Invariant Image Recognition of Objects Using the Radon Transform

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Abstract - One of the types of tasks solved using computer vision may be such a task as determining and isolating a test object from a series of images. For this, a very important point is the definition of invariant features. The aim of the study is to develop a method for quick image verification using invariant projection features. For this, the principles of finding invariant signs of images are examined using medical electrocardiographic images using the Monte Carlo method and the possibility of recognizing it using the Radon transform. Invariant features that are independent of the physical and psychoemotional state of a person are identified, allowing biometric identification of a person. It was determined that the most informative invariant features of electrocardiographic images are the amplitude values in the S- and T-regions of the electrocardiogram. The Monte Carlo method was used to sample the significance of the considered features characterizing the electrocardiogram. The combined use of these features allows biometric identification of the person with high accuracy.

Keywords: computer vision, invariant signs, Radon distribution, cardio signal, signal verification, personality identification.

1 Introduction

The task of computer vision is currently considered to be an urgent problem. One of the types of tasks solved using computer vision may be such a task as determining and selecting a test object from a series of images [1,2,3,4]. For this purpose, it was necessary to determine the signs being invariant to rotation, shift, and scaling, since real images can be subject to various transformations and be noisy, and then select those images that are similar to the test image [5-10]. The standard information processing scheme for solving the recognition problem is presented in fig.1 [11,12].

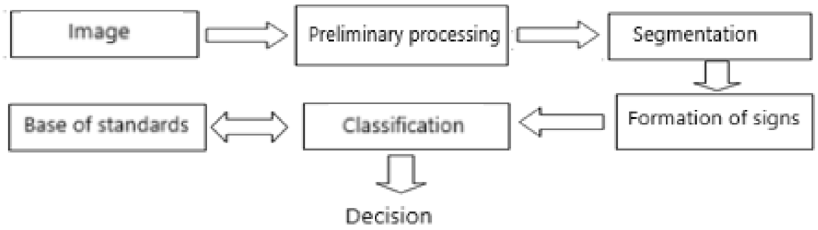

Fig.1. The standard information processing scheme for solving the recognition problem

In the field of image recognition, an important place is occupied by the problem of ensuring the invariance of recognition with respect to the shift, scale and rotation of images, i.e., the system must recognize an object regardless of its orientation, size and location in the field of view. There are three main approaches to the construction of invariant recognition systems [13,14]. The first of them is associated with the use of a large set of training images, which sufficiently fully displays the recognized images in all possible situations. The second is associated with the preliminary transformation of images and the formation of invariant features, which are then used in the classification of images. And the third approach is associated with the creation of a neural network recognition system, in which the invariance of features is provided by a special structure of the neural network. Studies in [13-16] have shown that in the first approach to ensure invariant recognition, the number of training images must be large. The number of training images increases with an increase in the desired invariant parameters. When recognizing images that are simultaneously invariant to three transformations (shift, scale and rotation), the number of training images will be too large. This approach is simple and intuitive and is suitable for a number of practical tasks. In practice, this approach can find application in conjunction with other approaches, for example, using pre-processing to ensure invariance to the shift and scale of images; in this case, in the training set, only images are required that adequately reflect the recognized images at all possible angles of rotation. Therefore, the number of training images is significantly reduced. Therefore, when creating an image recognition system, this method is one of the alternative approaches and should be investigated. In the second approach, invariant features are created with using mathematical transformations. Some transformations, for example, the Fourier transform were applied [17] to provide shear and rotation invariance. Linear interpolation and Hotelling transform are used to provide translation and scale invariance in [18]. Orthogonal transformation for obtaining recognition features invariant to affine transformation is considered in [19]. The method of moments was used in [17-21]. Note that the method of moments for the formation of invariant recognition features is used most often. The theoretical foundations of the method of moments are detailed in [17-21]. Studies in [20] also showed that Zernike and pseudo-Zernike moments are more effective than other moments in terms of sensitivity to image noise, the amount of useful information and the ability to reproduce the image. It was shown in [22, 23] that the reproduction and classification of English symbols by means of Zernike moments give better results than by means of geometric moments. Note, however, that in this study, noise at different levels was added to the normalized images, which are invariant to shift and zoom, and not to the original images. Therefore, the results obtained do not fully reveal the effect of noise on the entire recognition process.

