# Using non-metric multidimensional scaling for assessment of regions' economy in the context of their sustainable development

Pavlo Hryhoruk[0000-0002-2732-5038], Svitlana Grygoruk[0000-0003-3047-2271],
Nila Khrushch[0000-0002-9930-7023] and Tetiana Hovorushchenko[0000-0002-7942-1857]

Khmelnytskyi National University, 11 Instytutska Str., Khmelnytskyi, 29016, Ukraine
violete@ukr.net, grygoruk.svitlana@gmail.commailto:,
nila.ukr@gmail.com, tat_yana@ukr.net

**Abstract.** Solving the problems of regions' socio-economic development is strategic and most important for any country. In particular, the implementation of a new, active role of the region as a subject of sustainable development is important for the direct implementation of current regional policy. An important component of such a policy is the assessment of sustainable development of regions, which contributes to the timely detection of internal and external threats, the development of necessary stabilizing measures to prevent their negative impact, the formation of strategies aimed at sustainable regional systems. The economic system is an important subsystem of the region. The article proposes an approach to assessing the regions' economic development in the context of ensuring their sustainable development. We used the methods of multidimensional nonmetric scaling to solve this problem. The study aims to determine the structure of regions in the context of their sustainable development. Based on non-metric data reflecting the economic development of Ukraine's regions, two-dimensional space of latent scales was built based on multidimensional measures of proximity between them, and the positioning of regions in this space was carried out. The results received a semantic interpretation, which was improved by using the procedure of rotation of the scale space. The use of multidimensional non-metric scaling confirms its usefulness for the study of economic development of regions in the region and allows for their comparison and dynamics of their structure in the context of sustainable development.

**Keywords:** sustainable development, non-metric multidimensional scaling, region, two-dimensional space.

## 1 Introduction

### 1.1 Problem description

Regional development issues remain relevant for every country, as unreasonable and sporadic regional development policies can lead to growing disparities and exacerbation of economic, political, environmental, and social problems. In the context of European integration processes, the main goal of the state regional policy is to create conditions for dynamic, balanced development of territories, eliminate the asymmetry of development of the regions of Ukraine, intensify the involvement of human resources to ensure the competitiveness of the region's economy.

Sustainable development is a modern worldview, political and practical model of development for all countries of the world, which have started the transition from a purely economic model of development to finding the optimal balance between the three components of development - economic, social and environmental. This category is perceived around the world as a model of civilized development. Implementation of this model requires the formation of a system for managing such development.

In September 2015, during the 70th session of the UN General Assembly in New York, the UN Summit on Sustainable Development took place and adopted the 2030 Agenda for Sustainable Development. It approved new development benchmarks [31]. Summit issues covered all aspects of socio-economic development, in particular, countries' competitiveness, environmental and energy security, a global partnership for development, and were based on the principle of "Leaving no one behind". Summit Outcome Document contains 17 Sustainable Development Goals and 169 Goals. This led to the update of the Sustainable Development Strategy of Ukraine until 2030 [25]. The Strategic vision of Sustainable Development of Ukraine is focused on overcoming the imbalances that exist in the economic, social, environmental spheres and is based on the vectors defined in the Sustainable Development Strategy "Ukraine 2020" [24], one of which is the vector of development. It foresees the sustainable development of the country, carrying out structural reforms, ensuring economic growth in an environmentally sustainable way, creating favorable conditions for economic activity [25].

Thus, at the present stage of development of Ukraine's economy, the problem of transition to the sustainable development of both the country as a whole and each of its regions is urgent. The balanced region's development should be oriented towards providing conditions that will allow each region of the country to have the needed and sufficient resources to ensure decent living conditions, comprehensive development and increase the competitiveness of the economy.

On the one hand, sustainable development of the region may be seen as a positively directed process of improving the economic, social, and environmental components. On the other hand, it is considered as a process to achieve a balanced state for all of these components.

The assessment of the level and state of sustainable development of regions' economy is necessary to identify internal and external threats, which will allow devising measures to prevent their negative impact to identify scenarios of development and to develop an optimal strategy for the functioning of the country's regional economic systems.

Given this, an important role in solving these issues belongs to the analytical tools, the main tool of which is economic and mathematical modeling and modern information technology. Their application will identify trends and imbalances in the economic development of regions, carry out their structuring, and, ultimately, will contribute to the development of sound management decisions.

