Superpixel-Based Filtering for Image Noise Reduction

Anna Egorova
Samara National Research University
Samara, Russia
2358anna@gmail.com

***Abstract*—The paper presents a superpixel-based image filtering algorithm for additive white Gaussian noise (AWGN) reduction. The algorithm processes an image by connected homogeneous regions of small size (superpixels). Each superpixel is restored using the least squares method. The mean square error (MSE) between a reconstructed image and an ideal image provided by the proposed algorithm is compared with the MSE provided by the Wiener filter. The experimental part shows that the proposed superpixel filtering algorithm outperforms the Wiener filter, providing lower MSE values.**

***Keywords*—additive white Gaussian noise, filtering, least squares method, mean square error, noise reduction, superpixel, Wiener filter**

I. Introduction

Various random noises are introduced in images at the forming and transmitting stages [1]. Noises decrease the visual quality of images and negatively affect the result of image processing and analysis. Thus, the problem of image noise reduction is important today.

In practice, the most widespread is additive white noise [2]. Most of existing image filtering algorithms are aimed at reducing noise having a Gaussian distribution since such a model well approximates many noises. The most popular algorithm for reducing white Gaussian noise (AWGN) in images is the Wiener filtering. It's the optimal linear processing technique for minimizing, in the statistical sense, the mean square error (MSE) between a restored image and an ideal image. It efficiently removes AWGN, but the degree of blurring of restored images can exceed the values allowed by the task [2].

In this paper, an algorithm for image AWGN filtering by superpixels – perceptually meaningful connected disjoint regions [3] is proposed. It has several advantages over pixel-based noise reduction algorithms. First, it processes images by objects or their parts, since no superpixel should include pixels of more than one object [4], whereas pixel-based algorithms often process images by "sliding window", which may consist of pixels belonging to various objects with different characteristics. Secondly, the number of superpixels of the image is much less than the number of pixels. Consequently, the computational complexity of the noise filtering task is reduced.

II. Superpixel Algorithm

For obtaining a superpixel representation of an image, the threshold region detection algorithm [5] is used. The algorithm in the order of progressive scanning divides the image into spatially connected disjoint homogeneous in intensity areas (superpixels) in such a way that the spread of pixel intensity values inside each of them is within the range of 2ε , where ε is the input parameter of the algorithm that is further designated as "superpixel threshold". This algorithm is chosen due to low computational complexity and ease of setup (one input parameter) compared to the popular graph superpixel segmentation algorithms [6-8] and the clustering algorithms [4, 9, 10].

III. The Proposed Superpixel-Wise Image Noise Filtering Algorithm

Let $x_0(n_1,n_2)$ be an original image and $v(n_1,n_2)$ be a random noise (AWGN). Then an observed image $x(n_1,n_2)$ is modeled as $x(n_1,n_2) = x_0(n_1,n_2) + v(n_1,n_2)$, where $n_1 = 1,..,N_1$, $n_2 = 1,..,N_2$, and $N_1 \times N_2$ is size of the original image. Let a partition of the observed image $x(n_1,n_2)$ into superpixels is given. Denote $D = \{D_m\}_{m=1,..,M}$ a set of all superpixels, where M is the total number of superpixels of the image $x(n_1,n_2)$.

The task of image reconstruction is to design a filter that takes as input the observed image $x(n_1,n_2)$ and outputs an estimate $x(n_1,n_2)$ that is close to the original image $x_0(n_1,n_2)$ [1]. The proposed algorithm filters the image superpixel-wise and finds for each superpixel a linear combination of some functions f_i, $i = 1,..,I$, where I is the number of functions:

$$x(n_1,n_2) = \sum_{i=0}^{I-1} a_i f_i(n_1,n_2), \quad (n_1,n_2) \in D_m , \tag{1}$$

$\{a_i\}$ are the expansion coefficients.

Then it uses the least squares method [11] to reconstruct each superpixel:

$$S = \sum_{(n_1,n_2)\in D_m} [x(n_1,n_2) - x(n_1,n_2)]^2 \to \min_{\{a_i\}} . \tag{2}$$

To find the expansion coefficients $\{a_i\}$ at which minimum of (2) is achieved, equate the partial derivatives taken of (1) to zero, differentiate and obtain the following system of linear equations:

$$\begin{gathered}\sum_{i=0}^{I-1} a_i \sum_{(n_1,n_2)\in D_m} f_i(n_1,n_2) f_j(n_1,n_2) = \\ = \sum_{(n_1,n_2)\in D_m} x(n_1,n_2) f_j(n_1,n_2), \quad 0 \le j \le I-1\end{gathered} . \tag{3}$$

