Differential method of multidimensional signals compression based on the adapted parameterized interpolation algorithm

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***Abstract*—In this paper, parameterized algorithms of multidimensional signal interpolation are adapted for use as part of differential compression methods. These methods are based on the efficient coding of quantized differences between the initial and interpolated signal samples during sequential signal scanning. The proposed interpolators are based on the classification of signal samples and the use of various interpolation formulas within the classes. The sample classifier and its training procedure and a set of interpolating functions for the compression method are described. The results of experimental research on real multidimensional signals confirm that the use of an adapted parameterized interpolator leads to an increase in the efficiency of the differential compression method.**

***Keywords*—comparative study, compression, low-level processing, filtering, enhancement, color mapping, remote sensing imagery, still images**

I. Introduction

Algorithms for interpolation of multidimensional signals can be divided into two groups [1]: adaptive algorithms and non-adaptive ones. The most common examples of non-adaptive algorithms have relatively low computational complexity due to the lack of use of local signal features. They are: rectangular interpolation from the nearest (or neighboring) sample, as well as bilinear and bicubic interpolation [2].

Adaptive algorithms, on the contrary, take into account the features of the local neighborhood of each sample, which usually allows improving accuracy. Examples of such algorithms include DCCI [3], NEDI [4-5], super-resolution algorithms based on neural networks [6-7], as well as many other algorithms [8-10]. In this paper, we consider adaptive parameterized interpolation algorithms [11] based on the classification of signal samples using local features and the use of a simple interpolating formula for each sample class.

The goal of this research is to adapt the parametrized interpolators for differential compression methods [2, 8] based on interpolation of signal samples during sequential sweep and compression of interpolation errors.

II. Differential compression of multidimensional signals

During differential compression, [2, 8] samples of a multidimensional signal $f(\vec{x})$ are processed sequentially. Each sample $f(\vec{x})$ is interpolated using the function R based on the nearest processed (compressed and decompressed) samples $\left\{ g(\vec{x}+\vec{\delta}) : \vec{\delta} \in \Delta \right\}$, after which the difference signal $v(\vec{x})$ is calculated, which is then quantized by the function W to calculate the quantized difference signal $w(\vec{x})$:

$$\begin{gathered} r(\vec{x}) = R\left(\left\{ g(\vec{x}+\vec{\delta}) : \vec{\delta} \in \Delta \right\}\right), \\ v(\vec{x}) = f(\vec{x}) - r(\vec{x}), \\ w(\vec{x}) = W\left(v(\vec{x})\right), \end{gathered} \tag{1}$$

where $r(\vec{x})$ – is the interpolated signal, Δ – is the array of reference sample displacements during interpolation. For quantization in this work, we used a quantizer with absolute error e_{abs} control:

$$\begin{gathered} W\left(v(\vec{x})\right) = sign\left(v(\vec{x})\right) \cdot \\ \cdot int\left(\left| v(\vec{x}) \right| + \frac{e_{abs}}{2e_{abs}+1} \right)(2e_{abs}+1), \end{gathered} \tag{2}$$

where function *int*(...) calculates the integer part of a value, and *sign*(...) calculates its sign.

Then, restoration (decompression) of the current sample is performed, i.e. calculation of the reference value, which will is calculated during decompression:

$$g(\vec{x}) = w(\vec{x}) + r(\vec{x}). \tag{3}$$

The described feedback (interpolation not according to the initial, but according to the decompressed values of the samples) is necessary to ensure the identity of the interpolator at the stages of compression and decompression (the source signal is no longer available during decompression). The quantized difference signal $w(\vec{x})$ is processed by a statistical encoder to reduce the amount of data and is sent to a communication channel or archive data storage.