In [24], an automatic classifier of geometric moment features was created using parametric and nonparametric classification algorithms. The recognition quality is low, the processing time is long, and the influence of interfering factors (noise, sampling of the image rotation angle ...) on the classification quality was not taken into account. Further, in [25], a classifier of Zernike moments is proposed based on the Kohonen self-organizing neural network; the classification accuracy turned out to be low. In [26], geometric moments were used to interpret ship images. However, the interpretation error is large. In [27], complex moments are used to normalize and classify images. However, the neural network was not applied in this work, and quantitative results are not presented. In [28], a set of normalized inertial moments and topological characteristics of objects was used as a feature invariant to rotation, shift, and image scale. The classification was performed based on the modification of the nearest neighbor rule. The application of this approach is relatively complex and requires a lot of computation. The creation of a neural network recognition system, in which the invariance of features is provided by a special structure of the neural network, is a new direction of research in the field of image recognition. Higher-order neural networks that make it possible to implement invariance to transformation groups are investigated in [29,30]. In practice, the application of this approach is limited due to the large dimension of the network; the study also showed that higher-order neural networks are significantly inferior to the method of moments. In [31, 32, 33], a model of the neo- cognitron neural network is presented, which provides invariance to shear and small deformation of images. The main disadvantage of this model is that the number of network elements increases with an increase in the number of recognized objects. This causes an increase in network learning time. It follows from the analysis that the creation of image recognition systems that are invariant to rotation, shift and scale remains an important and urgent task. There are several main groups of methods for analyzing and recognizing medical images [34-37]. Most of the methods are focused on the selection of characteristic features and their comparison with each other both in a numerical and in a structural context, which is often an implementation of methods and algorithms being quite computationally difficult. One of these approaches is associated with the Trace/Radon transformations [36,37], based on the calculation of projecting image functional. The aim of the research is to develop a method for prompt verification (or recognition) of a cardio-signal image using invariant projection signs. Verification means a special case of recognition, i.e. testing the hypothesis about whether the input and reference images are representatives of the same equivalence class. The decomposition of the image (scanning) into a set of projections has become widespread in terms of computer vision problems, because projections contain significant potential for obtaining the necessary set of invariant signs without essential computational expenditures in comparison with other methods [34,35]. At the moment, for the stages of segmentation and formation of signs, there is no strict theoretical solution, since in the presence of noise (interference) these tasks are incorrect. And therefore, a high-quality solution to the image recognition problem at these stages is not possible, especially in real time scale [34,35]. Determining the signs invariant to geometric transformations in recognition problems of two-dimensional graphic objects is an important task. The task was to find signs being independent on scale, shifts and rotations, in other words, invariant signs. Invariants allow for the correct comparison of images subjected to geometric transformations. And this in turn leads to the right decision making. In order to determine the object signs, it is convenient to use the Radon transformation.

2 Theory

The Radon transformation can be used to select the characteristic features of images, which in the future can be used to recognize the desired image. We will consider a binary image. The physical meaning of the transformation for a two-dimensional image of Radon is to find the sum of the pixels forming this image along a straight line in the direction of transformation. The results of these transformations will be a two-dimensional array of numbers [36,37].

The Radon transformation R (k, b) of the continuous function f (x, y) is calculated by integrating (adding) the values of *f* along the inclined line, as shown in fig. 2 [6]:

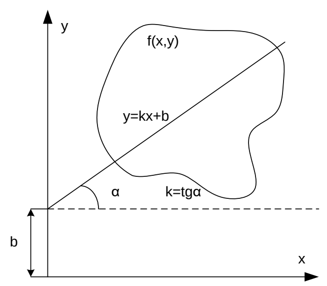

Fig. 2. Linear Radon Transformation

Then the expression for the Radon transformation can be written as follows (1):

$$R(k,b)=\int_{-\infty}^{\infty} f(x,kx+b)dx \quad (1)$$

Or using the Dirac δ-function, the following expression can be obtained:

$$R(k,b)=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y)\delta(y-kx-b)dxdy \quad (2)$$

It should be noted that transformations (1) or (k, b) have some properties that are very important for working with images, such as the property of linearity (3), shift (4), and scaling (5). The linearity property can be formulated as follows: “The Radon transformation of the suspended sum of functions is equal to the suspended sum of transformations of each function”:

$$R\{\sum_i w_i f_i(x,y)\} = \sum_i w_i R\{f_i(x,y)\} \tag{3}$$

Properties (4) and (5) (shift and scaling) show how the transformation (k, b) is calculated when the arguments of the integrable function change.

