Cyber Object State Maximal Probability Timing Obtained Through Multi-Optional Technique

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Abstract. In this publication a Doctrine for the Conditional Extremization of the Hybrid-Optional Effectiveness Functions Entropy is discussed as a tool for the Cyber Object State Maximal Probability Assessments. Traditionally, most of the problems having been dealt with in this area must relate with the probabilistic problem settings. Regularly, the optimal solutions are obtained through the probability extremizations. It is shown a possibility of the optimal solutions "derivation", with the help of a model implementing a variational principle which takes into account objectively existing parameters and components of the Markovian process. The presence of an extremum of the objective state probability is observed and determined on the basis of the proposed Doctrine with taking into account the measure of uncertainty of the hybrid-optional effectiveness functions in the view of their entropy. Such approach resembles the well known Jaynes' Entropy Maximum Principle from theoretical statistical physics adopted in subjective analysis of active systems as the subjective entropy maximum principle postulating the subjective entropy conditional optimization. The developed herewith Doctrine implies objective characteristics of the process rather than subjective individual's preferences or choices, as well as the states probabilities maximums are being found without solving a system of ordinary linear differential equations of the first order by Erlang corresponding to the graph of the process.

Keywords: cyber hygiene, conflict management, global information networks, effectiveness functions entropy, hybrid-optional effectiveness, multi-optionality, optimal distribution, variational principle, entropy maximum principle.

1 Introduction

1.1 Literature Survey

Cyber hygiene and conflict management in global information networks can be considered from the point of view of the theoretical developments for reliability [1].

The analogy of the hygiene to the maintenance procedures is very good. Therefore, the apparatus of theoretical physics related with the uncertainty measures [2-4] is quite applicable here. Thus, in the field of the Social Networking Services it is critical to take into considerations subjective entropy of preferences [5, 6]. The similar to the aircraft maintenance and repair approaches [7], combinations with the rationality of the choice behavior [8], [9], inspire global science and social science entropy research [10]. In addition, economic issues [11] in respect of risk [12], like in aviation [13], are complicated with the group decision making [14, 15].

All this initiated search for a new explanation of the described process. The presented doctrine, like developed in [16-30], is to demonstrate the possibilities of the entropy paradigm use to the variety of the problems solutions, for example discussed in works [31-39]. Mathematical means intended to be used are of the regular calculus [40]. Also, adjacent and similar formalism scientific areas, let us say mentioned in publications of [41-49], can implement the presented doctrine results.

1.2 The Problem Statement

Management of cyber incidents, warfare and conflicts are considered in terms of the mass service theory [37-39].

The considered cyber object (space) can change its states. Illustration of that is in the simplified graph (see Fig. 1).

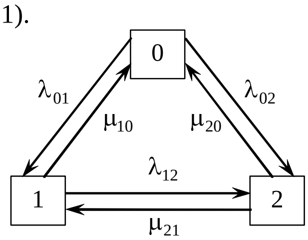

Fig. 1. A graph of three states of a cyber space.

Here, in Fig. 1, “0”, “1”, and “2” designate the states of the cyber object. The corresponding values of the rates λ_{ij} and μ_{ji} will determine the process going on in the system.

The problem is to find the timing for the maximal values of the states probabilities, for instance of $P_1(t)$, analytically and in an easier than the traditional way. The proposed is the multi-optional way.

2 Main Content

2.1 Traditional Methods

Even for the simplified (partial to Fig. 1) case, although implying the possible return of the system from the state “D” into the state of “A” without the transition into the state “F” (this transition is carried out with the parameter of μ_1 illustrated on the graph, see Fig. 2) the procedure is quite challenging analytically.

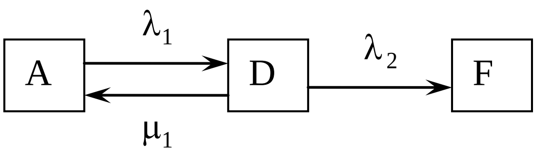

Fig. 2. A simplified graph of three states of a cyber space.