In matrix form, the system (3) can be written as follows:

$$\boldsymbol{BA} = \boldsymbol{C} , \tag{4}$$

where $\boldsymbol{B} = \{b_{ij}\}_{i,j=0}^{I-1} \left\{ \sum_{(n_1,n_2)\in D_m} f_i(n_1,n_2) f_j(n_1,n_2) \right\}_{i,j=0}^{I-1}$ is a symmetric matrix, $\boldsymbol{A} = \{a_i\}_{i=0}^{I-1}$ and $\boldsymbol{C} = \{c_i\}_{i=0}^{I-1} \left\{ \sum_{(n_1,n_2)\in D_m} x(n_1,n_2) f_i(n_1,n_2) \right\}_{i=0}^{I-1}$ are column-vectors of the sought coefficients and absolute terms of the system, respectively.

Let consider polynomials as expansion functions.

- If the degree of polynomials $I = 1$, the proposed superpixel-based image filtering represents intensity averaging operation inside each superpixel:

$$f_0(n_1,n_2) = 1,$$

$$b_{00} = \sum_{(n_1,n_2)\in D_m} 1,$$

$$c_0 = \sum_{(n_1,n_2)\in D_m} x(n_1,n_2),$$

$$a_0 = \frac{\sum_{(n_1,n_2)\in D_m} x(n_1,n_2)}{\sum_{(n_1,n_2)\in D_m} 1}.$$

- If the degree of polynomials $I = 3$, the proposed algorithm solves the system of linear equations to find the coefficients $\{a_i\}$:

$$f_0(n_1,n_2) = 1,$$

$$f_1(n_1,n_2) = n_1,$$

$$f_2(n_1,n_2) = n_2,$$

$$\begin{pmatrix} \sum_{(n_1,n_2)\in D_m} 1 & \sum_{(n_1,n_2)\in D_m} n_1 & \sum_{(n_1,n_2)\in D_m} n_2 \\ \sum_{(n_1,n_2)\in D_m} n_1 & \sum_{(n_1,n_2)\in D_m} n_1^2 & \sum_{(n_1,n_2)\in D_m} n_1 n_2 \\ \sum_{(n_1,n_2)\in D_m} n_2 & \sum_{(n_1,n_2)\in D_m} n_1 n_2 & \sum_{(n_1,n_2)\in D_m} n_2^2 \end{pmatrix} \begin{pmatrix} a_0 \\ a_1 \\ a_2 \end{pmatrix} =$$

$$= \begin{pmatrix} \sum_{(n_1,n_2)\in D_m} x(n_1,n_2) \\ \sum_{(n_1,n_2)\in D_m} x(n_1,n_2)\, n_1 \\ \sum_{(n_1,n_2)\in D_m} x(n_1,n_2)\, n_2 \end{pmatrix}.$$

IV. Experimental Research

For experimental research, piecewise-constant images of size 512×512 were generated. Such images represent a set of regions with random intensity values formed by dividing the plane by random lines [12]. The experiments were carried out on three sets of synthesized data, each of which included images with a fixed value of the correlation coefficient between neighboring pixels ρ : 0.90, 0.95, and 0.99. An example of generated piecewise-constant images is shown in Fig. 1.

The source images were noised by putting into them AWGN with zero mean. Further, the signal-to-noise ratio (SNR) is denoted as $d = D_x / D_v$, where D_x is the variance of the original image and D_v is the noise variance. The following d values were considered: 10 dB, 15 dB, 20 dB, 30 dB, 50 dB, 100 dB, 200 dB, 500 dB, and 1000 dB. For each pair of values (ρ, d) 10 images were generated.

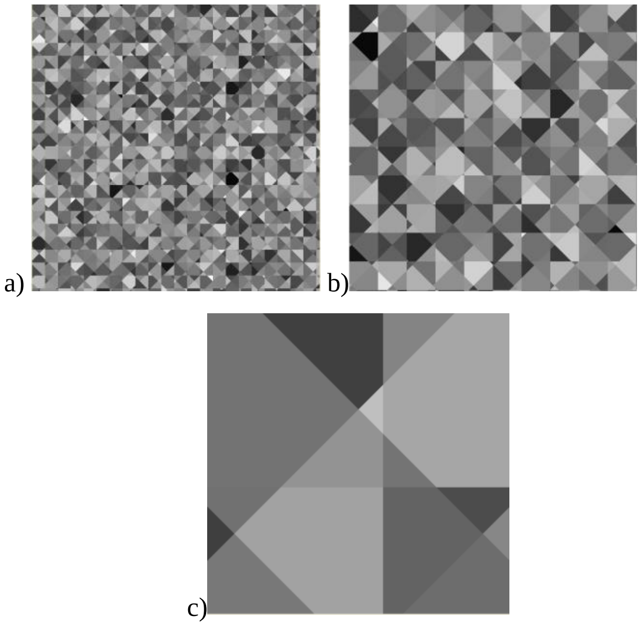

Fig. 1. Example of generated piecewise-constant images: a) $\rho = 0.90$, b) $\rho = 0.95$, c) $\rho = 0.99$.

