Some problems for the processes with compensation of the change-point event

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***Abstract*—In this paper, we will introduce definitions of three types of compensation of change-point (adaptive response): discontinuous, continuous, and combined (mixed). For each type of change-point we will use one of three methods of identification, based on the principles of filtering, extrapolation, or interpolation in partially observable schemes. Also, we will look at the task setting for all three types of compensation and the ways to solve it. The result of this work is a group of mathematical models that include a set of methods used to identify moments of different types of adaptive response in the terms of optimal control problems.**

Keywords—change-points, compensation, optimal control, extrapolation, interpolation

I. Introduction

In this paper, we will consider model groups defined by the method (type) of change-points identification (filtering, extrapolation, or interpolation). Three models are provided for each of them: “discontinuous”, “continuous” and “combined” (mixed) compensation. The optimization problems for compensation parameter analysis are defined.

In [1,2], the problems of detecting optimal values of process indicators with “discontinuous” compensation (intensity and jumps’ values) are solved. In this paper, similar problems are solved for other types of compensation. The work is the development of methods [1].

The investigated object is characterized by a system of processes which can correlate with each other. In the latter case, the independent detection of both, violations and compensations for individual indicators, can lead to the risk of an incorrect decision. As a result, we can either have a situation of an unjustified compensation or a missing malfunction of the system.

An important issue is the choice of compensatory conditions, since frequent compensation of violations can lead to both, a depletion of resources and an inability to physically carry them out.

Models of this type provide a reliable control of the examined processes with the aim of not only to detect changes leading to a deterioration in the further system operation but also to determine adaptive reactions and external influences which compensate violations.

Investigations of the system destruction moments, in other words, the retirement of the system from its working state, will make it possible to predict the behavior of an object during its functioning and judge the quality of compensation measures. In the analysis, methods of stochastic description and computer modeling are used (see, for example, [3], [4]).

In [5], we can say that a compensation in the biological system is a particular type of adaptation that occurs under pathological conditions in each damaged organ, when its functional tension takes place in the body. In [6], an algorithm method for compensating elastic mechatronic system vibrations is presented. In [7], a nonlinear compensation method and a construction of perceptron models of impulse noise filters are presented. In [8], algorithms for compensating external deterministic disturbances in linear systems are presented. In [9], a modified method for interference compensation was considered and experimentally verified. Source [10] presents an innovative frame dynamic rapid adaptation and noise compensation technique for tracking highly non-stationary noises and its applications for on-line ASR.

In technical systems, for example, a replacement or a repair of an object, or its change of use, can be a compensation.

Compensation research methods also depend on the presentation of information about the change-point.

The result of this work is a classification of identification of change-points and compensation types. For each model a method that allows us to solve the formulated optimization problems is presented.

We consider a closer look at the groups of models that correspond to the method of getting information about the change-points.

II. Groups of models with the compensation of the change-point based on the filtering principle

Let us consider three groups of processes with change-point compensation: “discontinuous”, “continuous”, “combined” (mixed), which are set on a stochastic of basis $\mathrm{B}^{(1)} = (\Omega, F, \mathrm{F} = (F_t)_{t \geq 0}, \mathrm{P})$ with the usual conditions of C. Dellacherie (e.g. [11-12]).

The system with function $X^{(1)} = (X_t^{(1)})_{t\geq 0}$ and process $Y^{(1)} = (Y_t^{(1)})_{t\geq 0}$ is defined as:

$$\begin{cases} X_t^{(1)} = \alpha^{(1)} \cdot I\left\{\theta^{(1)} \leq t\right\} \\ Y_t^{(1)} = \int_0^t X_s^{(1)} ds + \sigma^{(1)} W_t^{(1)} \end{cases}, \quad (1)$$

where variables $\alpha^{(1)} > 0$, $\sigma^{(1)} > 0$ ($\alpha^{(1)}, \sigma^{(1)} \in R^+$) are known. Change-point moment $\theta^{(1)}$ is also supposed to be known and $\theta^{(1)} \in [0;T]$, $0 < T < \infty$. Process $W^{(1)} = (W_t^{(1)})_{t\geq 0}$ is a standard Wiener one.

Function $X^{(1)} = (X_t^{(1)})_{t\geq 0}$ is a change-point "indicator", and $Y^{(1)} = (Y_t^{(1)})_{t\geq 0}$ is an observed process with a change-point.