The corresponding, to the graph of Fig. 2, system of differential equations by Erlang will have the view of

$$\left.\begin{aligned} \frac{dP_A}{dt} &= -\lambda_1 P_A + \mu_1 P_D; \\ \frac{dP_D}{dt} &= \lambda_1 P_A - (\lambda_2 + \mu_1) P_D; \\ \frac{dP_F}{dt} &= \lambda_2 P_D. \end{aligned}\right\} \tag{1}$$

The characteristic equation for system (1) will be similarly [40]:

$$\begin{vmatrix} -\lambda_1 - k & \mu_1 & 0 \\ \lambda_1 & -(\lambda_2 + \mu_1) - k & 0 \\ 0 & \lambda_2 & 0 - k \end{vmatrix} = 0 . \tag{2}$$

Determinant (2) yields

$$(-\lambda_1 - k)[-(\lambda_2 + \mu_1) - k](0 - k) + \lambda_1 \lambda_2 \cdot 0 + \mu_1 \cdot 0 \cdot 0 -$$

$$-[-(\lambda_2 + \mu_1) - k] \cdot 0 \cdot 0 - \lambda_1 \mu_1 (0 - k) - (-\lambda_1 - k)\lambda_2 \cdot 0 = 0 . \tag{3}$$

Which means

$$-(\lambda_1 + k)[\lambda_2 + \mu_1 + k]k + \lambda_1 \mu_1 k = 0 . \tag{4}$$

$$k[\lambda_1 \mu_1 - (\lambda_1 + k)(\lambda_2 + \mu_1 + k)] = 0 . \tag{5}$$

Thus, we have already known at least one root:

$$k_1 = 0 . \tag{6}$$

Then, for finding two other roots from Eq. (5)

$$\lambda_1 \mu_1 - \lambda_1 \lambda_2 - \lambda_1 \mu_1 - \lambda_1 k - k \lambda_2 - k \mu_1 - k^2 = 0 . \tag{7}$$

Reducing Eq. (7) and cancelling the similar members $\lambda_1 \mu_1$ and $-\lambda_1 \mu_1$,

$$-k^2 - k(\lambda_1 + \lambda_2 + \mu_1) - \lambda_1\lambda_2 = 0 . \qquad (8)$$

The sought roots are

$$k_2 = \frac{-b + \sqrt{b^2 - 4ac}}{2a} ; \qquad k_3 = \frac{-b - \sqrt{b^2 - 4ac}}{2a} ; \qquad (9)$$

where $a = -1$, $b = -(\lambda_1 + \lambda_2 + \mu_1)$, $c = -\lambda_1\lambda_2$ are corresponding coefficients of (8).

For each root k_i of Eq. (2)-(5), (7), (8), namely k_1, k_2, k_3 Eq. (6) and (9) we will write down the system of linear algebraic equations for $\alpha_1^{(i)}$, $\alpha_2^{(i)}$, $\alpha_3^{(i)}$ [40]:

$$\left.\begin{array}{lll} (-\lambda_1 - k)\alpha_1 & +\mu_1\alpha_2 & +0\cdot\alpha_3 = 0; \\ \lambda_1\alpha_1 & +[-(\lambda_2 + \mu_1) - k]\alpha_2 & +0\cdot\alpha_3 = 0; \\ 0\cdot\alpha_1 & +\lambda_2\alpha_2 & +(0-k)\alpha_3 = 0. \end{array}\right\} \qquad (10)$$

The system of Eq. (10) derives from an assumption of a partial solution existence

$$P_A = \alpha_1 e^{kt} ; \qquad P_D = \alpha_2 e^{kt} ; \qquad P_F = \alpha_3 e^{kt} ; \qquad (11)$$

for the system of Eq. (1).

