Algorithm for Verifying the Stability of Signal Separation for Objects with Varying Characteristics

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***Abstract*—This paper proposes an algorithm for verifying the stability of a solution to the inverse problem of separating individual signals from an additive mixture of several signals. The algorithm is designed for objects whose characteristics vary depending on a certain parameter vector. The paper also considers a version of the algorithm for objects whose changes in characteristics are described by deterministic functions. A feature of the proposed algorithm is preliminary learning, which can help reduce by far its computational complexity and the stability verification time by building a singularity boundary to separate the spaces of stable and unstable solutions. This paper also presents the computer modeling results for the proposed algorithm.**

***Keywords*—signals, mixture, separation, algorithm, characteristics, determination, variation, parameter, stability, boundary, calculation, complexity, learning**

I. Introduction

Signal separation involves solving the problem of extracting individual signals from an additive mixture of several signals that come to measurement points from various sources inaccessible for direct measurement.

The problem of signal separation relates to the class of inverse problems, which may be ill-posed, generally. From that it follows that a solution to the problem may be unstable [1]. For a stable solution to exist, parameters of the object described by the signal formation model (parameters of the mixing matrix $\mathbf{H}$ [1]) must satisfy several prior restrictions [2,3].

Under real operating conditions, the prior restrictions assumed in developing signal separation algorithms may fail to be satisfied. This leads to solutions that are unstable and therefore unsuitable for practical applications.

At present, the stability of a solution to the inverse problem of signal separation is verifiable by using the condition numbers $cond(\mathbf{H})$ [4] and the matrix norm $\|\Delta\mathbf{H}\|_2$ [5] of the mixing matrix $\mathbf{H}$; the singular-direction method [6]; and the algorithm for calculating singular intervals, as well as through comparison with given intervals of stable separation [7].

These methods and the algorithm are effective for static objects, the parameters and characteristics of which virtually do not change during operation or slowly change because of unstable environmental conditions, wear, and the like.

But for dynamic objects whose characteristics vary during operation, applying the methods and the algorithm [7] is inefficient because of their high computational complexity. Indeed, in this case, for each of the many varying states of objects, complicated and time-consuming calculations are necessary to verify stability, and this constrains the application of the methods and the algorithm in real-time systems.

Therefore, developing algorithms to verify the stability of solutions to the signal separation problem in objects with varying characteristics is a relevant problem.

II. Research Area

To state the problem formally, we will consider a mathematical signal formation model presented as a linear multivariable system that has N inputs and M outputs. The model's input signals are $s_n(k)$ and $n = 1,2,\ldots,N$; output signals, $x_m(k)$ and $m = 1,2,\ldots,M$.

The mathematical model of signal formation is described by equations of discrete convolution type (1), where the m th observed signal is an additive mixture of channel-distorted source signals [1]:

$$x_m(k) = \sum_{n=1}^{N}\sum_{g=0}^{G-1} h_{mn}(g,\mathbf{I})\, s_n(k-g), \tag{1}$$

where $h(g,\mathbf{I})$ is the element $N \times M$ of the mixing matrix $\mathbf{h}(g,\mathbf{I})$ for the impulse characteristics of channels; and $g = 0,\ldots,G-1$ and $k = 0,\ldots,K-1$ are the counts for the impulse characteristics of channels and signals, respectively.

Generally, the solution to the inverse problem of separating source signals is the solution to (1), and it can be expressed as

$$\hat{s}_n(k) = \sum_{m=1}^{M}\sum_{g=0}^{G-1} w_{nm}(g,I)\, x_m(k-g), \tag{2}$$

where $w_{nm}(g,I)$ are the impulse characteristics of the separating filters that form the separating matrix $\mathbf{w}(g,\mathbf{I})$, which is equal or close, by a given criterion (in the case of ill-posedness), to the matrix inverse to the matrix $\mathbf{h}(g,\mathbf{I})$.