The accumulated "discontinuous" (or "jump-like") change-point compensation $K^{(1)} = (\mathrm{K}_t^{(1)})_{t\geq 0}$ can be defined as:

$$\mathrm{K}_t^{(1)} = \int_0^t I\left\{ \int_0^s X_u^{(1)} du - K_s^{(1)} \geq \beta^{(1)} \right\} I\{\theta^{(1)} \leq \mathrm{t}\} \cdot \beta^{(1)} d\pi_s^{(1)}, \quad (2)$$

where $\beta^{(1)} > 0$ is a compensation level, $\pi^{(1)} = (\pi_t^{(1)})_{t\geq 0}$ is a Poisson process with an intensity $\lambda^{(1)}$, allowing a decomposition:

$$\pi_t^{(1)} = \lambda^{(1)} t + m_t^{\lambda^{(1)}}, \quad (3)$$

where intensity is $\lambda^{(1)} > 0$ and martingale is $m_t^{\lambda^{(1)}}$.

Thus, the process with "discontinuous" compensation of the change-point $Z^{(1)} = (\mathrm{Z}_t^{(1)})_{t\geq 0}$ can be written in the following form:

$$Z_t^{(1)} = Y_t^{(1)} - K_t^{(1)} . \quad (4)$$

The process with the compensation of "continuous" type of the change-point is supposed to have the stochastic differential (5):

$$dM_t^{(11)} = \gamma_0^{(11)} M_t^{(11)} \cdot I\{t \geq \theta^{(1)}\} dt - \\ -\gamma_1^{(11)} \cdot M_t^{(11)} / (\mathrm{t} - \gamma_2^{(11)}) I\{t \geq \eta^{(11)}\} dt + \gamma_3^{(11)} dW_t^{(11)}, \quad (5)$$

where the variables are $\gamma_0^{(11)}, \gamma_1^{(11)}, \gamma_2^{(11)}, \gamma_3^{(11)} \in R^+$ and are known.

The moment $\eta^{(11)}$ is determined as:

$$\eta^{(11)} = \inf\{t : \mathrm{M}_t^{(11)} \geq A^{(11)}\}, \quad (6)$$

where $A^{(11)}$ is a known constant ($A^{(11)} \in R^+$).

The stochastic differential of process with the "combined" type of change-point compensation has the formula:

$$dN_t^{(1)} = -\omega_0^{(1)}(\mathrm{t}) N_t^{(1)} dt + \omega_1^{(1)}(t) dW_t^{(12)}, \quad (7)$$

where

$$\omega_1(t)^{(1)} = \overline{\omega}_1^{(1)} \cdot I\{t \leq \theta^{(1)}\} + \overline{\omega}_2^{(1)} \cdot I\{\theta^{(1)} < t \leq \eta^{(1)}\} + \\ + \overline{\omega}_3^{(1)} \cdot I\{t > \eta^{(1)}\},$$

$$\omega_0(t)^{(1)} = \overline{\overline{\omega}}_1^{(1)} \cdot I\{t \leq \eta^{(1)}\} + \overline{\overline{\omega}}_2^{(1)} \cdot I\{t > \eta^{(1)}\},$$

variables are $\overline{\omega}_1^{(1)}, \overline{\omega}_2^{(1)}, \overline{\omega}_3^{(1)}, \overline{\overline{\omega}}_1^{(1)}, \overline{\overline{\omega}}_2^{(1)} \in R^+$ and known, here with $\overline{\omega}_2^{(1)} > \overline{\omega}_1^{(1)}$.

In the situation where $\overline{\overline{\omega}}_1^{(1)} = \overline{\overline{\omega}}_2^{(1)}$ three cases can be considered:

1). if $\overline{\omega}_2^{(1)} > \overline{\omega}_3^{(1)} > \overline{\omega}_1^{(1)}$, the compensation is partial

2). if $\overline{\omega}_2^{(1)} > \overline{\omega}_3^{(1)} = \overline{\omega}_1^{(1)}$, the compensation is full;

3). if $\overline{\omega}_2^{(1)} > \overline{\omega}_1^{(1)} > \overline{\omega}_3^{(1)}$, the overregulation occurs.

The moment $\eta^{(1)}$ is determined in the following way:

$$\eta^{(1)} = \inf\{t : N_t^{(1)} \geq A^{(12)}\}, \quad (8)$$

where $A^{(12)}$ is a known constant ($A^{(12)} \in R^+$).