$$R\{f(x-\tilde{x}, y-\tilde{y})\} = R(k, b-\tilde{y}+k\tilde{x}) \tag{4}$$

$$R\{f(\frac{x}{n}, \frac{y}{m})\} = nR(\frac{kn}{m}, \frac{b}{m}) \tag{5}$$

Since the aim of the research is the medical curve signal, it should be said that any signal consists of a set of points in the general case, and straight lines in the particular case. Therefore, we consider these cases.

Any point can be represented as a product of 2 δ-functions (6):

$$f(x,y) = \delta(x)\delta(y) \tag{6}$$

Then the Radon transformation for a point can be represented as follows (7):

$$R(k,b) = \int_{-\infty}^{\infty} \delta(x)\delta(kx+b)dx = \delta(b) \tag{7}$$

Using the shift property, we obtain the following expresison (8):

$$f(\tilde{x}, \tilde{y}) = \delta(x-\tilde{x})\delta(y-\tilde{y}) \tag{8}$$

The Radon transformation in this case will be considered as follows

$$R(k,b) = \delta(b-\tilde{y}+k\tilde{x}) \tag{9}$$

Thus, the Radon transformation of a point has a straight form (fig. 3).

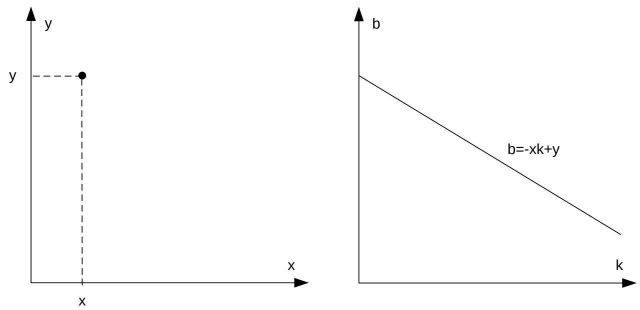

Fig.3. Single point transformation

Accordingly, for a straight line (Fig. 4) defined by the equation y = kx + b, we get the following expression:

$$f(x,y) = \delta(y-\tilde{k}x-\tilde{b}) \tag{10}$$

Radon transformation in this case will be as follows: (11)

$$R(k,b) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\delta(y - \tilde{k}x - \tilde{b})\delta(y - kx - b)dxdy = \int_{-\infty}^{\infty}\delta((k - \tilde{k})x + b - \tilde{b})dx \qquad (11)$$

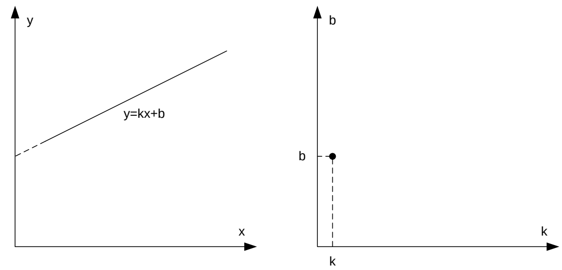

Fig. 4. Straight line transformation

As invariant values for verification, we will use the signs based on projection analysis of the image and geometric moments [34,35,36]:

Where *B (x, y)* is the analyzed image; *R (p, θ)* is the Radon transformation; *k* is the order of the moment (depends on the problem being solved, as well as on the computational and time constraints of the project); θ is the angle of projection. Signs (6) combine structural information about the image with a numerical description, which allows training and recognition based on a comparison of numerical data [36]. Signs (6) are resistant to scaling distortions and object displacements within the field of view. Invariant signs can be constructed both directly on the basis of expression (6) using the brightness function *B (x, y)*, and using modifications of the form $\Psi(B(x, y))$, where Ψ is some transforming function. The first method corresponds to the Radon transformation, the second is called the Trace transformation, which extends the set of possible invariant signs. According to expression (6), the main characteristic of invariants I_k, is the order of k. For example, signs of the first and seventh order may differ in value by more than a thousand times, which makes it impossible to compare them without preliminary normalization. In order to normalize invariants I_k, we introduce a transformation to a certain fixed interval [-10; 10]:

$$F = 10^{f-1},\ \ f = \begin{cases} \min\limits_{i}(I_k \times 10^i < -10), I_k < 0 \\ \min\limits_{i}(I_k \times 10^i > 10), I_k \geq 0 \end{cases}$$

where $i \in [0;100]$ is an integer (integer-valued) parameter. Due to the proposed transformations, we obtain normalized signs of the same order.