In the frequency domain, equation (2) can be written as

$$\hat{\mathbf{S}}(\omega) = \mathbf{W}(\omega,\mathbf{I})\,\mathbf{X}(\omega),$$

where $\mathbf{W}(\omega,\mathbf{I}) \cong \mathbf{H}^{-1}(\omega,\mathbf{I})$.

We propose using the singular intervals for the parameters of the mixing matrix $\mathbf{H}(\omega,\mathbf{I})$, whose calculation algorithms are given in [7], as parameters that make the solution stable.

We assume that the current state of the mathematical model is unequivocally determined by the state vector $\mathbf{l}$, whose parameters are set by the current characteristics of the object.

For example, in mobile communication systems, the parameters $\mathbf{l}$ specify the distance between mobile receivers and base transmitters; in vibroacoustic diagnosis systems, these parameters specify the relative positions of mechanisms, such as those determined by the rotation angle of a shaft. Thus, the impulse characteristics of channels $h_{mn}(g,\mathbf{l})$ in the signal formation model change depending on a certain vector, $\mathbf{l}$ [1,7]. The parameters of this vector are determined by on-site sensors measuring displacements, rotation angles, distances, coordinates, and the like.

Therefore, the states of the object (its characteristics) vary during operation as shown in Fig. 1, and they take values corresponding to those of the state vector $\mathbf{l}_0, \mathbf{l}_1, \ldots, \mathbf{l}_d$.

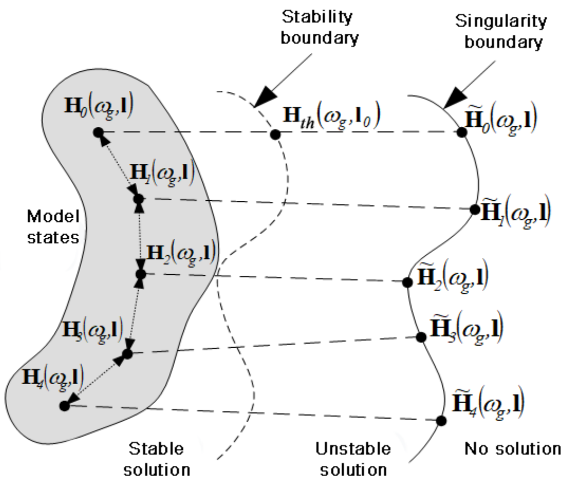

Fig. 1. Graphical representation of an object with varying characteristics.

The region of possible states for the mathematical model is described by the discrete set $\mathbf{H}_0(\omega_g,\mathbf{l}_0), \ldots, \mathbf{H}_d(\omega_g,\mathbf{l}_d)$, which we assume is bounded and finite. We also assume that the matrix of the maximum allowable variation intervals for the parameters $\Delta\mathbf{H}(\omega,\mathbf{l})$ is known beforehand for each state of the object set by the parameter vector $\mathbf{l}$. In Fig. 1 the area highlighted in gray represents the maximum allowable variation interval for object parameters set by prior restrictions.

Let us consider objects for which the elements of the set $\mathbf{H}_0(\omega_g,\mathbf{l}_0), \cdots, \mathbf{H}_d(\omega_g,\mathbf{l}_a)$, which defines the possible states of the mathematical model for the object, and the parameters of the vector $\mathbf{l}$ are linked by a functional relationship. For purposes of further discussion, we will divide objects into two groups. In group 1 objects, variation in characteristics is described by deterministic functions, as in radio communication systems in which mobile receivers follow routes such as roads or railways. In group 2 objects, variation in characteristics is described by random functions.

The purpose of this paper is to develop an algorithm to verify the stability of solutions to the problem of signal separation through calculating singular intervals—an algorithm differing from the known one [1,7] in that it offers extended functionality, allowing signal separation to be verified in objects with varying characteristics.

III. Algorithm to Verify the Stability of Signal Separation for Objects Whose Changes in Characteristics Are Described by Deterministic Functions

The algorithm consists of two stages—learning and verification—which include the following steps.