III. Groups of Models with the Compensation of the Change-Point Based on the Principle of Interpolation

In this paragraph the moment of change-point is unknown and is determined by the interpolation method and the processes considered here are defined on a stochastic structure [13-14]:

$$S = (\Omega, F, \mathrm{F}, \mathcal{P}), \quad (9)$$

where $\mathcal{P}$ is a family of probability distributions (measures) and $\mathcal{P}=\{P_{\theta^{(2)}}, \theta^{(2)} \in \Theta\}$, where $\Theta = [0,T]$, $0 < T < \infty$.

In the first model two objects are considered: the process of change-point accumulation ($Y^{(2)} = (\mathrm{Y}_t^{(2)})_{t\geq 0}$) and the "indicator" function that shows changes in the main characteristics ($X^{(2)} = (X_t^{(2)})_{t\geq 0}$). Function $X^{(2)} = (X_t^{(2)})_{t\geq 0}$ and process $Y^{(2)} = (Y_t^{(2)})_{t\geq 0}$ are set on the basis (9).

Function $X^{(2)} = (X_t^{(2)})_{t\geq 0}$ and process $Y^{(2)} = (Y_t^{(2)})_{t\geq 0}$ are determined by the system:

$$\begin{cases} X_t^{(2)} = \alpha^{(2)} \cdot I\left\{\theta^{(2)} \le t\right\} \\ Y_t^{(2)} = \int_0^t X_s^{(2)} ds + \sigma^{(2)} W_t^{(2)} \end{cases}, \quad (10)$$

where variables $\alpha^{(2)} > 0$, $\sigma^{(2)} > 0$ ($\sigma^{(2)}$, $\alpha^{(2)} \in R^+$) are known and $\theta^{(2)}$ is a priori unknown moment of the change-point, which takes values from $[0, T]$ $(0 < T < \infty)$. The process $W^{(2)} = (W_t^{(2)})_{t \ge 0}$ is Wiener standard. The system (10) is partially observed.

The accumulated "jump-like" compensation of the change-point $K^{(2)} = (K_t^{(2)})_{t \ge 0}$ we present as:

$$K_t^{(2)} = \\ = \int_0^t (I\left\{\alpha^{(2)} \int_0^s (u - \tilde{\theta}) I\{\tilde{\theta} \le u\} du - K_s^{(2)} \ge \beta^{(2)}\right\} \cdot \\ \cdot I\{\tilde{\theta} \le t\} \beta^{(2)}) d\pi_s^{(2)} \quad (11)$$

where $\beta^{(2)} > 0$ is a compensation level, $\pi^{(2)} = (\pi_t^{(2)})_{t \ge 0}$ is a Poisson process with intensity $\lambda^{(2)}$, allowing decomposition: $\pi_t^{(2)} = \lambda^{(2)} t + m_t^{\lambda^{(2)}}$ (with intensity $\lambda^{(2)} > 0$ and martingale $m_t^{\lambda^{(2)}}$).

The moment $\tilde{\theta}$ is defined as $\tilde{\theta} = \mathrm{E}\{\theta^{(2)} | F_t^{Y^{(2)}}\}$.

As a result, the process of "jump-like" compensation of the change-point $Z^{(2)} = (Z_t^{(2)})_{t \ge 0}$ has the formula:

$$Z_t^{(2)} = Y_t^{(2)} - K_t^{(2)}. \quad (12)$$

In this paper the change-point moment is unknown and is determined by using the least-squares method with an estimation error:

$$\delta_\theta = E\{(\theta^{(2)} - \tilde{\theta})^2 \mid F_t, \theta^{(2)} \le t\} \to \min_\theta. \quad (13)$$

The subproblem (13) was solved by simulation.

The stochastic differential of a process with the change-point compensation for the "continuous" type has the form:

$$dM_t^{(21)} = \gamma_0^{(21)} M_t^{(21)} \cdot I\{t \ge \tilde{\theta}\} dt - \\ -\gamma_1^{(21)} \cdot M_t^{(21)} / (t - \gamma_2^{(21)}) I\{t \ge \eta^{(21)}\} dt + \gamma_3^{(21)} dW_t^{(21)} \quad (14)$$

where variables $\gamma_0^{(21)}, \gamma_1^{(21)}, \gamma_2^{(21)}, \gamma_3^{(21)} \in R^+$ are known.