III. ADAPTATION OF THE PARAMETERIZED ALGORITHM FOR DIFFERENTIAL COMPRESSION

A. Parameterized interpolation algorithm for differential compression

Before interpolating, we will classify the signal samples based on a local feature $\vec{\alpha}(\vec{x})$:

$$c\left(\vec{x}\right) = C\left(\vec{\alpha}(\vec{x}), \vec{\beta}\right), \tag{4}$$

where $c\left(\vec{x}\right)$ – is the number of sample's class, a sample has coordinates $\vec{x}$, $\vec{\alpha}(\vec{x})$ – is the local feature, $C\left(\vec{\alpha}(\vec{x}), \vec{\beta}\right)$ – classifier, $\vec{\beta}$ – classifier parameter, which is calculated for each signal anew by the training procedure based on the optimization of some criterion.

Each class with a number $c\left(\vec{x}\right)$ has its own interpolation function R_c, the interpolation procedure can be expressed in the following way:

$$\begin{gathered} r\left(\vec{x}\right) = R\left(\left\{g(\vec{x}+\vec{\delta}) : \vec{\delta} \in \Delta\right\}\right) = \\ = R_c\left(\left\{g(\vec{x}+\vec{\delta}) : \vec{\delta} \in \Delta\right\}\right), \\ c = C\left(\vec{\alpha}(\vec{x}), \vec{\beta}\right). \end{gathered} \tag{5}$$

During classifier $C\left(\vec{x}\right)$ training procedure decompressed signal $g\left(\vec{x}\right)$ is used both as a training set and a test set.

To adapt a parametrized interpolator to differential compression, the following elements of the interpolation algorithm need to be specified: the classifier of samples, the optimization criterion of the classifier, the optimization procedure for the classifier, a set of interpolating functions.

B. Sample classifier for parameterized interpolation.

We will classify the signal samples based on the severity of the directed artifacts in the vicinity of the current sample, which we will calculate using a set of partial derivative estimates $\left\{g'_m\left(\vec{x}\right), m \in [0, M)\right\}$ along different directions (M – is the number of directions), which is calculated using the basic samples $\left\{g(\vec{x}+\vec{\delta}) : \vec{\delta} \in \Delta\right\}$ and neighboring processed samples (these estimates can be easily calculated based on discrete differences of already processed samples).

Let us sort the derivatives $g'_m\left(\vec{x}\right)$ in the ascending order and rename them, creating the variation series $\hat{g}'_1\left(\vec{x}\right) \le \hat{g}'_2\left(\vec{x}\right) \le \ldots \le \hat{g}'_M\left(\vec{x}\right)$. We assume that there is a directed artifact in the vicinity, if the least derivative $\hat{g}'_1\left(\vec{x}\right)$ is significantly different from others. We will estimate the significance via local feature $\alpha(\vec{x})$, which can be calculated by the following three rank filters:

$$\alpha_1(\vec{x}) = \hat{g}'_2\left(\vec{x}\right) - \hat{g}'_1\left(\vec{x}\right),$$

$$\alpha_2(\vec{x}) = \frac{\hat{g}'_1\left(\vec{x}\right)}{E\left(\left\{\hat{g}'_m\left(\vec{x}\right) : m \in [1, M]\right\}\right)},$$

$$\alpha_3(\vec{x}) = \frac{\alpha_1(\vec{x})}{E\left(\left\{\hat{g}'_m\left(\vec{x}\right) - \hat{g}'_{m-1}\left(\vec{x}\right) : m \in [3, M]\right\}\right)},$$

where E performs averaging:

$$E\left(\left\{\hat{g}'_m\left(\vec{x}\right) : m \in [1, M]\right\}\right) = \frac{1}{M}\sum_{m=1}^{M} \hat{g}'_m\left(\vec{x}\right). \tag{6}$$

Classifier $C\left(\alpha(\vec{x}), \beta\right)$ is based on a thresholding function $C\left(\alpha(\vec{x}), \beta\right) = 1 + Bin\left(\beta - \alpha(\vec{x})\right)$ and depends on parameter β. The function chooses one of the interpolation functions depending on the presence of artifact inside the vicinity.