Since having three roots in the stated problem setting [40], the solution of (1):

$$P_A^{(1)} = \alpha_1^{(1)} e^{k_1 t} ; \qquad P_D^{(1)} = \alpha_2^{(1)} e^{k_1 t} ; \qquad P_F^{(1)} = \alpha_3^{(1)} e^{k_1 t} ; \qquad (12)$$

$$P_A^{(2)} = \alpha_1^{(2)} e^{k_2 t} ; \qquad P_D^{(2)} = \alpha_2^{(2)} e^{k_2 t} ; \qquad P_F^{(2)} = \alpha_3^{(2)} e^{k_2 t} ; \qquad (13)$$

$$P_A^{(3)} = \alpha_1^{(3)} e^{k_3 t} ; \qquad P_D^{(3)} = \alpha_2^{(3)} e^{k_3 t} ; \qquad P_F^{(3)} = \alpha_3^{(3)} e^{k_3 t} . \qquad (14)$$

In the way of direct substitution of partial solutions (12)-(14) into equations, one can be convinced that the system of functions, similarly to [40]:

$$\left.\begin{array}{l} P_A = C_1 P_A^{(1)} + C_2 P_A^{(2)} + C_3 P_A^{(3)} = C_1\alpha_1^{(1)} e^{k_1 t} + C_2\alpha_1^{(2)} e^{k_2 t} + C_3\alpha_1^{(3)} e^{k_3 t}; \\ P_D = C_1 P_D^{(1)} + C_2 P_D^{(2)} + C_3 P_D^{(3)} = C_1\alpha_2^{(1)} e^{k_1 t} + C_2\alpha_2^{(2)} e^{k_2 t} + C_3\alpha_2^{(3)} e^{k_3 t}; \\ P_F = C_1 P_F^{(1)} + C_2 P_F^{(2)} + C_3 P_F^{(3)} = C_1\alpha_3^{(1)} e^{k_1 t} + C_2\alpha_3^{(2)} e^{k_2 t} + C_3\alpha_3^{(3)} e^{k_3 t}; \end{array}\right\} \qquad (15)$$

where C_1; C_2; C_3 are arbitrary constants; also is the solution of the differential equations system (1). This is the general solution of the differential equations system (1), [40].

Satisfying the condition of Eq. (6) for root $k_1 = 0$ from the system of Eq. (10)

$$\left.\begin{array}{lll}\left(-\lambda_1-k_1\right)\alpha_1^{(1)} & +\mu_1\alpha_2^{(1)} & +0\cdot\alpha_3^{(1)}=0;\\ \lambda_1\alpha_1^{(1)} & +\left[-\left(\lambda_2+\mu_1\right)-k_1\right]\alpha_2^{(1)} & +0\cdot\alpha_3^{(1)}=0;\\ 0\cdot\alpha_1^{(1)} & +\lambda_2\alpha_2^{(1)} & +\left(0-k_1\right)\alpha_3^{(1)}=0.\end{array}\right\} \qquad (16)$$

$$\left.\begin{array}{lll}-\lambda_1\alpha_1^{(1)} & +\mu_1\alpha_2^{(1)} & +0\cdot\alpha_3^{(1)}=0;\\ \lambda_1\alpha_1^{(1)} & -\left(\lambda_2+\mu_1\right)\alpha_2^{(1)} & +0\cdot\alpha_3^{(1)}=0;\\ 0 & +\lambda_2\alpha_2^{(1)} & -0\cdot\alpha_3^{(1)}=0.\end{array}\right\} \qquad (17)$$

From where, immediately the coefficients are

$$\alpha_2^{(1)}=0; \qquad \alpha_1^{(1)}=0; \qquad \alpha_3^{(1)}=1; \qquad (18)$$

since $\alpha_3^{(1)}$ is an arbitrary number, supposedly $\alpha_3^{(1)}=1$, [40].

