Abstract — The paper investigates the problem of risk control in the regional industrial complex. We consider the risk distribution among the industrial firms, the insurance sector, and the recovering enterprises. We study the model of the interaction in this multi-agent system. We develop the algorithm for the choice of the number of the waste utilization firms and the number of the insurers, which provide the minimum of the industrial firm’s risk costs.

Keywords — industrial risk control, insurance, waste utilization, Pareto equilibrium

I. INTRODUCTION

The regional economy includes the industrial firms, and often their number achieves tens of thousands. Each firm is a source of the industrial risk for the environment, legal entities and individuals, including firm’s employees. The effective risk control in the industrial firms is based on the correct risk assessment and the reasonable choice of the control methods, such as the insurance, the waste utilization and the self-insurance.

The risk control issues were considered in wide range of studies [1-10]. The risk of industrial firms was analyzed by using various mathematical tools: the game theory [11], [12], the penalties mechanisms [13], the simulation modeling [14], [15]. The industrial risk was investigated at various levels, including the regional level [16] – [18] and the firms’ level [1], [3], [4] – [6].

The risk control in the regional industrial complex combines the regional insurance sector, and the waste utilization firms of the regional recovering sector. In the region, the industrial firms may interact with many waste utilization companies and insurers. In turn, the regional recovering firms and the regional insurers may interact with many industrial firms.

The number of the industrial firms, the insurance companies and the waste utilization firms is quite great. Consequently, we consider the problem of determining the interaction parameters in the big data framework. Further, on the basis of the mathematical methods and tools [19 - 22], we search for the solution of this problem.

II. METHODS

We introduce the following assumptions, which determine the applicability limits of our model.

Assumption 1. The industrial firms sell their products in the perfect competitive markets. The product price \( p_i \) is an exogenous constant for the \( i \)-th firm in the industrial regional complex and the price \( p \) does not depend on the production volume \( Q \).

\[
\frac{dp_i}{dQ_i} = 0, \quad \forall \ i = 1, n ,
\]

where \( n \) is the number of the firms in the industrial regional complex.

The waste utilization firms and the insurance companies work in the monopolistic competitive market with a falling inverse demand curve

\[
\frac{dp_{ij}}{dY_j} < 0, \quad j = 1, m, \quad \frac{dT_k}{dX_k} < 0, \quad \frac{dT_k}{dY_k} < 0, \quad k = 1, l ,
\]

where \( p_{ij} \) is the utilization price of the conventional waste unit in the \( j \)-th waste utilization firm (WUF), \( Y_j \) is the external damage accepted for utilization by the \( j \)-th WUF, \( m \) is the number of WUFs, \( T_k \) is the insurance rate of the \( k \)-th insurance company, \( l \) is the number of the insurance companies in the region, \( X \) is the internal damage, \( Y \) is the external damage.

Assumption 2. The production growth leads to a decreasing in return:

\[
C_{Q_i, Q_i}^* < 0, \quad \forall \ i = 1, n ,
\]

where \( C_i \) is the value of the \( i \)-th firm’s costs.

Assumption 3. An increase in the production volume \( Q_i \) leads to an increasing in the possible internal damage \( X_i \); the internal damage \( X_i \) is reduced with an increase in the voluntary risk costs; the internal damage \( X_i \) is limited from above due to technology features and production volume

\[
\frac{\partial X_i}{\partial Q_i} > 0, \quad \frac{\partial X_i}{\partial f_i} < 0, \quad X_i \in [0, X_{i, max}], \quad X_{i, max} > 0 ,
\]

where \( X_{i, max} \) is the limit of the internal damage, \( f_i \) is the voluntary risk costs (VRC) of the \( i \)-th firm.

