g, [15]) According to the used genetic algorithm,

adjustment of customizable parameters of fuzzy degrees of impact is performed $\left\{\widetilde{w}_{ij}^{(t-l_i^j)} \mid l_i^j = 0, ..., L_i^j\right\}$

between NFCTM concepts thus, to ensure maximum increase in accuracy of forecasting each of the components of MTS without deterioration of prediction accuracy of at least one of the other MTS components.

Steps 1-3 are iteratively repeated, and the procedure for matching all NFCTM is considered successful if the total error for each of these models does not exceed a certain set threshold (For well-aligned MTS components), or for these models, the Ejworth-Pareto principle will be implemented, [14], which, in relation to consistent NFCTM training, is expressed in that it is impossible to maximize the prediction accuracy of any MTS component without deteriorating the prediction accuracy of at least one of the other MTS components.

Stage 4. MTS forecast based on trained NFCTM.

MTS forecasting is performed based on a trained NFCTM and consists in calculating the values of output model variables FS_i , i = 1, ..., N by the corresponding sets of values of the input variables of these models that are set each time.

Experiments were carried out and the results of using the proposed method on the example of multidimensional and forecasting the state of the urban environment in Moscow were obtained. Figure 3 illustrates the results obtained.



Figure 3. Illustration of the results of multidimensional forecasting of the state of the urban environment of Moscow based on NFCTM

Table 2 presents a comparative assessment of the results of multidimensional forecasting of the state of the urban environment in Moscow using an artificial neural network (ANN) and the developed NFCTM. As a comparison, a multilayer perceptron with a hidden layer of 24 neurons was used, which showed the best among various ANN variants.

The comparative assessment showed that the use of the proposed method based on **NFCTM** in small sample conditions allows to increase the accuracy of the forecast of MTS by an average of 10-15% compared to the best-performing ANN.

Table 2

Comparative evaluation of the multidimensional forecasting results

No	MTS components	Forecasting error, MAPE, %		
INU	W15 components	ANN	NFCTM	
1.	Ecology of the urban environment	7,40	6,91	
2.	The infrastructure power of the urban environment	1,51	0,13	
3.	Income level of the population	8,72	9,85	
4.	Industrial consumption of fuel and energy resources	2,35	1,62	
5.	Population life quality	2,12	0,55	
6.	Sanitary and epidemiological situation	5,35	5,31	

The article describes Neuro-Fuzzy Cognitive Temporal Models focused on predicting multidimensional time series and providing for the direct and indirect mutual impact of all components of the MTS with their time delays relative to each other under conditions of uncertainty.

To implement fuzzy temporal transformations of NFCTM concepts, Neuro-Fuzzy Component Temporal Models are used, which are modified ANFIS-type models, and provide the formation of predicted values of MTS components with the required time delays.

The proposed method of consistent training of NFCTM is described, which consists, firstly, in training Neuro-Fuzzy Component Temporal Models for each concept of NFCTM, and secondly, in the coordination of these NFCTM.

A method for MTS predicting under conditions of uncertainty, non-linearity of mutual impact, partial inconsistency and interdependence of MTS components, based on NFCTM, has been developed.

Experimental studies are conducted and the results of using the proposed method are presented on the example of the problem of multidimensional forecasting of the state of the urban environment in Moscow. A comparative assessment showed that using this method based on NFCTM in small sample conditions allows to improve the accuracy of the MTS forecast by an average of 10-15% compared to the best ANN results.

The use of the proposed method may also be in demand to ensure reliable forecasting of the state of the urban environment in different regions of Russia and other countries, including taking into account the difficult epidemiological situation.

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