Step 1. Identify the possible path of variation in the object's state corresponding to the values of the state vector $\mathbf{l}_0, \mathbf{l}_1, \ldots, \mathbf{l}_d$ —that is, describe the region of possible model states with the discrete set $\mathbf{H}_0(\omega_g,\mathbf{l}_0), \ldots, \mathbf{H}_d(\omega_g,\mathbf{l}_a)$.

Step 2. Calculate the norms $\left\|\mathbf{H}(\omega_g,\mathbf{l})\right\|_E$ of mixing matrices for various object states, and determine the parameters of vector $\mathbf{l}$ for which the matrix norms differ by the given value γ.

Thus, a list of object states is compiled for which variation in characteristics is substantial, calling for stability to be verified.

Step 3. For the object states determined in step 2 and the selected type of perturbation (absolute, relative, critical, or their combinations), calculate the following parameter matrices using the algorithm proposed in [1,7]:

The singular matrices $\tilde{\mathbf{H}}(\omega)$, which set a singularity boundary for the region of stable solutions

Matrices of singular intervals for model parameters, $\Delta\tilde{\mathbf{H}}(\omega_g)$, which determine the intervals of model parameters from the initial $(\mathbf{H}(\omega_g))$ to the singular $(\tilde{\mathbf{H}}(\omega_g))$ state

The threshold matrices $\mathbf{H}_{th}(\omega_g,\mathbf{l}_0), \ldots, \mathbf{H}_{th}(\omega_g,\mathbf{l}_a)$— mixing matrices for each of which the condition number $cond\mathbf{H}(\omega_g,\mathbf{l})$ exceeds a given threshold

The matrices $\Delta\tilde{\mathbf{H}}_R(\omega_g)$ and $\Delta\tilde{\mathbf{H}}_S(\omega_g,\mathbf{l})$ for the intervals of model parameters corresponding to stable and unstable separation of signals

The parameters of these matrices and the parameters of the associated state vectors $\mathbf{l}$ are written to a database.

Step 4. For each object state determined in step 2, verify the condition

$$\left|\Delta\mathbf{H}_{max}(\omega_g,\mathbf{l})\right| \le \left|\Delta\tilde{\mathbf{H}}_R(\omega_g,\mathbf{l})\right|. \qquad (3)$$

This verifies whether the model with the preset matrix $\Delta\mathbf{H}_{max}(\omega_g,\mathbf{l})$ for maximum allowable parameter variation intervals falls into the stability region determined by the matrix $\Delta\tilde{\mathbf{H}}(\omega,\mathbf{l})$. If condition (3) is not fulfilled, a message is displayed that a stable separation of signals in the object is impossible and that the given mathematical model cannot be used.

In step 4 the algorithm completes learning, resulting in the parameters for singularity and stability boundaries being calculated and stored in the database according to the parameters of the vector $\mathbf{l}$.

Thus, a function is calculated that determines the stability boundary for signal separation when the object's characteristics vary.

Learning is completed in free time and involves averaging the measured parameters of the mixing matrices $\mathbf{H}(\omega,\mathbf{l})$, allowing a diagnostic model to be obtained for the object. This model is then used at the second stage, described in step 5, to monitor in real time whether signal separation is stable during object operation.

Step 5. For each object state identified in step 2, verify the following condition for stable separation:

$$\left|\Delta\mathbf{H}_{per}\left(\omega_g,\mathbf{l}\right)\right| \le \left|\Delta\tilde{\mathbf{H}}_R\left(\omega_g,\mathbf{l}\right)\right|. \qquad (4)$$

Under this condition, the matrix $\Delta\mathbf{H}_{per}\left(\omega_g,\mathbf{l}\right)$ for parameter perturbation intervals is determined from

$$\Delta\mathbf{H}_{per}\left(\omega_g,\mathbf{l}\right) = \mathbf{H}_{var}\left(\omega_g,\mathbf{l}\right) - \mathbf{H}\left(\omega_g,\mathbf{l}\right),$$

where $\mathbf{H}\left(\omega_g,\mathbf{l}\right)$ and $\mathbf{H}_{var}\left(\omega_g,\mathbf{l}\right)$ correspond to the parameter matrices for the model and the object at a frequency of ω_g for the given state vector $\mathbf{l}$. For the same state vectors $\mathbf{l}$, the matrices $\left|\Delta\tilde{\mathbf{H}}_R\left(\omega_g,\mathbf{l}\right)\right|$ of parameter intervals for stable separation are retrieved from the database for verification under condition (4).