Assumption 4. The external damage \( Y_i \) is proportional to the internal damage \( X_i \):

\[
\frac{\partial Y_i}{\partial X_i} > 0, \quad i = 1, n .
\]

Assumption 5. The voluntary combination insurance is considered, the wear is not included. The insurance indemnity is proportional to the insured damage, the indemnity does not exceed the damage:
Data

\[
\frac{\partial W_k}{\partial X_k} > 0, \quad \frac{\partial W_k}{\partial Y_k} > 0, \quad W_k \leq X_k + Y_k, \quad k = 1, l. \quad (6)
\]

**Assumption 6.** The utilization cost of the conventional waste unit is constant.

\[ c_Y = \text{const.} \quad (7) \]

**Assumption 7.** The external and internal damages of the \(i\)-th firm in the regional industrial complex consist of three elements:

\[
\begin{align*}
\sum_{k=1}^{l} \delta_k^S + \sum_{j=1}^{m} \delta_{ij}^U + \delta_{i}^{\text{res}} &= 1, \\
\sum_{k=1}^{l} \gamma_k^S + \sum_{j=1}^{m} \gamma_{ij}^U + \gamma_{i}^{\text{res}} &= 1, \quad i = 1, n, \quad (8)
\end{align*}
\]

where \(\delta_k^S, (\gamma_k^S)\) are the fractions of the external or internal damages, which are insured in the \(k\)-th insurance company, \(\delta_{ij}^U, (\gamma_{ij}^U)\) are the fractions of the external or internal damages, which are accepted for utilization by the \(j\)-th WUF, \(\delta_{i}^{\text{res}}, (\gamma_{i}^{\text{res}})\) are the rest fractions of the external or internal damages, which are rectified by the \(i\)-th firm.

According to the assumption 2, the production costs function has the following form [23, 24].

\[ C_{Q_i}(Q) = B_i Q_i^{\beta_i}, \quad \beta_i \in (1, \beta_i^{\max}), \beta_i^{\max} \in (1, 2), B_i > 0. \quad (9) \]

The internal damage function satisfies the assumption 3, and it has the following form:

\[ X(Q_i, f_i) = \chi(Q_i) e^{-\xi f_i}, \quad \xi \in (0, \xi^{\max}), \xi^{\max} \in (0, 1), \chi(Q_i) \geq 0. \quad (10) \]

The function \(\chi(Q_i)\) expresses the relationship between the internal damage and the production volume. The parameter \(\xi\) characterizes the effectiveness of the measures to reducing in the internal damage. The function \(\chi(Q_i)\) expresses an exponential distribution of the damage, which corresponds to man-made accidents.

The external damage function satisfies the assumption 4:

\[ Y(X) = \mu X, \quad \mu \geq 0. \quad (11) \]

The coefficient of the accident consequences expansion \(\mu\) expresses the ratio of the external damage to the internal damage, taking into account the specifics of the regional industrial complex, the geographical features, etc.

The insurance indemnity satisfies the assumption 5:

\[ W(X, Y) = \alpha(X + Y), \quad 0 \leq \alpha \leq 1. \quad (12) \]

The penalty function has the following form:

\[ H_i = aY_i = a\mu X_i, \quad a > 0, \quad i = 1, n. \quad (13) \]

If the regional industrial complex consists of \(n\) firms, the profit function of the industrial complex \(\Pi_i\) is

\[ \Pi_i = \sum_{i=1}^{n} (Q_i p_i + W_i) - \sum_{i=1}^{n} (C_{Q_i} + f_i + X_i^{\text{res}} + V_i + H_i + F_i) . \quad (14) \]

The problem of searching for the optimal production volume vector \(Q^*=\{Q_1^*, Q_2^*, \ldots Q_n^*\}\) and the optimal VRC vector \(f^*=\{f_1^*, f_2^*, \ldots f_n^*\}\) is based on a maximization of the profit criterion

\[ \{ f^*, Q^* \} = \arg\max_{f_i \in A_f, Q_i \in A_Q} \Pi_i. \quad (15) \]

\[ A_Q = \{ Q_i \in R^+ : Q_i \leq Q_i^{\max}, Q_i^{\max} > 0 \}, \quad (16) \]

\[ A_f = \{ f_i(\bullet) \in R^+ : f_i(\bullet) \leq f_i^{\max}, f_i^{\max} (0, R_i) \}, \quad (17) \]

where \(R_i\) is the limit value of VRC.