For the Eq. (8) roots of k_2 and k_3, Eq. (9), the system of Eq. (10) analogous to the system of Eq. (16) it yields

$$\left.\begin{array}{lll}\left(-\lambda_1-k_{2,3}\right)\alpha_1^{(2,3)} & +\mu_1\alpha_2^{(2,3)} & +0\cdot\alpha_3^{(2,3)}=0;\\ \lambda_1\alpha_1^{(2,3)} & +\left[-\left(\lambda_2+\mu_1\right)-k_{2,3}\right]\alpha_2^{(2,3)} & +0\cdot\alpha_3^{(2,3)}=0;\\ 0\cdot\alpha_1^{(2,3)} & +\lambda_2\alpha_2^{(2,3)} & +\left(0-k_{2,3}\right)\alpha_3^{(2,3)}=0.\end{array}\right\} \qquad (19)$$

The system of Eq. (19) can be solved for unknown sought coefficients.

Since one of the alpha coefficients can be chosen arbitrary, [40], let us assume

$$\alpha_2^{(2,3)}=1. \qquad (20)$$

Then, from the first equation of system (19)

$$\left(-\lambda_1-k_{2,3}\right)\alpha_1^{(2,3)}+\mu_1=0; \qquad \alpha_1^{(2,3)}=\frac{\mu_1}{\lambda_1+k_{2,3}}. \qquad (21)$$

Or from the second equation

$$\lambda_1\alpha_1^{(2,3)}-\left(\lambda_2+\mu_1\right)-k_{2,3}=0; \qquad \alpha_1^{(2,3)}=\frac{\lambda_2+\mu_1+k_{2,3}}{\lambda_1}. \qquad (22)$$

Or summing the first and second equations

$$-k_{2,3}\alpha_1^{(2,3)}-\left[\lambda_2+k_{2,3}\right]\alpha_2^{(2,3)}=0; \qquad \alpha_1^{(2,3)}=-\frac{\lambda_2+k_{2,3}}{k_{2,3}}. \qquad (23)$$

All three expressions for $\alpha_1^{(2,3)}$, i.e. Eq. (21)-(23) are equivalent because all of them use the roots k_2 and k_3, Eq. (9) of the initial quadratic equation Eq. (8).

Indeed. Equalizing Eq. (21) and (22) we get

$$\lambda_1\mu_1 = \lambda_1\lambda_2 + \lambda_1\mu_1 + \lambda_1 k_{2,3} + \lambda_2 k_{2,3} + \mu_1 k_{2,3} + k_{2,3}^2 . \tag{24}$$

And cancelling for $\lambda_1\mu_1$ in both parts of Eq. (24) it yields Eq. (8):

$$\lambda_1\lambda_2 + (\lambda_1 + \lambda_2 + \mu_1)k_{2,3} + k_{2,3}^2 = 0 . \tag{25}$$

The same result is obtained if make equal Eq. (21) and (23):

$$\mu_1 k_{2,3} = -\lambda_1\lambda_2 - \lambda_1 k_{2,3} - \lambda_2 k_{2,3} - k_{2,3}^2 ; \qquad k_{2,3}^2 + (\lambda_1 + \lambda_2 + \mu_1)k_{2,3} + \lambda_1\lambda_2 = 0 . \tag{26}$$

When equalling Eq. (22) and (23) it gives the same. Indeed:

$$(\lambda_2 + \mu_1)k_{2,3} + k_{2,3}^2 = -\lambda_1\lambda_2 - \lambda_1 k_{2,3} ; \qquad k_{2,3}^2 + (\lambda_1 + \lambda_2 + \mu_1)k_{2,3} + \lambda_1\lambda_2 = 0 . \tag{27}$$

For coefficient $\alpha_3^{(2,3)}$, from the third equation of Eq. (19) and condition (20),

$$\lambda_2\alpha_2^{(2,3)} - k_{2,3}\alpha_3^{(2,3)} = 0 ; \qquad \alpha_3^{(2,3)} = \frac{\lambda_2}{k_{2,3}} . \tag{28}$$