C. Classifier optimization criterion

As the optimization criterion, we have decided to use an entropy minimum criterion $h(\beta)$ of the quantized differential signal $w\left(\vec{x}\right)$:

$$\begin{gathered} h(\beta) = -\sum_{\tilde{w}=w_{\min}}^{w_{\max}} \hat{w}(\beta, \tilde{w}) \log_2 \hat{w}(\beta, \tilde{w}) \to \min_{\beta}, \\ \hat{w}(\beta, \tilde{w}) = card\left\{w\left(\vec{x}\right) : w\left(\vec{x}\right) = \tilde{w}\right\} \end{gathered} \tag{7}$$

where $\hat{w}(\beta, \tilde{w})$ is the number of values of quantized differential signal $w\left(\vec{x}\right)$ which are equal $\tilde{w}$. Parameter β determines the choice of interpolating functions at each sample of the signal, thereby influencing the difference signal. The choice of this criterion was made due to the fact that the entropy well approximates the size of the compressed data; this makes the criterion the most suitable for the compression problem.

To solve the optimization task (7), the statistics $\hat{W}(\tilde{\alpha}, c, \tilde{w})$ of quantized differential signal $w\left(\vec{x}\right)$ values for every class $c(\vec{x})$ and every feature value $\alpha(\vec{x})$ is obtained:

$$\begin{gathered} \hat{W}(\tilde{\alpha}, c, \tilde{w}) = \\ card\left\{\begin{array}{l} \vec{x} : \alpha(\vec{x}) = \tilde{\alpha}, \\ W\left(f(\vec{x}) - R_{c(\vec{x})}\left(\left\{g(\vec{x}+\vec{\delta}) : \vec{\delta} \in \Delta\right\}\right)\right) = \tilde{w} \end{array}\right\} \end{gathered} \tag{8}$$

The number of $\widehat{w}(\beta, \tilde{w})$ values for the minimum of β, equal $\alpha_{\min}$, can be calculated as follows:

$$\widehat{w}(\alpha_{\min}, w) = \sum_{\alpha=\alpha_{\min}}^{\alpha_{\max}} \widehat{W}(\alpha, 2, w), \tag{9}$$

since in this case the same interpolation function is used for all samples.

Values $\widehat{w}(\beta, \tilde{w})$ for other β values are calculated as follows:

$$\begin{gathered} \widehat{w}(\beta, w) = \widehat{w}(\beta - 1, w) - \\ -\widehat{W}(\beta - 1, 2, w) + \widehat{W}(\beta - 1, 1, w). \end{gathered} \tag{10}$$

After the calculation of the number of $\widehat{w}(\beta, \tilde{w})$ values, entropy $h(\beta)$ is calculated via expression (7) for every parameter β. Since there are not many of these values, brute forcing β among $h(\beta)$ values will give the result of optimization task.

D. Interpolation functions of the parameterized interpolator.

Classifier (4) based on the feature $\alpha(\vec{x})$ allows determining at each point whether an artifact exists in the vicinity. If there is no artifacts, then averaging over the nearest reference samples interpolation is used:

$$R_1\left(\left\{g(\vec{x}+\vec{\delta}) : \vec{\delta} \in \Delta\right\}\right) = \frac{\sum_{\vec{\delta} \in \Delta} g(\vec{x}+\vec{\delta})}{card\left\{g(\vec{x}+\vec{\delta}) : \vec{\delta} \in \Delta\right\}}. \tag{11}$$