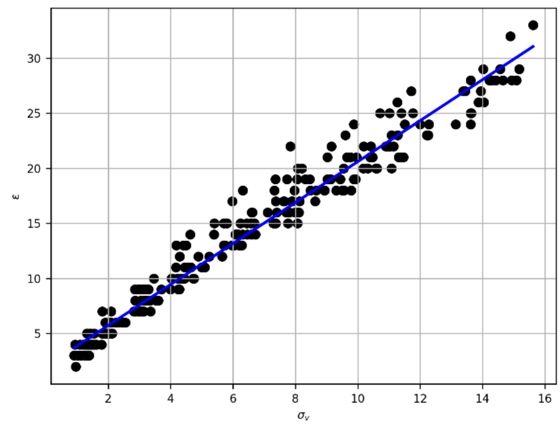

Fig. 2. The dependence of superpixel threshold values ε minimizing MSE between the reconstructed image and the ideal image on noise standard deviation σ_v.

First of all, the effectiveness of the proposed filtering algorithm was tested. To automate the stage of searching for the superpixel threshold values minimizing MSE the dependence of superpixel threshold values on noise standard deviation $\varepsilon(\sigma_v)$ was investigated. Note the MSE σ between a reconstructed image and an ideal image was calculated as follows:

$$\sigma = \left(\frac{1}{N_1 N_2} \sum_{n_1=1}^{N_1} \sum_{n_2=1}^{N_2} \left(x_0(n_1,n_2) - x(n_1,n_2) \right)^2 \right)^{1/2}.$$

Superpixel segmentation was performed at various threshold values from 2 to 25 in increments of 1. Fig. 2 illustrates that the dependence is linear. To figure out the dependence experimental data were approximated using the least squares method. The obtained dependence has the following form: $\varepsilon(\sigma_v) = 1.9\sigma_v + 2$. Thus, the higher the value of noise standard deviation, and, therefore, the lower the SNR, the higher the superpixel threshold ε value that minimizes reconstruction error. It was also found that the threshold values of the superpixel segmentation algorithm [5], which provides the minimum MSE, don't depend on the correlation between the pixels of the original image.

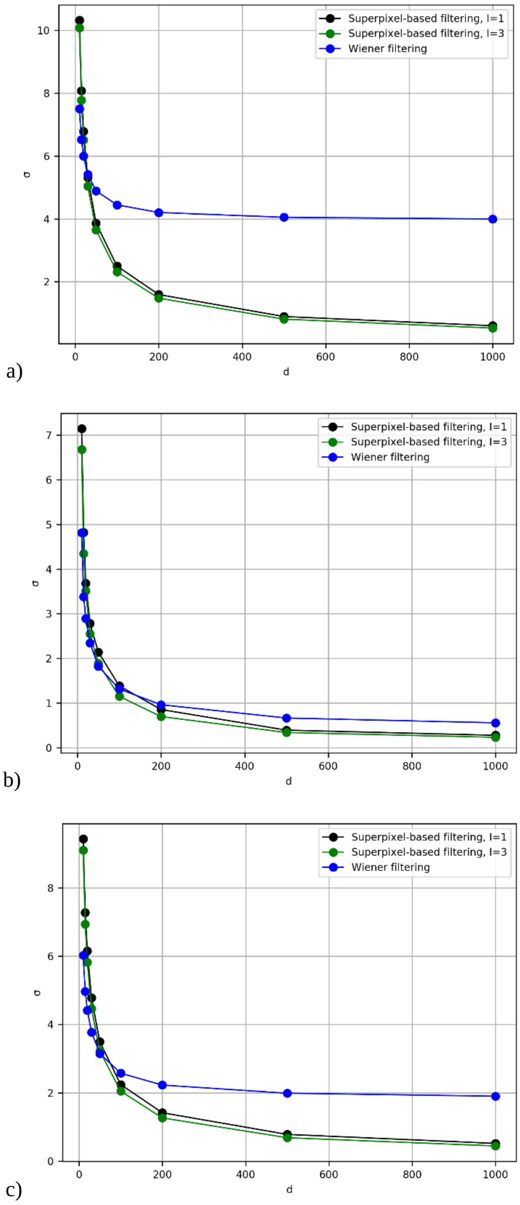

Fig. 3. The dependence of MSE σ between the reconstructed image and the ideal image on the signal-to-noise ratio d : a) $\rho = 0.90$, b) $\rho = 0.95$, c) $\rho = 0.99$.