Thus, turning back to the system of Eq. (15), we determine the unknown coefficients of the general solution of the differential equations system (1), [40], satisfying the initial conditions: $t_0 = 0$; $P_A\big|_{t=t_0} = 1$; $P_D\big|_{t=t_0} = 0$; $P_F\big|_{t=t_0} = 0$; and have already known the coefficients of alpha; i.e. Eq. (18); (20); (21); (28):

$$\left.\begin{aligned} P_A &= C_1\alpha_1^{(1)}e^{k_1 t} + C_2\alpha_1^{(2)}e^{k_2 t} + C_3\alpha_1^{(3)}e^{k_3 t};\\ P_D &= C_1\alpha_2^{(1)}e^{k_1 t} + C_2\alpha_2^{(2)}e^{k_2 t} + C_3\alpha_2^{(3)}e^{k_3 t};\\ P_F &= C_1\alpha_3^{(1)}e^{k_1 t} + C_2\alpha_3^{(2)}e^{k_2 t} + C_3\alpha_3^{(3)}e^{k_3 t}; \end{aligned}\right\}\Bigg|_{t_0=0} = \left\{\begin{aligned} &1 = 0 + C_2\frac{\mu_1}{\lambda_1 + k_2} + C_3\frac{\mu_1}{\lambda_1 + k_3};\\ &0 = 0 + C_2 + C_3;\\ &0 = C_1 + C_2\frac{\lambda_2}{k_2} + C_3\frac{\lambda_2}{k_3}. \end{aligned}\right\} \tag{29}$$

From the second equation of the system of Eq. (29) it yields

$$C_2 = -C_3 . \tag{30}$$

Substituting the values of Eq. (30) for the corresponding members into the first equation of the system of Eq. (29) we get

$$1 = C_3\left(\frac{\mu_1}{\lambda_1 + k_3} - \frac{\mu_1}{\lambda_1 + k_2}\right); \qquad C_3 = \frac{1}{\dfrac{\mu_1}{\lambda_1 + k_3} - \dfrac{\mu_1}{\lambda_1 + k_2}} . \tag{31}$$

In order to make the notations shorter let us put down the indications with the alpha symbolizations:

$$1 = -C_3\alpha_1^{(2)} + C_3\alpha_1^{(3)} = C_3\left[\alpha_1^{(3)} - \alpha_1^{(2)}\right]; \qquad C_3 = \frac{1}{\alpha_1^{(3)} - \alpha_1^{(2)}}. \tag{32}$$

$$C_2 = -\frac{1}{\alpha_1^{(3)} - \alpha_1^{(2)}}. \tag{33}$$

From the third equation of the system of Eq. (29) we obtain

$$C_1 = -C_2\alpha_3^{(2)} - C_3\alpha_3^{(3)}. \tag{34}$$

Now, all coefficients are expressed through the given values, hence, the system of Eq. (1) is successfully solved. The Laplace integral transformation methods give the same results. For the general case described with the graph shown in Figure 1

$$P_0(t) = \frac{k_1 e^{k_1 t} - k_2 e^{k_2 t}}{k_1 - k_2} + a_1 \frac{e^{k_1 t} - e^{k_2 t}}{k_1 - k_2} +$$

$$+ \frac{b_1}{k_1 k_2} + \left(-\frac{b_1}{k_2(k_2 - k_1)} - \frac{b_1}{k_1 k_2}\right) e^{k_1 t} + \left(\frac{b_1}{k_2(k_2 - k_1)}\right) e^{k_2 t}. \tag{35}$$

$$P_1(t) = \lambda_{01} \frac{e^{k_1 t} - e^{k_2 t}}{k_1 - k_2} + \frac{c_1}{k_1 k_2} + \left(-\frac{c_1}{k_2(k_2 - k_1)} - \frac{c_1}{k_1 k_2}\right) e^{k_1 t} + \left(\frac{c_1}{k_2(k_2 - k_1)}\right) e^{k_2 t}. \tag{36}$$

$$P_2(t) = \lambda_{02} \frac{e^{k_1 t} - e^{k_2 t}}{k_1 - k_2} + \frac{d_1}{k_1 k_2} + \left(-\frac{d_1}{k_2(k_2 - k_1)} - \frac{d_1}{k_1 k_2}\right) e^{k_1 t} + \left(\frac{d_1}{k_2(k_2 - k_1)}\right) e^{k_2 t}. \tag{37}$$