The vector \(f^*\) is the solution of problem (14)-(18), and it has the following coordinates:

\[ f_i^* = \frac{1}{\xi} \ln |\xi X(Q_i) K_j|, \quad (19) \]

where

\[ K_i = - \sum_{k=1}^{l} \alpha_k \gamma_a^k - \mu \sum_{k=1}^{l} \alpha_k \delta_a^k + \gamma_{a}^{\text{con}} + \frac{1}{m} \sum_{k=1}^{l} \gamma_{a}^k + \]

\[ + \mu \sum_{k=1}^{l} \alpha_k \delta_a^k + \mu a \mu \xi^{\max} + \mu \sum_{j=1}^{m} \rho_{ij} \delta_{ij} + \mu \sum_{j=1}^{m} \rho_{ij} \gamma_{ij} \]

The coordinates of the vector \(Q^*\) are the solution of the following equation

\[ p_i - B_i \beta_i Q_i^{\beta_i - 1} - \frac{\chi(Q_i^*)}{\xi} = 0, \quad i = 1, n. \quad (20) \]

For \(f^*, Q^*\), the value \(\Pi_i^*\) is

\[ \Pi_i^* = \sum_{i=1}^{n} (Q_i^* p_i - B_i Q_i^{\beta_i} - f_i^* - \frac{1}{\xi}) . \quad (21) \]

If the regional recovering sector includes \(m\) of WUFs, the profit function of this sector \(\Pi_y^*\) has the following form
The problem of searching for the optimal price vector \( p_Y^* = \{p_Y^1, p_Y^2, \ldots, p_Y^m\} \) is based on the recovering sector’s profit criterion

\[
p_Y^* = \arg \max_{p_Y \in \mathbb{R}^+} \Pi_H
\]  

(23)

For \( p_Y^* \), the value of the profit function \( \Pi_H^* \) is

\[
\Pi_H^* = \frac{1}{4} \sum_{j=1}^{m} \left( p_Y - c_j \right)^2
\]  

(24)

If the regional insurance sector includes \( l \) of insurers, the profit function of this sector \( \Pi_{II}^* \) has the following form

\[
\Pi_{II}^* = \sum_{k=1}^{l} (V_k - W_k).
\]  

(27)

The problem of searching for the optimal insurance rate vector \( T^* = \{T_1^*, T_2^*, \ldots, T_l^*\} \) is based on the regional insurance sector’s profit criterion

\[
T^* = \arg \max_{T_{II} \in (0, 1)} \Pi_{II}^*
\]  

(28)

The problem (27)-(29) has the solution \( T^* = \{T_1^*, T_2^*, \ldots, T_l^*\} \), where \( T^*_k \) is

\[
T^*_k = \frac{T + \alpha_k}{2}, \quad k = 1, l
\]  

(30)

For \( T_k^* \), the value of the insurance regional sector profit is

\[
\Pi_{II}^* = \frac{1}{4T} \sum_{k=1}^{l} \left( \frac{X_k}{T} - \frac{\alpha_k}{2} \right)^2.
\]  

(31)

Further, we consider the problem of searching for the interaction parameters. We search for the Pareto equilibrium set of the compromise utilization prices in the following form

\[
p_Y^{com} = \arg \max_{p_Y \in G} \{ \Pi_I, \Pi_H \},
\]  

(32)

\[
G = \{ p_Y | \Pi_I (p_Y) > 0 \land \Pi_H (p_Y) > 0 \}.
\]  

(33)

Additionally, we analyze the Pareto set of the compromise insurance rates in the following form

\[
T^{com} = \arg \max_{T \in \Omega} \{ \Pi_I, \Pi_{II} \},
\]  

(34)

\[
\Omega = \{ T | T_k \in (0, 1) \land \Pi_I (T) > 0 \land \Pi_{II} (T) > 0 \}.
\]  

(35)

This problem and their solutions enable us to determine the interaction parameters of the regional risk-control system for a variety of the regional industrial firms, the regional insurers and WUFs in the regional recovering sector.

III. RESULTS AND DISCUSSION

We investigate our model on the basis of the regional industrial complex of Volga Federal District, which includes 14 regions and republics of Russian Federation. In each region (or republic) of this District, tens of thousands industrial firms emit the waste (Table I).