## 1.2 Literature review

At the initial stage of its development, the concept of sustainable development was associated with environmental sustainability, but soon it was also used to describe social and economic sustainability. In the modern sense, sustainable development is perceived as a socio-economic system that meets human needs, but also long-term progress towards prosperity and improving the overall quality of life.

We support the opinion of the authors of [16] that the concept of sustainable development concerns not only the well-being of people but also the world where people live, so it can be understood as a holistic philosophy that includes classical philosophical perspectives and harmonizes and integrates economic, social-political and environmental system.

The issues of assessing the level of sustainable development at both the state and regional levels remain the subject matter of the attention of many scholars. For regional systems, economic, social, and environmental components are traditionally taken into account. In our study, the main attention will be paid to assessing the sustainable development of regions by their economic component of development. In this case, it is advisable to consider sustainable development in the context of its sustainability, which according to [34] we will understand the property or quality that determines the ability of the regional system to be in a state of dynamic equilibrium in the presence of external and internal influences. It can be inherent not only in the fixed state of development of the regional system but also considered in dynamics. As the main types of such stability can be distinguished:

- the stability of development, characterized by a systematic increase in the result, which is not lower than the acceptable minimum and not higher than the objectively determined maximum;
- as permanent stability when changes, including positive ones, occur only occasionally and for a short time;
- as hyper-sustainability like a state where regions are not susceptible to development but can adapt to changes, including positive ones.

The regions' economic development is characterized by a large number of different indicators, and therefore is essentially multidimensional. This presupposes the use of multidimensional economic and statistical modeling tools to reduce the state of economic development and reduce the space dimensionality of the original characteristics.

The analysis of publications showed a variety of methodological approaches to assess sustainable development, which is determined by the identified goals to obtain evaluations. At the same time, most of the methodological approaches involve calculating a composite (integrated, comprehensive) index of sustainable development based on the use of additive or multiplicative convolution. In some cases, the author's techniques involve the analysis of the output without the convolution, in particular, the indicative method of evaluation. In our view, narrowing the assessment outcome to a single indicator makes it easier to conclude, but on the other hand, this approach causes a "compensation effect" when the low values of some indicators are offset by the high values of other ones. This disadvantage can be partially eliminated by using a weighted convolution of the initial indicators.

The paper [21] proposes a methodological approach to the assessment of sustainable development of Ukrainian regions, in which the overall assessment of its level is carried out using a composite index, based on the additive convolution of indicators of social, economic and environmental components. An integrated assessment of the sustainable development of Ukraine's regions is carried out and problems of regional development in the social, economic, and environmental fields are identified. It should be noted that the author proposes to use the financial statements, which, in our opinion, limits the application of the proposed methodology.

Paper [27] proposes to assess sustainable development on generalized indices, such as Green GDP; Human development index; Genuine Progress Indicator; Index of Sustainable Economic Welfare; Happy Planet Index; Environmental Sustainability Index; Environmental Performance Index, which is converted into a generalized index of sustainable development by weighted additive convolution. The authors provide a scale for determining the level of sustainable development, but the authors do not justify the boundaries of the levels of such a scale. Also, in our opinion, the practical use of this approach is limited by the difficulty of obtaining statistics for the selected system of baseline indicators.

The paper [9] proposes an approach to the construction of an integrated indicator, which allows assessing the level and dynamics of economic development of the region in the context of ensuring its sustainable development. A feature of the proposed approach is the simultaneous use of both metric and non-metric indicators, and the calculation of weights for components is based on the values of factor loads of the first main component, calculated for a set of metrics. Clarification of weights is carried out with the help of non-metric indicators. However, the article does not substantiate the limits for determining the levels of sustainable development.

I. V. Horiana [8] distinguishes infrastructure and innovation components in addition to the traditional part of sustainable development to design a composite index. The author defines the rules of composite index calculation based on the multiplicative convolution and formulates the conditions under which sustainable development is reached. A similar approach is implemented in the papers [7; 10]. However, these approaches leave open the issues to identify the required initial data set.

The study of O. O. Nesterenko [20] uses an approach based on the scoring model to assess the sustainable development composite index. At the same time, the author proposes to use both the statistical reporting indicators and questionnaire outcomes of the evaluation of the several components of sustainable development. The commonly proposed method uses recognized indicators of human development like the Knowledge Index, the Human Capital Development Index, and the Human Development Index. It should be noted, that some of the used indicators have a non-numerical origin and therefore their mathematical processing is not always correct.

Papers [1; 3] proposed a set of criteria for assessing the sustainable development of a region based on the use of both metric and non-metric indicators. But authors do not specify the rules for processing data and constructing the resulting metric.