If there is an artifact, then as the interpolated value $R_2\left(\left\{g(\vec{x}+\vec{\delta}) : \vec{\delta} \in \Delta\right\}\right)$ the sample along the artifact direction is. The direction is defined by the minimum value of derivative $g'_m(\vec{x})$. The general interpolation function (5) will look as follows:

$$\begin{gathered} r(\vec{x}) = \begin{cases} R_1\left(\left\{g(\vec{x}+\vec{\delta}) : \vec{\delta} \in \Delta\right\}\right), \alpha(\vec{x}) < \beta \\ R_2\left(\left\{g(\vec{x}+\vec{\delta}) : \vec{\delta} \in \Delta\right\}\right), \alpha(\vec{x}) \geq \beta \end{cases} \\ R_1\left(\left\{g(\vec{x}+\vec{\delta}) : \vec{\delta} \in \Delta\right\}\right) = \frac{\sum_{\vec{\delta} \in \Delta} g(\vec{x}+\vec{\delta})}{card\left\{g(\vec{x}+\vec{\delta}) : \vec{\delta} \in \Delta\right\}}, \\ R_2\left(\left\{g(\vec{x}+\vec{\delta}) : \vec{\delta} \in \Delta\right\}\right) = g(\vec{x}+\vec{\delta}^{(k)}), \\ k = \arg\min_m g'_m(\vec{x}), \end{gathered} \tag{12}$$

We specify the described interpolation functions for an important special case when the signal dimension is three. The displacements of the reference samples in this case can be written as follows:

$$\begin{gathered} \delta^{(0)}, \ldots, \delta^{(9)} = \\ \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \\ \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix}, \begin{bmatrix} 0 \\ -1 \\ 1 \end{bmatrix}. \end{gathered} \tag{13}$$

Next, we will use the auxiliary difference of the processed samples:

$$d_m\left(\begin{bmatrix} x \\ y \\ z \end{bmatrix}\right) = \left| g\left(\begin{bmatrix} x \\ y \\ z \end{bmatrix}\right) - g\left(\begin{bmatrix} x \\ y \\ z \end{bmatrix} - \delta^{(m)}\right) \right|, m \in [0, 9], \tag{14}$$

on the basis of which it is possible to write abnormal estimates of partial derivatives in directions

$$g'_0\left(\begin{bmatrix} x \\ y \\ z \end{bmatrix}\right) = d_0\left(\begin{bmatrix} x \\ y-1 \\ z \end{bmatrix}\right) + d_0\left(\begin{bmatrix} x-1 \\ y-1 \\ z \end{bmatrix}\right) + d_0\left(\begin{bmatrix} x-1 \\ y+1 \\ z \end{bmatrix}\right), \tag{15}$$

$$g'_1\left(\begin{bmatrix} x \\ y \\ z \end{bmatrix}\right) = d_1\left(\begin{bmatrix} x-1 \\ y-1 \\ z \end{bmatrix}\right) + d_1\left(\begin{bmatrix} x-1 \\ y \\ z \end{bmatrix}\right) + d_1\left(\begin{bmatrix} x-1 \\ y+1 \\ z \end{bmatrix}\right), \tag{16}$$

$$g'_2\left(\begin{bmatrix} x \\ y \\ z \end{bmatrix}\right) = d_2\left(\begin{bmatrix} x \\ y-1 \\ z \end{bmatrix}\right) + d_2\left(\begin{bmatrix} x-1 \\ y \\ z \end{bmatrix}\right) + d_2\left(\begin{bmatrix} x-1 \\ y-1 \\ z \end{bmatrix}\right), \tag{17}$$

$$g'_3\left(\begin{bmatrix} x \\ y \\ z \end{bmatrix}\right) = d_3\left(\begin{bmatrix} x \\ y-1 \\ z \end{bmatrix}\right) + d_3\left(\begin{bmatrix} x-1 \\ y \\ z \end{bmatrix}\right) + d_3\left(\begin{bmatrix} x+1 \\ y-1 \\ z \end{bmatrix}\right), \tag{18}$$

$$g'_4\left(\begin{bmatrix} x \\ y \\ z \end{bmatrix}\right) = d_4\left(\begin{bmatrix} x-1 \\ y \\ z \end{bmatrix}\right) + d_4\left(\begin{bmatrix} x \\ y-1 \\ z \end{bmatrix}\right) + d_4\left(\begin{bmatrix} x-1 \\ y-1 \\ z \end{bmatrix}\right), \tag{19}$$