**TABLE I. NUMBER OF INDUSTRIAL FIRMS AND INSURERS IN VOLGA FEDERAL DISTRICT [25]**

<table>
<thead>
<tr>
<th>Region</th>
<th>Number of Firms</th>
<th>Number of Insurers</th>
</tr>
</thead>
<tbody>
<tr>
<td>Republic of Bashkortostan</td>
<td>128025</td>
<td>85</td>
</tr>
<tr>
<td>Republic Of Mari El</td>
<td>20299</td>
<td>63</td>
</tr>
<tr>
<td>Republic of Mordovia</td>
<td>20462</td>
<td>65</td>
</tr>
<tr>
<td>Republic of Tatarstan</td>
<td>160009</td>
<td>93</td>
</tr>
<tr>
<td>Udmurt Republic</td>
<td>57393</td>
<td>75</td>
</tr>
<tr>
<td>Chuvash Republic</td>
<td>45665</td>
<td>68</td>
</tr>
<tr>
<td>Perm region</td>
<td>102906</td>
<td>73</td>
</tr>
<tr>
<td>Kirov region</td>
<td>48160</td>
<td>76</td>
</tr>
<tr>
<td>Nizhny Novgorod region</td>
<td>128667</td>
<td>84</td>
</tr>
<tr>
<td>Orenburg region</td>
<td>57584</td>
<td>68</td>
</tr>
<tr>
<td>Penza region</td>
<td>45395</td>
<td>67</td>
</tr>
<tr>
<td>Samara region</td>
<td>135063</td>
<td>80</td>
</tr>
<tr>
<td>Saratov region</td>
<td>75304</td>
<td>71</td>
</tr>
<tr>
<td>Ulyanovsk region</td>
<td>43765</td>
<td>71</td>
</tr>
</tbody>
</table>

The volumes of the waste in Volga Federal District is presented in table II. In these regions, as a rule, the volumes of the waste grow.

We calculate the interaction parameters of the insurances sector and WUF sector by using formulas (32) - (35).
Next, we analyze the number of WUFs and the number of the insurers, which provide the minimum of the industrial firm’s risk costs. The number of WUFs is determined on the basis of the WUF’s capability $V_j$ for the waste volume $X_{ij}^U + Y_{ij}^U$, and taking into account a minimum of the utilization expenses $F_i$. Similarly, the number of the insurers is chosen on the basis of the minimal insurance rate criterion $V_i$ among insurers that meet the condition $X_{ik}^S + Y_{ik}^S \leq X_k$. The optimal WUFs number selection procedure is presented as the algorithm in Figure 1. The iteration procedure allows us to calculate the number $m$ according to a fulfillment of the condition $X_{ij}^U + Y_{ij}^U \leq V_j$. Among WUFs that meet this condition, we search for the best WUF according to the minimal waste utilization costs criterion. The choice of the insurers is organized in the same way.

If WUF satisfies the condition $\exists j | X_{ij}^U + Y_{ij}^U \leq V_j$, then the parameters of the interaction between the industrial firm and WUF correspond to the Pareto equilibrium set (Fig. 2).

Figure 2 indicates the solution of problem (32) – (33). Therefore, the compromise utilization price $p_{Yj}^{\text{com}}$ belongs to the following set

$$p_{Yj}^{\text{com}} = \left[ \frac{p_Y + c_{Yj}}{2} - \frac{1}{2} \left( \frac{p_Y + c_{Yj}}{2} - \frac{4A_jp_Y}{Y_{ij}} \right)^2, \frac{p_Y + c_{Yj}}{2} \right],$$

that is not empty if

$$\frac{p_Y + c_{Yj}}{2} - \frac{1}{2} \left( \frac{p_Y + c_{Yj}}{2} - \frac{4A_jp_Y}{Y_{ij}} \right)^2 < \frac{p_Y}{2}.$$
The problem of searching for the parameters of the interaction with the regional insurers is solved similar. The solution of problem (34) – (35) has the following form

1) for one insurer provided \( \exists k_0 \mid X_{ik0}^S + Y_{ik0}^S \leq \bar{X}_{ik0} \) the solution is \( T^\text{com}_{k_0} \in \left[ a_{k_0} : \frac{T}{2} \right] \) for \( a_{k_0} < \frac{T}{2} \).