Paper [14] presents the author's methodology for assessing sustainable development for the Czech Republic according to four components: Political area, Social area, Economic area and Environmental area, which uses 101 output indicators and provides for the calculation of a hierarchical integral index system, which includes 12 partial indicators, 4 partial composite indicators and one aggregate integrated index of sustainable development.

The authors of the paper [35] compared the situation of Central and Eastern Europe in terms of sustainable development based on calculating the integrated indicator. The comparison was based on indicators that emerged from the thematic areas of the EU Sustainable Development Strategy, in particular, socio-economic development, demographic change, health status, climate change, energy, and others. According to the results obtained, the countries were ranked, and the gap between the countries of Central and Eastern Europe in comparison with the average level of the EU countries was assessed.

The study [26] presents an original approach to the calculation of a comprehensive sustainable development index based on the case of Indonesia's regions. Authors propose three measures for indices: arithmetic, geometric, and entropy-based. Indices are aggregated to use for comparing regions in terms of their stability. The article also analyzes the sensitivity of the results obtained. It should be noted that despite the ranking of regions by the value of the integrated indicator of sustainable development, the authors were not offered a scale to estimate the level of sustainable development of regions.

Studies [2; 32; 11] have presented approaches for assessing the degree of achievement of the 17 Sustainable Development Goals identified in [31] in the case of Asian countries.

The conducted analysis of publications makes it possible to conclude that the presented approaches are based on the UN Sustainable Development Concept using some differences in the structure of components and the number of partial indicators. The advantages of the composite indexes for evaluation of various aspects of sustainable development include the simplicity of their calculation and the ease of results interpretation. However, the approaches don't contain criteria for identifying the level of sustainable development. A significant disadvantage of these methods is the use of an overloaded set of partial indicators, which, moreover, don't always correspond to the system of national statistics. This fact creates a multiplier effect and complicates the assessment in dynamics. There is also a methodological problem to select indicators to be included in the index and with the identification of weights of partial indicators.

Considerable attention of researchers in assessing sustainable development is paid to the application of methods of multidimensional statistical analysis. The authors of [12] considered the construction of cluster models describing the development of agritourism in the context of ensuring sustainable development of rural areas. This allowed us to identify the leading factors of the sustainable development of these areas.

The article [23] presents an approach to assessing the sustainable development of regions, which is based on a combination of methods of factor and cluster analysis. The first is used to construct integrated partial indices of sustainable development for each of its components, and the second - to group regions by the level of sustainable development. The authors propose to use the obtained cluster map of regents and municipalities for decision-makers to take action, to take measures, and determine appropriate policies to solve problems in each region.

Separately in the toolkit of multidimensional statistical analysis, it is necessary to allocate methods of multidimensional scaling (MDS) which are directed on the identification of the structure of the set of studied objects. To this end, they are reflected in some space of latent characteristics, which adequately models reality. It is built based on a matrix of measures that reflect the pairwise similarity between objects. The resulting configuration of objects allows us to conclude their set internal structure. For a long time, this area of multidimensional statistical analysis was not given due attention, primarily due to the complexity of computational procedures, especially for the group of methods of non-metric multidimensional scaling. However, the development of software tools that implement these methods has eliminated this shortcoming and opened the horizons of their application, including for economic research.

J. J. de Jongh and D. F. Meyer in [5] used MDS tools to build a multidimensional regional development index (MREDI). The authors presented the practical implementation of the developed tools for rural municipalities in the North-West province of South Africa, identified trends in the proposed indicator. We support the conclusions made in the article on the feasibility of using the MDA to measure the economic development of regions.

The application of MDA technology to study the dynamics of economic growth in the world and the impact on it of globalization, scientific and technological progress, competition is presented in the works of J. A. Tenreiro Machado and M. E. Mata [29; 30; 28]. The authors substantiated the evolution of the main indicators of economic growth, globalization, prosperity, and development of the human world economy, established periods of prosperity and crisis, growth and stagnation

Paper [33] considers the procedure for assessing the social cohesion of the counties of Lower Silesia in the period 2005-2015 based on MDS tools in combination with linear ordering and Theil decomposition. The application of MDS together with cluster analysis to assess the development of farms in Kenya and Zimbabwe is presented in the study [22]. The method was implemented in two samples using expert data processing.

Note that the analyzed studies use methods of metric multidimensional analysis, which are focused on the use of quantitatively measurable indicators. In practice, it is often necessary to deal with non-metric indicators, measured in particular in the ranking scale or obtained from the expert evaluation. In such cases, it is advisable to use methods of non-metric multidimensional scaling.