Let *P* be a sign or set of signs, *M* be the set of signs of the image $Q: M \rightarrow P$ is called invariant with respect to the group of geometric transformations *R* if $Q(rB)$ $=Q(B)$, $r \in R$ is true for it. The following signs may be invariant signs.

1. The figure area (the number of pixels inside the figure). It is invariant to displacements, but not invariant to scaling.

2. The contour length requires much more computation than area. The contour length is invariant to displacements and rotations, but it also depends on scale,

including the area sign. Let S (B) be the area, L (B) be the contour length. Then the sign can be introduced as $V(B) = \frac{\sqrt{S(B)}}{L(B)}$ -: it is invariant to displacements, rotations, and scaling.

3. The sign is also invariant introduced as follows:

$$V_1(B) = \frac{\sqrt{S(B)}}{L^2(B)}$$

however, it cannot be used together with sign 2, because these two signs are interconnected.

4. Consider the shortest d_{min} and the largest d_{max} distance from the center of the figure mass (hbc/5). These signs are invariant to displacements and rotations, but depend on scale. However the ratio of these distances does not depend on scale.

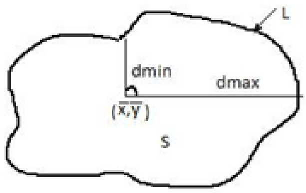

Fig. 5. The shortest d_{min} and largest d_{max} distance from the center of the figure mass

Then the invariants to the operation of scaling and rotation are like: and $\frac{L}{2S}$. If a circle is described around an individual image, then ratio $\frac{S}{S_R}$ _R will be an invariant to rotation and scaling. Here *S* is the image area, S_R is the area of the described circle.

3 Sign selection

Based on the aforementioned statement, a method for comparing and recognizing a complex image can be proposed. For this, individual images can be represented as a set of circles inscribed in it. Thus, the reference and tested images are compared with a vector, the components of which are the radii and coordinates of the corresponding circles centers. Based on this, the distance between the images can be found.

In order to do this, similarity measures in the sign space are determined.

Q_l (P_1, P_2 , …, P_r) - sign vector describing an object.

Q_0 (P_1^0 ,P_2^0 ,…, P_r^0) - sign vector describing another reference object.

If the objects are the same, then the signs will coincide. But if there is noise (interference), the signs may vary. From this we can derive the following image recognition rule. The sign vector of the input image is compared with reference vectors. The object should be assigned to a class where the similarity will be the greatest. Let k = 1,2, ..., s be a finite set of classes. Each class has a reference, i.e. it has totally s standards. The selection of signs allows us to simplify the implementation of recognition or identification of objects. When selecting the most informative signs, it is necessary to take into account both the properties of the objects themselves and the capabilities of primary driver image signals. Sign selection is carried out by the sample of processing monochrome (single-layer) images. In colour images, the considered algorithms can be applied to each colour separately. When processing, the following geometric signs of objects are usually preferred: area and perimeter of the object image; dimensions of the inscribed simple geometric figures (circles, rectangles, triangles, etc.); number and relative position of angles; moments of inertia of object images. An important feature of most geometric signs is invariance with respect to the reversal of the object image, and by normalizing the geometric signs with respect to each other, invariance with respect to the scale of the object image is achieved. In the early 2000s, it was established that electrocardiograms (ECG) contain invariant signs that are independent on the physical and psycho-emotional state of a person, allowing biometric identification of an individual [36,37]. It is believed that ECG is practically impossible to fake [37].

Despite the great prospects for identifying an individual by ECG, there are a number of problems. In particular, there is no consensus of opinion on which biometric signs are best used for identification. Existing methods of identifying an individual by ECG nowadays can be divided into two groups: those that use characteristic ECG points (fiducial marks) and those that do not use them. The first category of methods includes approaches using temporal, amplitude, and morphological features of an electrocardiogram, and the second one includes the methods based on the analysis of autocorrelation, phase, and frequency properties of an ECG. Most often, temporary signs are used, including the duration of various phases of the cardio-cycle (Fig. 6) and time intervals between them, sometimes the distances between the R-peaks are used [36,37]. A number of authors indicate interpersonal variability of the cardio-cycle peaks amplitude, measured relative to the R-peak (Fig. 7) [38,39,40]. As a result of the abovementioned statement, we will consider the values of the amplitude and time of the cardio-signal points as signs. Next, we will process the obtained material using the Radon distribution in order to check for the invariance of these informative signs.