The values of the parameters in (35) - (37) have the mathematical expressions corresponding to the general case (see Fig. 1). Then, it has to be found the possible extreme values of the probabilities. For distinctness, let it be $P_1(t)$.

$$\frac{dP_1(t)}{dt} = \frac{\lambda_{01}}{k_1 - k_2}\left(k_1 e^{k_1 t} - k_2 e^{k_2 t}\right) + k_1\left(-\frac{c_1}{k_2(k_2 - k_1)} - \frac{c_1}{k_1 k_2}\right) e^{k_1 t} + k_2\left(\frac{c_1}{k_2(k_2 - k_1)}\right) e^{k_2 t}. \tag{38}$$

After equalizing (38) to zero, the needed timing is

$$t_p^* = \frac{\ln(\lambda_{01} k_1 + c_1) - \ln(\lambda_{01} k_2 + c_1)}{k_2 - k_1}. \tag{39}$$

2.2 The Proposed Approach

Herein it is suggested to formulate the own concept (idea, problem, hypotheses).

In such respect [1-40], the considered example may be given an attention to in regards with the Multi-Optional Hybrid-Effectiveness Functions Uncertainty Measure Conditional Optimization Doctrine (method, approach, concept) applicable (used, implemented) to the cyber object state maximal probability timing determination [17, 20, 22, 25].

The values can be obtained not only in the entire probabilistic way, but also in a hybrid partially probabilistic partially optional way [17, 20, 22, 25].

The essence of the doctrine (method, idea, approach, concept) is to consider the process developing in the system from the position of some hybrid optional functions distribution optimality.

Consider the options essential to the general view three state system (see Fig. 1).

Objective functional, like proposed in references [17, 20, 22, 25], is as follows:

$$\Phi_h = -\sum_{i=1}^{3}\left[xF_1^{(i)}\right]\ln\left[xF_1^{(i)}\right] - \frac{t_p^*}{\lambda_{01}}\sum_{i=1}^{3}\left[xF_1^{(i)}\right]\left(M_{12}^{(i)}\right) + \gamma\left[\sum_{i=1}^{3}\left[xF_1^{(i)}\right] - 1\right],$$

$$F_1^{(i)} = \frac{M_{12}^{(i)}}{\Delta(\mathbf{M})} = \frac{k_i\lambda_{01} + c_1}{p\left(p^2 + pe_1 + b_1 + c_1 + d_1\right)},$$

$$M_{12}^{(i)} = k_i\lambda_{01} + c_1, \qquad \Delta(\mathbf{M}) = p\left(p^2 + pe_1 + b_1 + c_1 + d_1\right), \tag{40}$$

where x is an unknown parameter; $h_i = xF_1^{(i)}$ is the multi-optional hybrid functions depending upon the options effectiveness functions of $F_1^{(i)}$; t_p^*/λ_{01} is the intrinsic parameter of the system and the process, which is the ratio of the timing (delivering the sought maximal value to the probability) t_p^*, it is unknown yet for such problem formulation and the time of t_p^* is going to be determined as a solution, i.e. it is not the Eq. (39) so far, however it will be, that is why the indication is the same, to the flow intensity λ_{01}; $M_{12}^{(i)}$ is the algebraic addition of the initial elementary intensities matrix $\mathbf{M}$ formed in the style likewise from the Erlang's system, Eq. (1), element of m_{12}; γ is the parameter, coefficient, function (uncertain Lagrange multiplier, weight coefficient) for the normalizing condition.