If condition (4) is not satisfied, then stable separation of signals for the frequency ω_g is not guaranteed.

The condition for the stable separation of signals can also be expressed as

$$\left|\Delta\mathbf{H}_{per}\left(\omega_g,\mathbf{l}\right)\right| \le \left|\Delta\mathbf{H}_{\max}\left(\omega_g,\mathbf{l}\right)\right| < \min\left|\Delta\tilde{H}_{mn}\left(\omega_g,\mathbf{l}\right)\right|, \quad (5)$$

where $\min\left|\Delta\tilde{H}_{mn}\left(\omega_g,\mathbf{l}\right)\right|$ is the module of the minimum singular interval for the matrix $\tilde{\mathbf{H}}\left(\omega_g\right)$.

A graphical interpretation of the proposed algorithm is shown in Fig. 2.

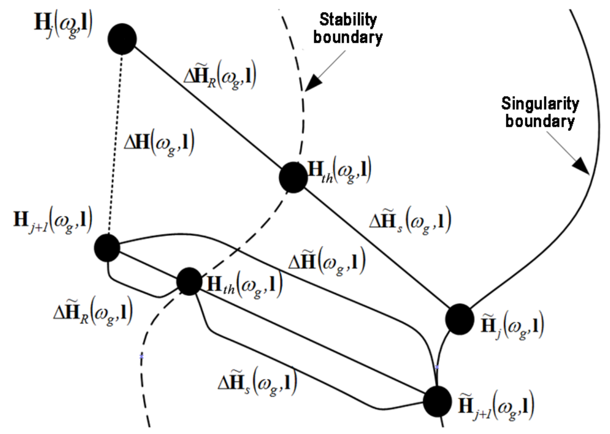

Fig. 2. Graphical interpretation of the algorithm for verifying the stability of signal separation.

Thus, in the proposed algorithm, time constraints (real-time requirements) are only imposed in step 5. This step is simple and comes down to comparing the intervals of perturbations for the object parameters and the parameter intervals for stable signal separation—that is, to verifying conditions (4) or (5).

This expands the possibilities of applying the algorithm to objects with dynamically changing characteristics. Complicated calculations of matrix intervals for the parameters of stable signal separation are removed from real-time constraints and are performed instead in free time at the learning stage as part of building an object model, which is updated rarely (when major changes are made to the object).

IV. Computer Modeling Results

Let us consider monitoring a railroad infrastructure facility by using specialized mobile laboratory cars, with the facility including a track, a contact network, a train radio communication system, and the like.

We assume that the signal generation model for a communication system with two transmitters (mounted at stations) and two mobile receivers (in cars) is described by the mixing matrix $M = N = 2$ with frequency-dependent channels. Signals from the two transmitters as well as reflected signals that form an additive mixture of signals can enter the mobile receivers. Therefore, to make messages encoded in signals accurate, the system should provide stable separation of signals according to their source.

The frequency response of the channels changes when the receivers are moving on the rail-track in relation to the transmitters. An example of the measured frequency response of communication channels for a specific track coordinate (the state parameter $\mathbf{l}$ is the distance) is shown in Fig. 3(a).

For certain track coordinates of the receivers, a change in the frequency response of the channels simulates a stable and unstable separation of signals, and the separation is confirmed by the condition number of the mixing matrix $cond\mathbf{H}\left(\omega_g,\mathbf{l}\right)$ (Fig. 3(b-1) and 3(b-2), respectively).