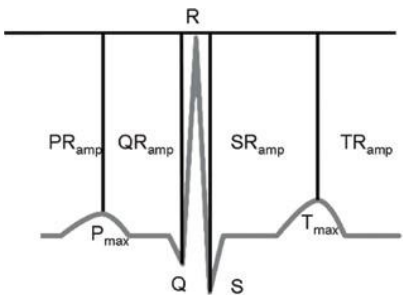

Fig.6. Temporary signs of a cardio-cycle

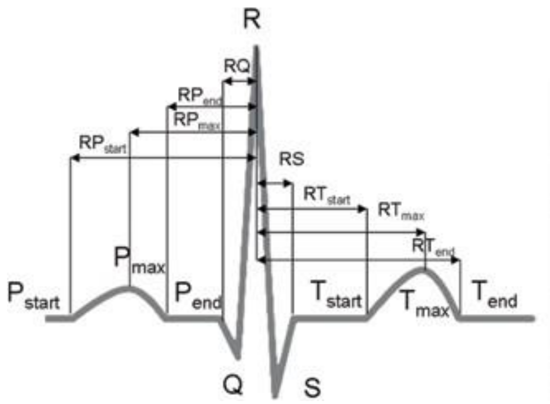

Fig. 7. Amplitude signs measured relative to the R peak

4 Experiment

When conducting the research, the database of digitized ECGs (Physikalisch-Technische Bundesanstalt, PTB) was used, provided by the professor of the cardiology department named after Benjamin Franklin University Hospital in Berlin, Germany, Michael Oeff, German National Institute of Metrology under the Physio - Net project [35]. PTB contains 549 ECG samples measured in 290 tests aged 17 to 87 years. Most of the tests suffered from various disorders of the cardiovascular system, the control group included 51 healthy tests. In the proposed research, ECG of the control group of the research (n = 51) was used. Each ECG record included 12 common diversions (recording) (i, ii, iii, avr, avl, avf, v1, v2, v3, v4, v5, v6) and three Frank diversions (recording) (vx, vy, vz). The sampling frequency was 1 kHz, and the resolution was 16 bits in the voltage range of ± 16.384 mV. The initial cardio-signal was proposed to be cut into fragments of 700 or 1000 ms, and then synchronize the fragments obtained by R-peaks [37]. At the preliminary processing stage, P, Q, S, and T- areas were determined on the fragments of the electro-cardio signal (electrocardiogram). The areas of R peaks were not determined, since fragments were synchronized from them (Fig. 8). As it is seen from Fig. 8, the point clouds have a rather large dispersion. As it is seen from figure 8, after preliminary processing of the electro-cardio signal (electrocardiogram), four point clouds were obtained corresponding to the P, Q, S and T-areas of the cardio-signal. Each point has two coordinates - one along the amplitude axis, the second along the time axis. That is, in total there are 8 signs (P_{value}, P_{index}, Q_{value}, Q_{index}, S_{value}, S_{index}, T_{value}, T_{index}) corresponding to the values of the amplitude and time of the PQST areas of the electro-cardio signals. New signs were generated using the Monte Carlo method on the base of the obtained signs. The most informative signs were the values of the amplitude in S and T areas of the electro-cardio signal (electrocardiogram). The combined use of these signs allows biometric identification of an individual with an accuracy of 100%.