The moment $\eta^{(21)}$ is defined as $\eta^{(21)} = \inf\{t : \mathrm{M}_t^{(21)} \ge A^{(21)}\}$. The constant $A^{(21)}$ $(A^{(21)} \in R^+)$ is known.

The stochastic differential of the process with the "mixed" type of compensation of change-point has the formula:

$$dN_t^{(2)} = -\omega_0^{(2)}(t) N_t^{(2)} dt + \omega_1^{(2)}(t) dW_t^{(22)}, \quad (15)$$

where

$$\omega_1(t)^{(2)} = \overline{\omega_1}^{(2)} \cdot I\{t \le \theta^{(2)}\} + \\ + \overline{\omega_2}^{(2)} \cdot I\{\theta^{(2)} < t \le \eta^{(22)}\} + \overline{\omega_3}^{(2)} \cdot I\{t > \eta^{(22)}\},$$

$$\omega_0(t)^{(2)} = \overline{\overline{\omega_1}}^{(2)} \cdot I\{t \le \eta^{(22)}\} + \overline{\overline{\omega_2}}^{(2)} \cdot I\{t > \eta^{(22)}\},$$

variables $\overline{\omega_1}^{(2)}$, $\overline{\omega_2}^{(2)}$, $\overline{\omega_3}^{(2)}$, $\overline{\overline{\omega_1}}^{(2)}$, $\overline{\overline{\omega_2}}^{(2)} \in R^+$ are known, here with $\overline{\omega_2}^{(2)} > \overline{\omega_1}^{(2)}$.

In the situation where $\overline{\overline{\omega_1}}^{(2)} = \overline{\overline{\omega_2}}^{(2)}$ three cases can be considered:

1). if $\overline{\omega_2}^{(2)} > \overline{\omega_3}^{(2)} > \overline{\omega_1}^{(2)}$, the compensation is partial;

2). if $\overline{\omega_2}^{(2)} > \overline{\omega_3}^{(2)} = \overline{\omega_1}^{(2)}$, the compensation is full;

3). if $\overline{\omega_2}^{(2)} > \overline{\omega_1}^{(2)} > \overline{\omega_3}^{(2)}$, the overregulation occurs.

Moment $\eta^{(2)}$ is defined in the following way:

$$\eta^{(22)} = \inf\{t : N_t^{(2)} \ge A^{(22)}\}, \quad (16)$$

where $A^{(22)}$ is a known constant ($A^{(22)} \in R^+$).

IV. Groups of Models with the Compensation of the Change-Point Based on the Principle of Extrapolation

The processes considered in this paragraph are defined on a stochastic structure [13-14]:

$$S = (\Omega, F, \mathrm{F}, \mathcal{P}), \quad (17)$$

where $\mathcal{P} = \{P_{\theta^{(3)}}, \theta^{(3)} \in \Sigma\}$, $\Sigma = [0, T]$, $0 < T < \infty$.

Function $X^{(3)} = (X_t^{(3)})_{t \ge 0}$ (change point "indicator") and process $Y^{(3)} = (Y_t^{(3)})_{t \ge 0}$ (damage accumulation) are set on a stochastic structure (17) and defined as:

$$\begin{cases} X_t^{(3)} = \alpha^{(3)} \cdot I\left\{\theta^{(3)} \le t\right\} \\ Y_t^{(3)} = \int_0^t X_s^{(3)} ds + \sigma^{(3)} W_t^{(3)} \end{cases}, \quad (18)$$

where variables $\alpha^{(3)} > 0$, $\sigma^{(3)} > 0$ $(\alpha^{(3)}, \sigma^{(3)} \in R^+)$ are known, $\theta^{(3)}$ is a change-point moment, which has the value $[0, T]$ ($0 < T < \infty$) and is unknown. Process $W^{(3)} = (W_t^{(3)})_{t \ge 0}$ is Wiener standard. The system (18) is partially observed, however, the observation is possible only after the moment of time N $(0 < N < T)$. An accumulated compensation of the change-point $K^{(3)} = (K_t^{(3)})_{t\geq 0}$ can be presented as:

$$K_t^{(3)} = \\ = \int_0^t (I\left\{\alpha^{(3)} \int_0^s (u-\tilde{\varsigma}) I\{\tilde{\tilde{\theta}} \leq u\} du - K_s^{(3)} \geq \beta^{(3)}\right\} \cdot \\ \cdot I\{\tilde{\tilde{\theta}} \leq t\} \cdot \beta^{(3)}) d\pi_s^{(3)} \quad (19)$$

where $\beta^{(3)} > 0$ is the compensation level, $\pi^{(3)} = (\pi_t^{(3)})_{t\geq 0}$ is the Poisson process with intensity $\lambda^{(3)}$ allowing decomposition: $\pi_t^{(3)} = \lambda^{(3)} t + m_t^{\lambda^{(3)}}$ (with intensity $\lambda^{(3)} > 0$ and martingale $m_t^{\lambda^{(3)}}$).