$$g'_5\left(\begin{bmatrix} x \\ y \\ z \end{bmatrix}\right) = d_5\left(\begin{bmatrix} x-1 \\ y \\ z \end{bmatrix}\right) + d_5\left(\begin{bmatrix} x \\ y-1 \\ z \end{bmatrix}\right) + d_5\left(\begin{bmatrix} x+1 \\ y-1 \\ z \end{bmatrix}\right), \tag{20}$$

$$g'_6\left(\begin{bmatrix} x \\ y \\ z \end{bmatrix}\right) = d_6\left(\begin{bmatrix} x-1 \\ y \\ z \end{bmatrix}\right) + d_6\left(\begin{bmatrix} x-1 \\ y-1 \\ z \end{bmatrix}\right) + d_6\left(\begin{bmatrix} x \\ y-1 \\ z \end{bmatrix}\right), \tag{21}$$

$$g'_7\left(\begin{bmatrix} x \\ y \\ z \end{bmatrix}\right) = d_7\left(\begin{bmatrix} x-1 \\ y \\ z \end{bmatrix}\right) + d_7\left(\begin{bmatrix} x+1 \\ y \\ z-1 \end{bmatrix}\right) + d_7\left(\begin{bmatrix} x-1 \\ y-2 \\ z \end{bmatrix}\right), \quad (22)$$

$$g'_8\left(\begin{bmatrix} x \\ y \\ z \end{bmatrix}\right) = d_8\left(\begin{bmatrix} x+1 \\ y-1 \\ z \end{bmatrix}\right) + d_8\left(\begin{bmatrix} x \\ y-1 \\ z \end{bmatrix}\right) + d_8\left(\begin{bmatrix} x \\ y-1 \\ z-1 \end{bmatrix}\right), \quad (23)$$

$$g'_9\left(\begin{bmatrix} x \\ y \\ z \end{bmatrix}\right) = d_9\left(\begin{bmatrix} x+1 \\ y-2 \\ z \end{bmatrix}\right) + d_9\left(\begin{bmatrix} x \\ y-1 \\ z \end{bmatrix}\right) + d_9\left(\begin{bmatrix} x-1 \\ y-1 \\ z \end{bmatrix}\right), \quad (24)$$

Then the general interpolation formula (12) takes the following form:

Fig. 1 Several contrasted signal channels of the test set.

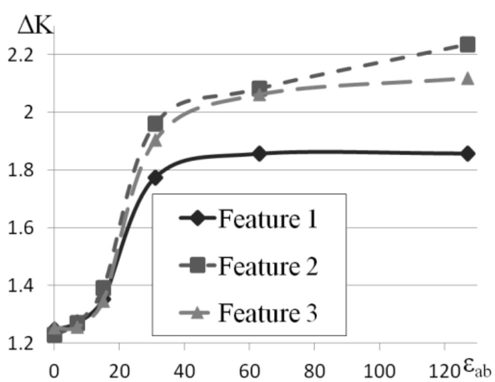

Fig. 2 Gain in compression ratio of the proposed interpolator over the averaging one.

$$r\left(\begin{bmatrix} x \\ y \\ z \end{bmatrix}\right) = \begin{cases} R_1\left(\begin{bmatrix} x \\ y \\ z \end{bmatrix}\right) = \left. \sum\limits_{m=0}^{9} g\left(\begin{bmatrix} x \\ y \\ z \end{bmatrix} + \delta^{(m)}\right) \middle/ 10, \right. \\ \alpha\left(\begin{bmatrix} x \\ y \\ z \end{bmatrix}\right) < \beta \\ R_2\left(\begin{bmatrix} x \\ y \\ z \end{bmatrix}\right) = g\left(\begin{bmatrix} x \\ y \\ z \end{bmatrix} + \delta^{(k)}\right), \\ k = \arg\min\limits_{0 \le m \le 9} g'_m\left(\begin{bmatrix} x \\ y \\ z \end{bmatrix}\right), \\ \alpha\left(\begin{bmatrix} x \\ y \\ z \end{bmatrix}\right) \ge \beta \end{cases} \quad (25)$$