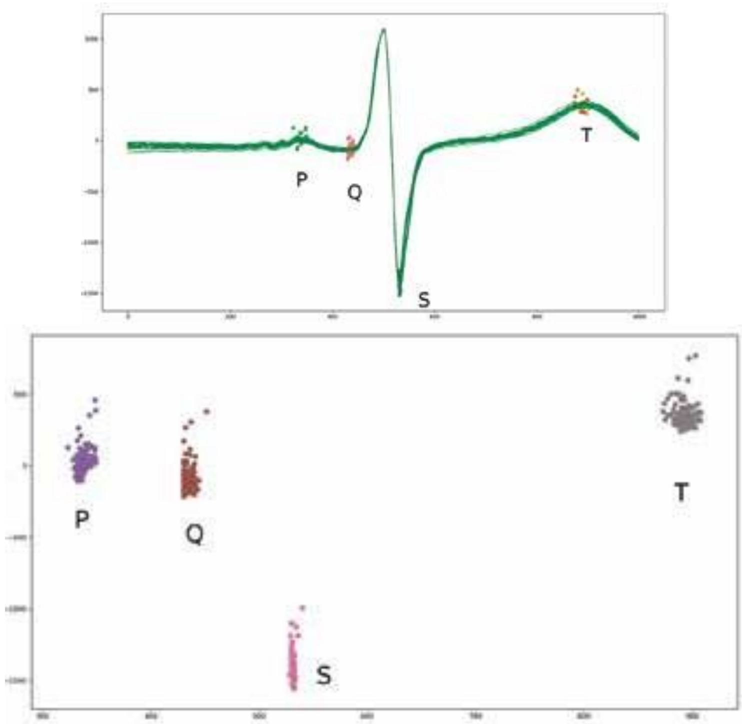

Fig. 8. Point clouds corresponding to P, Q, S and T-areas of the electro-cardio signal (electrocardiogram)

The next step was to apply the Radon transformation using selected invariant signs. Direct transformation work will create projections at angles from 0 to 179 degrees. The inverse Radon transformation will collect the desired image.