Moment $\tilde{\tilde{\theta}}$ is defined as: $\tilde{\tilde{\theta}} = \mathrm{E}\{\theta^{(3)} \mid F_t\}$. The "discontinuous" process of the change-point compensation $Z^{(3)} = (Z_t^{(3)})_{t\geq 0}$ has the form:

$$Z_t^{(3)} = \alpha^{(3)} \int_0^N (u-\tilde{\tilde{\theta}}) I\{\tilde{\tilde{\theta}} \leq u\} du + Y_t^{(3)} I\{N \leq t\} - K_t^{(3)} \quad (20)$$

The change-point moment is determined by using the least-squares method with an estimation error:

$$\delta_\theta = E\{(\theta - \tilde{\tilde{\theta}})^2 \mid F_t,\ \theta \leq t, t \geq N\} \to \min_{\tilde{\theta}} \quad (21)$$

The problem (21) is solved by simulation.

A stochastic differential of the process with a "continuous" type of the change-point compensation can be defined as:

$$dM_t^{(31)} = \gamma_0^{(31)} M_t^{(31)} \cdot I\{t \geq \tilde{\tilde{\theta}}\} dt - \\ - \gamma_1^{(31)} \cdot M_t^{(31)} / (t - \gamma_2^{(31)}) I\{t \geq \eta^{(31)}\} dt + \\ + \gamma_3^{(31)} dW_t^{(31)}, \quad (22)$$

where variables $\gamma_0^{(31)}, \gamma_1^{(31)}, \gamma_2^{(31)}, \gamma_3^{(31)} \in R^+$ are known.

Moment $\eta^{(31)}$ is defined as: $\eta^{(31)} = \inf\{t : M_t^{(31)} \geq A^{(31)}\}$. Constant $A^{(31)}$ is known $(A^{(31)} \in R^+)$.

The stochastic differential of the process with the "mixed" type of the change-point compensation has the formula:

$$dN_t^{(3)} = -\omega_0^{(3)}(t) N_t^{(3)} dt + \omega_1^{(3)}(t) dW_t^{(32)}, \quad (23)$$

where

$$\omega_1(t)^{(3)} = \overline{\omega}_1^{(3)} \cdot I\{t \leq \theta^{(3)}\} + \\ + \overline{\omega}_2^{(3)} \cdot I\{\theta^{(3)} < t \leq \eta^{(32)}\} + \overline{\omega}_3^{(3)} \cdot I\{t > \eta^{(32)}\},$$

$$\omega_0(t)^{(3)} = \overline{\overline{\omega}}_1^{(3)} \cdot I\{t \leq \eta^{(32)}\} + \overline{\overline{\omega}}_2^{(3)} \cdot I\{t > \eta^{(32)}\},$$

variables $\overline{\omega}_1^{(3)}, \overline{\omega}_2^{(3)}, \overline{\omega}_3^{(3)}, \overline{\overline{\omega}}_1^{(3)}, \overline{\overline{\omega}}_2^{(3)} \in R^+$ are known, here with $\overline{\omega}_2^{(3)} > \overline{\omega}_1^{(3)}$.

In the situation where $\overline{\overline{\omega}}_1^{(3)} = \overline{\overline{\omega}}_2^{(3)}$ three cases can be considered:

1). if $\overline{\omega}_2^{(3)} > \overline{\omega}_3^{(3)} > \overline{\omega}_1^{(3)}$, the compensation is partial;

2). if $\overline{\omega}_2^{(3)} > \overline{\omega}_3^{(3)} = \overline{\omega}_1^{(3)}$, the compensation is full;

3). if $\overline{\omega}_2^{(3)} > \overline{\omega}_1^{(3)} > \overline{\omega}_3^{(3)}$, the overregulation occurs.