Fig. 3 shows the dependence of MSE on the signal-to-noise ratio for the proposed superpixel filtering algorithm with threshold values ε defined in the previous step. It can be seen that the proposed algorithm can be applied to filter piecewise-constant images at $d \geq 50$ dB. Approximation by polynomials of degree $I = 3$ isn't much more efficient than approximation by polynomials of degree $I = 1$. Thus, to reconstruct piecewise-constant images by the proposed filtering algorithm, it's sufficient to use a polynomial of degree $I = 1$.

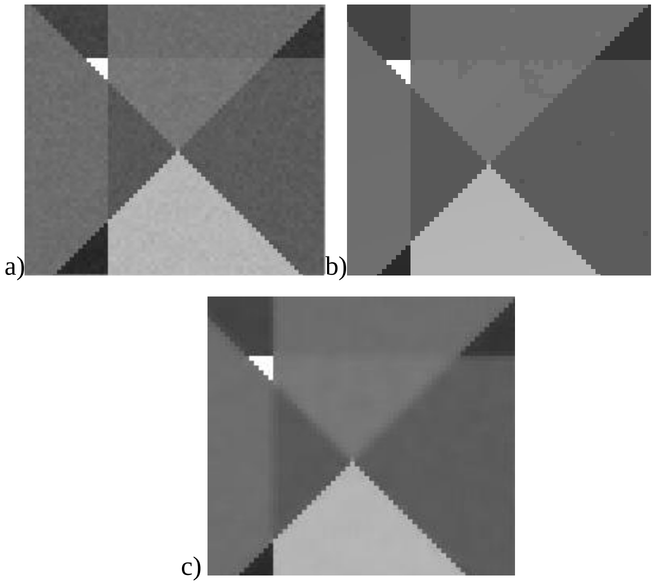

Fig. 4. Piecewise-constant image reconstruction: a) the noisy image fragment $(\rho = 0.95,\ d = 200\ \text{dB})$, b) the image fragment reconstructed using the proposed superpixel-based filtering algorithm $(I = 1)$, c) the image fragment reconstructed using the Wiener filtering.

Fig. 3 also illustrates the dependence $\sigma(d)$ for the Wiener filter. It's worth noting that the Wiener filter reconstruction error can be calculated using the power spectral density of the image and noise. It's known that piecewise-constant images have an isotropic exponential autocorrelation function [13]. The calculation of the energy spectrum of such signals is presented in [14].

By comparing the proposed filtering algorithm with the Wiener filter, the following conclusions can be drawn.

- At signal-to-noise ratio $d \leq 50$ dB, the Wiener filter provides lower MSE values (however, they are high), whereas at $d > 50$ dB the proposed superpixel filtering performs better regardless of the value of I .
- The higher the value of the correlation coefficient between the pixels of the original image ρ , the smaller MSE obtained for the proposed algorithm and the Wiener filter.
- The proposed algorithm is more efficient than the Wiener filter at $\rho \leq 0.95$.
- The higher the correlation between the original image pixels, the lower MSE, regardless of the filtering method used.

An example of a noisy image fragment reconstructed by each of the compared algorithms is shown in Fig. 4. The reconstruction errors of the proposed algorithm are local and are observed at the boundaries of similar in intensity regions. In turn, the Wiener filtering is characterized by a blurring of reconstructed images.

V. Conclusion

The paper presents a superpixel-based filtering algorithm and compares it with the Wiener filtering. The experimental part of the research shows that at signal-to-noise ratios higher than 50 dB, the proposed superpixel-based filtering algorithm provides lower reconstruction errors than the

Wiener filter. Moreover, unlike the Wiener filter, the proposed method proved to be good at various values of the correlation coefficient between the pixels of the original image. The superpixel-based filtering algorithm is more efficient than the Wiener filter at the correlation coefficient between neighboring pixels less than 0.95.

It's also shown that it's sufficient to approximate superpixels with polynomials of the first degree, since at higher degrees the reduction in MSE between the reconstructed image and the ideal image isn't significant.

The disadvantage of the proposed algorithm is the effect of the obtaining superpixel representation stage on the final result. In other words, an incorrectly selected superpixel threshold results in pixels of different objects are merged into a single superpixel. Conversely, when the noise level in the observed image is high, the oversegmentation may occur, and, as a result, the noise after filtering remains partially.

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