In this study, we propose a new approach for application of the method of non-metric method of multidimensional scaling (NMMDS) to determine the configuration of regions of Ukraine by indicators of economic development, which have a non-metric nature and study their structure in the context of sustainable development.

## 2 Research methodology

In multidimensional scaling, the source data matrix is a square symmetric matrix of objects' differences measures:

$$\Delta = \begin{bmatrix} \delta_{11} & \delta_{12} & \dots & \delta_{1m} \\ \delta_{21} & \delta_{22} & \dots & \delta_{2m} \\ \dots & & & \\ \delta_{m1} & \delta_{m2} & \dots & \delta_{mm} \end{bmatrix}, \quad (1)$$

where $\delta_{ij}$ is a measure of the difference between *i*-th and *j*-th points (corresponding to objects), what is proportional to the distance between them;

*m* – number of objects, to be analyzed.

The NMMDA assumes that the measure of difference is a monotonic function of distance:

$$\delta_{ij} = \mathrm{f}(d_{ij}) = f\left(\sqrt[s]{\sum_{k=1}^{p} |u_{ik} - u_{jk}|^{s}}\right), \quad (2)$$

where *U* is an of objects' coordinates in the new space;

*p* is a dimension of a new scale space;

*s* – Minkowski metric index for calculating the distance in space.

Neither the coordinates of the $u_{ij}$ j objects in this space nor the dimension of the space *p* itself are known. The value of s is chosen in the study process based on substantive considerations. Equation (2) means that the measures of differences are calculated from the original data corresponding to the distances between objects in the new space. Therefore, the problem of metric scaling is that based on the known matrix of differences of objects (1), which is calculated by the values of the original features $X_1$, $X_2$, …, $X_n$ we need to find the coordinates of objects in the new scale-space $U_1$, $U_2$, …, $U_p$ under the condition that the ranking order of the estimates of distances calculated by them is as close as possible to the ranking order of the initial values.

An important component of the nonmetric scaling algorithm is the degree of correspondence, which shows how the estimates of the coordinates of the stimuli reproduce the rank order of the data $\delta_{ij}$. Typically, it contains three sets of parameters. The first includes the estimates of the coordinates $u_{ij}$. The second set contains estimates of the distances $d_{ij}$. The third set consists of fictitious parameters called rank data images, or deviations. They are calculated to be as close as possible to the distance estimates. Deviations to determine the degree of compliance can be calculated using the stress formulas proposed by J. B. Kruskal [13]:

$$S_1 = \sqrt{\frac{\sum_{i=1}^{m}\sum_{j=1}^{m}(\delta_{ij}-d_{ij})^2}{\sum_{i=1}^{m}\sum_{j=1}^{m}(d_{ij})^2}}, \quad (3)$$

$$S_2 = \sqrt{\frac{\sum_{i=1}^{m}\sum_{j=1}^{m}(\delta_{ij}-d_{ij})^2}{\sum_{i=1}^{m}\sum_{j=1}^{m}(d_{ij}-d)^2}}, \quad (4)$$

$$d = \frac{1}{m^2} \sum_{i=1}^{m} \sum_{j=1}^{m} d_{ij}. \quad (5)$$

The algorithm of NMMDA consists of four stages:

— Determining the starting configuration;
— Standardization of distances and coordinates' estimates;
— Calculation of deviations;
— Calculation of new coordinates' estimates.

The first stage is performed only once. It sets the initial estimates of the $x_{ik}$ stimulus coordinates in multidimensional scale space. These estimates can be calculated in different ways. One of them is the use of values obtained by the algorithm of metric scaling Torgerson [4]. A necessary condition is that the estimates obtained by any method must be standardized.

The second stage is the standardization of coordinate estimates and the standardization of distances between stimuli. If this is the first time this step has been performed, the coordinate estimates are not standardized because they are already standardized, which reduces the likelihood of obtaining a degenerate solution.

The third stage involves the calculation of deviations. It is assumed that the values of $\delta_i$ are ordered by increasing their values. This stage is often called the non-metric stage of NMMDA. It does not change estimates of stimulus coordinates and distance estimates. It only provides for a change (recalculation) of estimates of differences.

In the first step of this stage, the data of the matrix of differences *D* are written in one column in ascending order (zero values of the matrix, ie the difference between the object itself, is not taken into account). If there is an equality of values of differences for two pairs of stimuli, ie $\delta_{ij} = \delta_{sr}$, then the first indicates the value of the difference for the pair of stimuli for which the corresponding value of the distance is less.