Consider an extremum existence necessary conditions for the objective functional of (40), [17, 20, 22, 25]:

$$\frac{\partial\Phi_h}{\partial h_i} = \frac{\partial\Phi_h}{\partial\left[xF_1^{(i)}\right]} = 0, \qquad \forall i \in \overline{1,3}. \tag{41}$$

$$\ln\left[xF_1^{(1)}\right] + \frac{t_p^*}{\lambda_{01}}\left(\lambda_{01}k_1 + c_1\right) = \gamma - 1 = \ln\left[xF_1^{(2)}\right] + \frac{t_p^*}{\lambda_{01}}\left(\lambda_{01}k_2 + c_1\right). \tag{42}$$

From where

$$\ln\left[xF_1^{(1)}\right]+\frac{t_p^*}{\lambda_{01}}\left(\lambda_{01}k_1+c_1\right)=\ln\left[xF_1^{(2)}\right]+\frac{t_p^*}{\lambda_{01}}\left(\lambda_{01}k_2+c_1\right). \tag{43}$$

After that, we have got the *law of subjective conservatism* on one hand and on the other hand

$$\ln\left[xF_1^{(1)}\right]-\ln\left[xF_1^{(2)}\right]=\frac{t_p^*}{\lambda_{01}}\left[\left(\lambda_{01}k_2+c_1\right)-\left(\lambda_{01}k_1+c_1\right)\right]. \tag{44}$$

$$\ln\left[xF_1^{(1)}\right]-\ln\left[xF_1^{(2)}\right]=t_p^*\left[\left(k_2+\frac{c_1}{\lambda_{01}}\right)-\left(k_1+\frac{c_1}{\lambda_{01}}\right)\right]. \tag{45}$$

After that likewise Eq. (39)

$$t_p^*=\frac{\ln\left[F_1^{(1)}(\cdot)\right]-\ln\left[F_1^{(2)}(\cdot)\right]}{k_2(\cdot)-k_1(\cdot)}\ . \tag{46}$$

And finally equivalent with Eq. (39) with taking into account the roots, i.e. the second, third, and fourth expressions of the Eq. (40)

$$t_p^*=\frac{\ln\dfrac{k_1\lambda_{01}+c_1}{p\left(p^2+pe_1+b_1+c_1+d_1\right)}-\ln\dfrac{k_2\lambda_{01}+c_1}{p\left(p^2+pe_1+b_1+c_1+d_1\right)}}{k_2(\cdot)-k_1(\cdot)}\ . \tag{47}$$

$$t_p^*=\frac{\ln\left(k_1\lambda_{01}+c_1\right)-\ln\left(k_2\lambda_{01}+c_1\right)}{k_2(\cdot)-k_1(\cdot)}. \tag{48}$$

3 Discussion

Thus, the result of Eq. (39) is obtained in absolutely not probabilistic rather in the Multi-Optional Hybrid-Effectiveness Functions Uncertainty Measure Conditional Optimization Doctrine way [17, 20, 22, 25].

The same approach is applicable to $F_2^{(i)}$ with yielding the parallel to the Eq. (39) and (48) results.

Now we ought to say that for the situation when the probability of $P_2(t)$ undergoes the extremum instead of the probability of $P_1(t)$, the problem, **due to the symmetry**, has a symmetrical solution:

$$t_p^*=\frac{\ln\left(\lambda_{02}k_1+d_1\right)-\ln\left(\lambda_{02}k_2+d_1\right)}{k_2-k_1}. \tag{49}$$

4 Conclusions

That is the system according to the developing stationary Poison flow process has the possible states optimal options related with either the system of parameters

$$\{k_i, \lambda_{02}, d_1\} \text{ or } \{k_i, \lambda_{01}, c_1\} \tag{50}$$

values for the initial moment probability of the state "0" being equaled to "1". The proposed optional method is more compact and applicable for a cyber object state maximal probability timing determination.

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