A moment $\eta^{(32)}$ is defined in the following way:

$$\eta^{(32)} = \inf\{t : N_t^{(3)} \geq A^{(32)}\}, \quad (24)$$

where $A^{(32)}$ is a known constant ($A^{(32)} \in R^+$).

V. Methods for Identifying Characteristics of the Compensation of the Change-Point

An objective function and an optimization problem are developed for each type of compensation of the change-point.

For "discontinuous" compensation, the target function has the formula:

$$\Phi_T^{(1)} = \beta^{(i)} \cdot (1 + \lambda^{(i)} \cdot T) + \overline{A}^{(1)} \cdot EZ_T^{(i)} I\{Z_T^{(i)} \geq 0\} \quad (25)$$

To find the compensation parameters, you need to solve an optimization problem:

$$\Phi_T^{(1)}(\lambda^{(i)}, \beta^{(i)}) \to \min_{\lambda^{(i)}, \beta^{(i)}}. \quad (26)$$

The solution of the problem (26) is found by simulation, where for the model built on the principle of filtering, the value of the target function (25) can be seen in [2], the solutions for the optimization problem (26) are $\alpha = 3.5$, $\beta = 34$.

For "continuous" compensation, the target function has the formula:

$$\Phi_T^{(2)} = \frac{A^{(i1)}}{\gamma_2^{(i1)}} + \overline{A}^{(2)} \cdot EM_T^{(i1)} I\{M_T^{(i1)} \geq 0\}, \quad (27)$$

where $\overline{A}^{(2)}$ is the model parameter.

To find the parameters of "continuous" compensation it is necessary to solve the optimization problem:

$$\Phi_T^{(2)}(\gamma_2^{(i1)}, A^{(i1)}) \to \min_{\gamma_2^{(i1)}, A^{(i1)}}. \quad (28)$$

The problem (28) is solved by computer simulation.

For "mixed" compensation, the target function has the formula:

$$\Phi^{(3)}_T = \lim_{t\to\infty}(D_1 N_t^{(i)} - D_2 N_t^{(i)})^2 + \overline{A}^{(3)}(\overline{\overline{\omega}}_2^{(i)} - \overline{\overline{\omega}}_1^{(i)})^2, \quad (29)$$

where $D_1 N_t^{(i)}$, $D_2 N_t^{(i)}$ are process variances $N_t^{(i)}$ before and after compensation.

To find the parameters of "mixed" compensation it is necessary to solve the optimization problem:

$$\Phi^{(3)}_T(\overline{\omega}_3^{(i)}, \overline{\overline{\omega}}_2^{(i)}) \to \min_{\overline{\omega}_3^{(i)}, \overline{\overline{\omega}}_2^{(i)}}. \quad (30)$$

In formulas (25)-(30), the index i corresponds to the type of change point identification (i =1,2,3).

The adequacy of the problem (30) is confirmed by a special case:

Statement: If in processes (7), (15), or (23) the coefficients before change point and after it (i.e. $\overline{\omega}_1^{(i)}$, $\overline{\omega}_2^{(i)}$) match, then task (30) will look like:

$$\left(\frac{(\overline{\overline{\omega}}_2^{(i)})^2}{2\overline{\omega}_3^{(i)}} - \frac{(\overline{\overline{\omega}}_1^{(i)})^2}{2\overline{\omega}_2^{(i)}}\right)^2 + \left(\overline{\omega}_3^{(i)} - \overline{\omega}_2^{(i)}\right)^2 \to \min_{\overline{\omega}_3^{(i)}, \overline{\overline{\omega}}_2^{(i)}} \quad (31)$$

The solution of the problem (31) is $\overline{\omega}_2^{(i)} = \overline{\omega}_3^{(i)}$, $\overline{\overline{\omega}}_2^{(i)} = \pm\overline{\overline{\omega}}_1^{(i)}$.

In the general case, the problem (29) is solved by simulation.

VI. Conclusion

The mathematical models based on descriptions in terms of processes with the change-point compensation can be used in many fields (technical, biological, meteorological, social), [15-18]. At the same time, for the constructed models it is possible to solve various optimization problems (the main part of which is presented in this paper). The classification described in the work and the presented group of models can be generalized. These developments can be applied in the following courses "Additional chapters of the theory of random processes", "Modeling of stochastic systems" and "Methods of computer simulation."

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