The next step of the third stage is the implementation of a series of views on the ordered data. The purpose of each pass is to divide the deviation estimates into blocks of equal values. Before the first view, these deviations are the values of the matrix of standardized distances obtained in the previous stage. If all values are different, then each of them forms a separate block. If some adjacent values are the same, they form one block. Next, the values of adjacent blocks are compared. If the value of the block with the larger number is less than the value of the block with the smaller number, then the blocks are combined into one, and their value is equal to the arithmetic mean of the values of the blocks. The data view is completed after analyzing the values of all blocks. If blocks have been merged, the next pass is performed. That is, the passes are completed if the next step the values of the blocks are ordered in ascending order. This completes the non-metric stage of multidimensional scaling.

In the fourth stage, the coordinates of the stimuli and the distances between the stimuli are recalculated. The new coordinates are calculated by the formula:

$$x_{ij}^{k+1} = x_{ij}^{k} - \frac{1}{m} \sum_{t=1}^{m} \left(1 - \frac{\delta_{it}^{k+1}}{d_{it}^{k}}\right)\left(x_{ij}^{k} - x_{tj}^{k}\right), \quad (6)$$

If $d_{it}^{k} = 0$, then relation $\frac{\delta_{it}^{k+1}}{d_{it}^{k}}$ is arbitrarily set equal to 1. New measures of distance are standardized so that the sum of the squares of their values is equal to 1. This step is not mandatory, but to some extent can simplify the calculations.

After the completion of the fourth stage, the value of the stress formula $S^{k+1}$ is calculated, which is then compared with the value calculated in the previous iteration $S^{k}$. If the deviation is less than a predetermined small value of ε> 0, the calculations are completed. Otherwise, steps 2, 3, 4 are repeated.

## 3 Results and discussion

Within the framework of the approach described, let us assess the economic development of Ukraine's regions in the context of their sustainable development. As mentioned above, the sustainable development of the region can be seen as a process of improving the functioning of all its subsystems, including economic. From these considerations, we will form a system of initial indicators. Recently, it is quite common to calculate and use a variety of ratings that reflect the level of regional development and are calculated based on the key performance indicators of all region's subsystems. In particular, such estimates are presented by the Ministry of Development of Communities and Territories of Ukraine [15]. They can be used as non-metric indicators of economic development. Thus, for the calculations we have chosen the following set of non-metric indicators, which are rank estimates: $X_1$ – Investment and innovation development and foreign economic cooperation; $X_2$ – Financial self-sufficiency; $X_3$ – Labor market efficiency.

The data source for determining indicators are materials of the Ministry of Development of Communities and Territories of Ukraine [18; 17; 19]. We will select data for the period from 2016 to 2019.

To compactly present the information, we will assign a code to each region. Relevant information is given in table 1. The values of the original data are presented in table 2.

**Table 1.** Relations between the name of regions and their codes.

| Code | Region | Code | Region | Code | Region |
|---|---|---|---|---|---|
| r_1 | Vinnytsia | r_9 | Kyiv | r_17 | Sumy |
| r_2 | Volyn | r_10 | Kyrovohrad | r_18 | Ternopil |
| r_3 | Dnipro | r_11 | Luhansk | r_19 | Kharkiv |
| r_4 | Donetsk | r_12 | Lviv | r_20 | Kherson |
| r_5 | Zhytomyr | r_13 | Mykolaiv | r_21 | Khmelnytskyi |
| r_6 | Zakarpattia | r_14 | Odesa | r_22 | Cherkasy |
| r_7 | Zaporizhzhia | r_15 | Poltava | r_23 | Chernivtsi |
| r_8 | Ivano-Frankivsk | r_16 | Rivne | r_24 | Chernihiv |