IV. An Experimental Study of a Parametrized Interpolator as Part of a Differential Compression Method

In this work, the proposed parametrized interpolator was examined on real multidimensional signals of the UAVSAR hyperspectral array [12] (see the example in Fig. 1) as part of the differential compression method. The compression coefficient K was obtained using a parameterized interpolator (with features $\alpha_1, \alpha_2, \alpha_3$), compression coefficient K' was obtained with the use of averaging interpolator. Their ratio $\Delta K = K' / K$ shows how the proposed method outperforms the averaging one. The dependence of the compression coefficient on the absolute error $\varepsilon_{abs} = \max\left|f(\vec{x}) - g(\vec{x})\right|$ and squared error ε^2 (normalized by

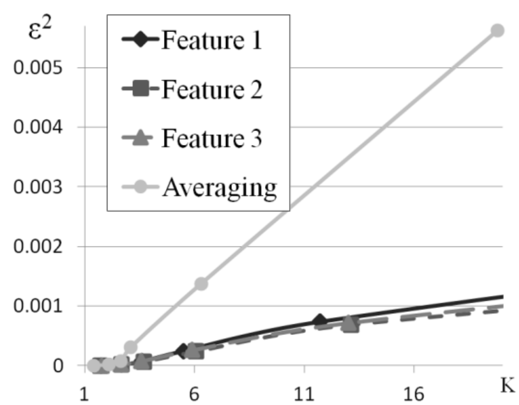

Fig. 3 Dependence of normalized MSE on compression ratio.

signal variance) were obtained.

As can be seen from the dependencies shown in figures.2-5, the use of the proposed interpolator gives a significant gain in compression ratio. Best results were obtained for the feature α_2 , however its usage is significantly time consuming. In general, obtained results show that proposed algorithm outperforms averaging one.

V. Conclusions

In this paper, the parametrized algorithms for interpolation of multidimensional signals were modified and adapted for use as part of differential compression methods based on the efficient coding of quantized differences between the initial and interpolating values of the samples during sequential signal sweep. The studied interpolators are based on the classification of signal samples and the use of various interpolation formulas within the classes. The classifier of readings and the procedure for its training are described, as well as a set of interpolating functions for the differential compression under consideration. Computational experiments on real multidimensional signals have confirmed that the use of an adapted parameterized interpolator has led to an increase in the efficiency of the differential compression method.

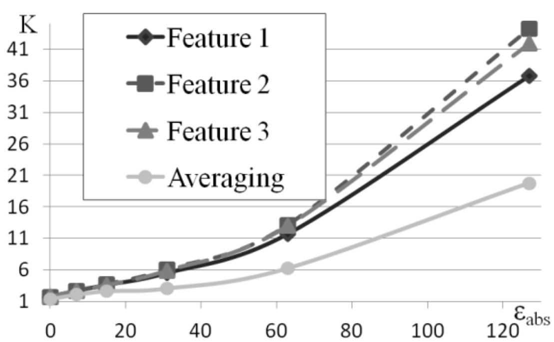

Fig. 4 Dependence of the compression ratio on the absolute error.

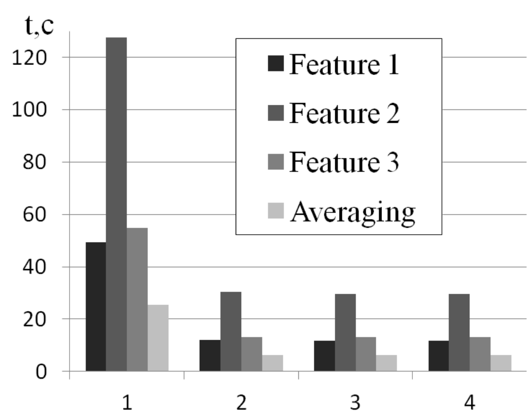

Fig. 5 Averaged time of processing of a test signal.

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