2) for multiple insurers provided \( \forall k = 1, \ldots \mid X_{ik}^S + Y_{ik}^S \geq \bar{X}_k \) the solution is

\[
T^\text{com} = (T^\text{com}_1, \ldots, T^\text{com}_n), \text{ where } T^\text{com}_k \in \left[ a_k : \frac{T}{2} \right] \text{ for } a_k < \frac{T}{2}.
\]

Thus, our results allow us to determine the compromise waste utilization prices and the compromise insurance rates that meet the requirements of the industrial firms, the recovering enterprises and the insurance regional sector. In addition, we construct the firm-insurer system, i.e., we calculate the number of insurers, which are necessary to insures the firm’s damage. This solution includes the big data as input parameters that reflect the operating conditions of all agents in the regional industrial risk control system.

### IV. CONCLUSION

The developed models describe the functioning of the regional industrial risk control system on the basis of big data regarding to the industrial firms, the insurance companies and the waste utilization organizations. The number of agents in the system varies from region to region, but generally exceeds tens of thousands. Each firm of the industrial regional complex interacts with one or multiple agents of the environmental protection and the insurance regional sector. The formulated problems and the presented solutions allow us to determine the parameters of the agents’ interaction in the regional system based on Pareto equilibrium. Our results may be used by the industrial firms to determine the terms of waste utilization and insurance contracts. In the strategies designing, the simulation results may be useful for WUF and insurers to develop the requirements for the industrial firms.

### REFERENCES


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**TABLE III. CALCULATION MINIMUM NUMBER OF INSURERS**

<table>
<thead>
<tr>
<th>Region</th>
<th>Average Waste per Firm, thousand tons</th>
<th>Average Insured Damage per Insurers, million rubles</th>
<th>Estimate Minimum Number of Insurers per Firm</th>
</tr>
</thead>
<tbody>
<tr>
<td>Republic of Bashkortostan</td>
<td>3.26</td>
<td>29 490.42</td>
<td>3</td>
</tr>
<tr>
<td>Republic of Mari El</td>
<td>1.72</td>
<td>3 332.67</td>
<td>1</td>
</tr>
<tr>
<td>Republic of Mordovia</td>
<td>2.63</td>
<td>4 970.68</td>
<td>1</td>
</tr>
<tr>
<td>Republic of Tatarstan</td>
<td>1.9</td>
<td>18 446.06</td>
<td>2</td>
</tr>
<tr>
<td>Udmurt Republic</td>
<td>2.43</td>
<td>11 136.08</td>
<td>1</td>
</tr>
<tr>
<td>Chuvash Republic</td>
<td>0.94</td>
<td>3 778.06</td>
<td>1</td>
</tr>
<tr>
<td>Perm region</td>
<td>3.02</td>
<td>25 548.58</td>
<td>3</td>
</tr>
<tr>
<td>Kirov region</td>
<td>2.04</td>
<td>7 743.24</td>
<td>1</td>
</tr>
<tr>
<td>Nizhny Novgorod region</td>
<td>1.17</td>
<td>10 738.57</td>
<td>1</td>
</tr>
<tr>
<td>Orenburg region</td>
<td>8.25</td>
<td>4 1920.85</td>
<td>1</td>
</tr>
<tr>
<td>Penza region</td>
<td>0.82</td>
<td>3 348.18</td>
<td>1</td>
</tr>
<tr>
<td>Samara region</td>
<td>1.86</td>
<td>18 845.55</td>
<td>2</td>
</tr>
<tr>
<td>Saratov region</td>
<td>1.63</td>
<td>10 359.38</td>
<td>1</td>
</tr>
<tr>
<td>Ulyanovsk region</td>
<td>0.78</td>
<td>2 875.61</td>
<td>1</td>
</tr>
</tbody>
</table>
Data Science