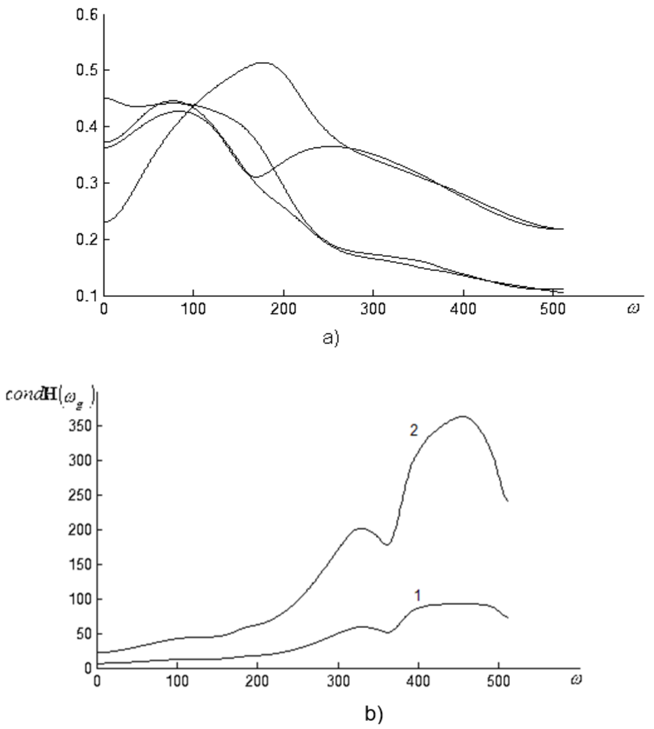

Fig. 3. a) Frequency response of the channels of matrix $\mathbf{H}\left(\omega_g,\mathbf{l}\right)$; b) the relationship between the condition number $cond\mathbf{H}\left(\omega_g,\mathbf{l}\right)$ of the mixing matrix and the frequency.

At the learning stage, the parameters of singularity and stability boundaries for a 75 km track section were calculated and stored in the database according to the track coordinate changed in 1 km increments. Thus, a function was determined that set a stability boundary for signal separation.

Next, at the verification stage, condition (5) for stable separation was verified for all values of track coordinates for the channels' randomly perturbed frequency responses.

The modeling showed that the time taken to monitor the stability of signal separation for each of the coordinates (object states) did not exceed 6 s. This time makes monitoring possible when the receivers are moving at a speed up to 100 km/h, as opposed to static monitoring with algorithm [7].

This enhances the algorithm's functionality and therefore reduces monitoring times. The receivers' speed was modeled on the speed of data transfer to a program that used the stability verification algorithm.

The reliability of the verification results obtained from the proposed algorithm was confirmed by comparing them with those of the known algorithm [7], shown in Fig. 4 and Fig. 5. tability was verified for two track coordinates for which the conditions of stable and unstable signal separation were simulated.

If condition (5) is fulfilled as shown in Fig. 4, then the solution to the problem of signal separation is stable, and triangular test signals are separated from the additive mixture. Otherwise (Fig. 5), the solution is unstable, and no signal separation takes place.

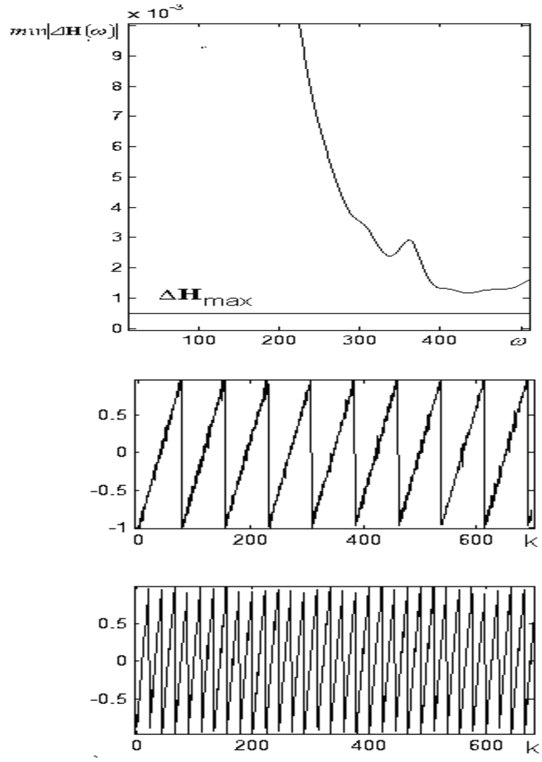

Fig. 4. Modeled stable signal separation [7].