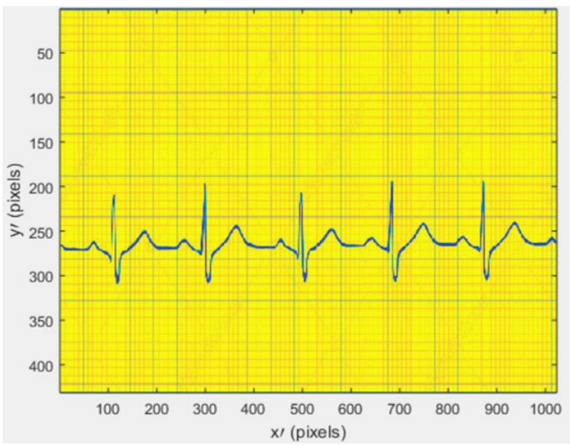

Fig.9. Initial images

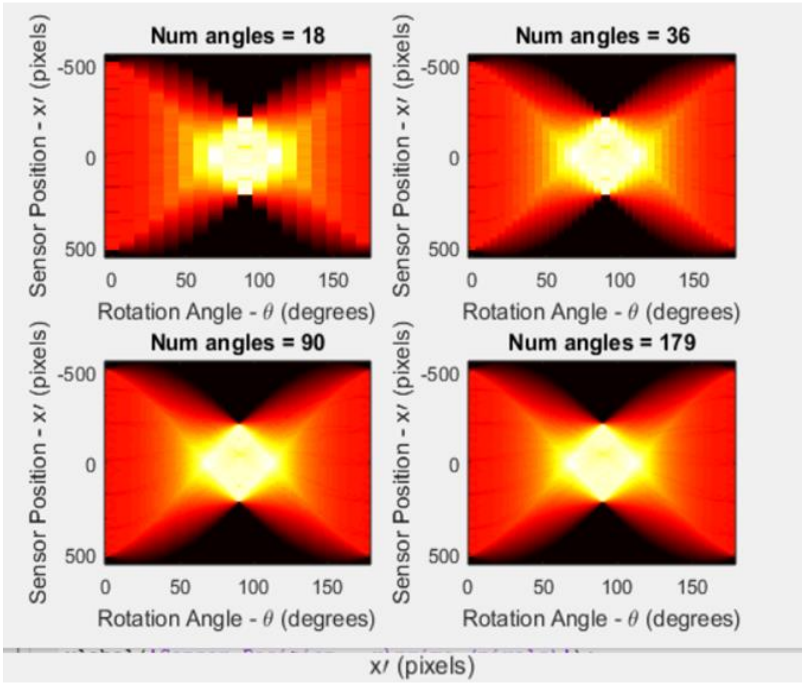

Fig. 10. Direct Radon transformations with various angles theta

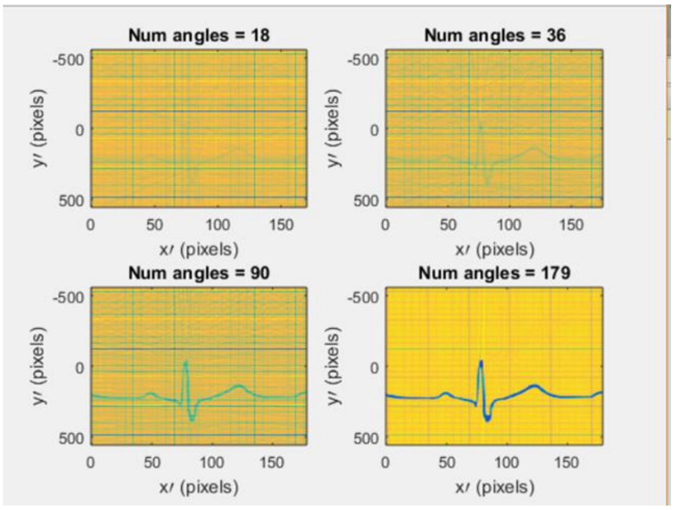

Fig. 11. The inverse Radon calculation with various angles theta

The root-mean-square relative error was calculated, root-mean-square error, maximum relative error, root-mean absolute standard error of the Radon transformations was normalized for various angles.

In order to test the hypothesis about whether the input and reference images are representatives of the same equivalence class, one of the simplest in the computational aspect for comparing projection descriptions is the Manhattan metric:

$$\rho_M(B, B_0) = \sum_k \sum_\theta | I_k(\theta) - I_k^0(\theta) |$$

Where B_0 is an image sample; B is a source image; $\{I_k^0(\theta)\}$ is a set of signs, B_0, $\{I_k(\theta)\}$ is a set of signs B.

As a result, a decision is made either on the measure of similarity, or the measure of difference. Depending on the task, the decision is to divide the studied set of objects into groups of “similar” objects, called clusters. In order to determine the “similarity” of objects, a proximity measure called distance is introduced.

The task of clustering is to build a set as follows:

$$C = \{c_1, c_2, \ldots, c_k, \ldots, c_g\}$$
$$c_k = \{i_j, i_p | i_j \in I, i_p \in I \text{ и } d(i_j, i_p|) < \sigma\}$$

Here c_k is the cluster, σ is the quantity determining the proximity measure for including objects in one cluster; $d(i_j,i_p)$ is the proximity measure between objects, called distance.

If the distance $d(i_j, i_p) < \sigma$, then the elements are said to be close and placed in one cluster, otherwise the elements are said to be different from each other and placed in different clusters.

5 Conclusion

1. The principles of finding invariant signs of images were examined using medical electrocardiographic images using the Monte Carlo method and the possibility of recognizing it using the Radon transform.
2. Studies have shown that electrocardiograms are indeed virtually impossible to fake and they contain invariant signs independent of the physical and psycho-emotional state of a person, allowing biometric identification of the person. The principles of finding the invariant signs of signals and cardio-signal in particular were considered. Using the Radon transformation, the possibility of signal recognition was realized, and using the Manhattan metric, the possibility of signal verification was shown.
3. Studies of the point cloud of P, Q, S, and T-regions obtained from the electrocardiosignals of the test control group of people showed that the most informative invariant signs of electrocardiographic images are the amplitude values in the S- and T-regions of the electrocardiogram. The Monte Carlo method was used to sample the significance of the considered features characterizing the electrocardiogram. The combined use of these features allows biometric identification of the person with high accuracy.
4. In the case of direct Radon conversion for the subjects, an electrocardiogram is constructed according to the results of the study. In the inverse problem, when,

according to the results of the study of the electrocardiogram, the amplitude values in the S and T regions of the electrocardiogram can verify the person's personality. In the version of the mathematical formulation, this is the restoration of the value of a function from the known values of the integrals of this function, calculated from the elements of a certain set of surfaces, i.e. the function itself is unknown; only a set of linear or surface integrals of this function obtained as a result of experiments are known.

5. The clarity of the image with the inverse Radon transform increases with an increase in the angle of rotation, because the projections of the electrocardiogram are formed by measuring the intensity of the radiation passing through the physical object at different angles.

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