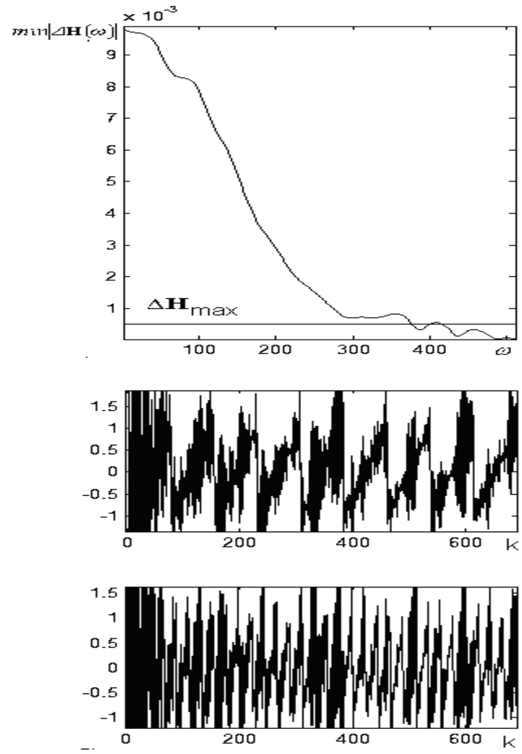

Fig. 5. Modeled unstable signal separation [7].

The modeling also showed that the verification results for the coordinates of stable and unstable signal separation in [7] (at rest) and the verification results for the same conditions obtained in the proposed algorithm (in postlearning motion) are virtually identical.

This provides a proof of continuity of the algorithm's proposed generalized version with its earlier published version [7].

The proposed algorithm is effective for verifying whether solutions to the signal separation problem are stable when object characteristics change anomalously.

The computational complexity involved and the time spent on learning are substantial and require special individual operating modes. As a result, the diagnostic model is only updated when major changes are made to the facility.

Therefore, the learning process (building a diagnostic model) should run when the facility is operating. One of the methods used to follow this approach is the adaptive parametric identification method [8]. Fig. 6. shows a block diagram for it.

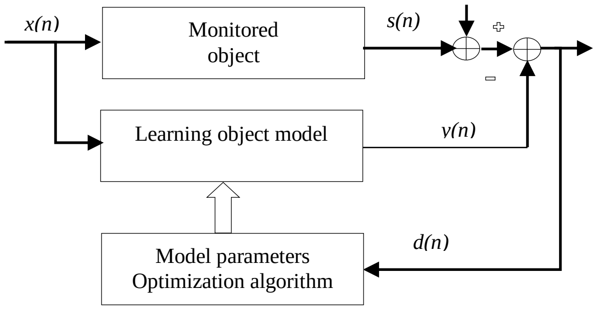

Fig. 6. Block diagram of adaptive parametric identification.

When information is incomplete, optimization problems are most efficiently solved through stochastic algorithms since the efficiency of deterministic algorithms largely depends on the conditions of the problem at hand [9].

The model calculated with the identification method is a digital twin of the object. This model can be used not only to verify the stability of solutions to the signal separation problem but also for predictive maintenance.

Using forecasting methods—for example, [10]—one can calculate variation trends for stability boundaries and predict the evolution of the diagnostic model.

V. PRIMARY CONCLUSIONS

We developed an algorithm to verify the stability of solutions to the problem of signal separation through calculating singular intervals. The algorithm is characterized by extended functionality that allows the stability of signal separation to be verified for objects whose characteristics are described by deterministic functions.

With learning incorporated in the proposed algorithm, it takes far less time to verify stability, making the algorithm suitable for use in real-time systems.

Our computer modeling results confirmed the efficiency of the solutions proposed.

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