**Table 2.** Values of initial data for 2016-2019.

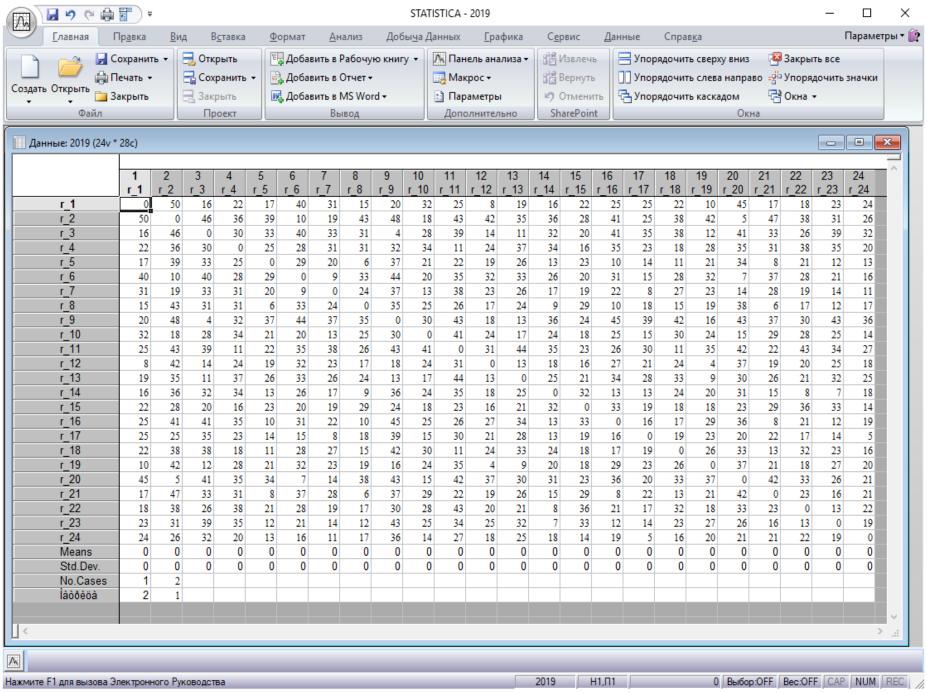
| Code | Values | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | 2016 | | | 2017 | | | 2018 | | | 2019 | | |
| | $X_1$ | $X_2$ | $X_3$ | $X_1$ | $X_2$ | $X_3$ | $X_1$ | $X_2$ | $X_3$ | $X_1$ | $X_2$ | $X_3$ |
| r_1 | 2 | 5 | 3 | 10 | 4 | 15 | 19 | 6 | 9 | 16 | 5 | 6 |
| r_2 | 16 | 24 | 17 | 21 | 5 | 23 | 20 | 18 | 21 | 1 | 24 | 22 |
| r_3 | 5 | 2 | 2 | 2 | 3 | 3 | 4 | 7 | 3 | 6 | 3 | 2 |
| r_4 | 1 | 4 | 23 | 3 | 23 | 24 | 9 | 23 | 24 | 14 | 2 | 23 |
| r_5 | 19 | 15 | 5 | 12 | 12 | 12 | 16 | 16 | 15 | 19 | 12 | 13 |
| r_6 | 3 | 22 | 20 | 13 | 11 | 17 | 21 | 1 | 16 | 8 | 22 | 21 |
| r_7 | 4 | 21 | 10 | 4 | 21 | 14 | 11 | 2 | 10 | 9 | 20 | 15 |
| r_8 | 23 | 14 | 9 | 9 | 19 | 9 | 24 | 19 | 11 | 20 | 13 | 9 |
| r_9 | 9 | 8 | 1 | 15 | 2 | 2 | 2 | 5 | 2 | 5 | 1 | 1 |
| r_10 | 13 | 23 | 15 | 18 | 16 | 20 | 10 | 20 | 20 | 2 | 15 | 14 |
| r_11 | 24 | 9 | 24 | 24 | 24 | 22 | 17 | 24 | 23 | 22 | 4 | 24 |
| r_12 | 17 | 11 | 7 | 11 | 10 | 5 | 6 | 3 | 5 | 11 | 7 | 7 |
| r_13 | 10 | 19 | 21 | 22 | 17 | 10 | 7 | 11 | 6 | 4 | 10 | 4 |
| r_14 | 20 | 20 | 11 | 8 | 1 | 4 | 3 | 4 | 4 | 17 | 18 | 8 |
| r_15 | 14 | 1 | 16 | 7 | 14 | 19 | 8 | 10 | 22 | 7 | 6 | 18 |
| r_16 | 22 | 6 | 8 | 14 | 6 | 18 | 22 | 17 | 12 | 24 | 16 | 12 |
| r_17 | 8 | 10 | 22 | 19 | 13 | 6 | 15 | 14 | 7 | 13 | 17 | 16 |
| r_18 | 7 | 16 | 12 | 5 | 15 | 21 | 18 | 22 | 19 | 21 | 9 | 19 |
| r_19 | 6 | 3 | 4 | 16 | 9 | 1 | 5 | 8 | 1 | 10 | 8 | 5 |
| r_20 | 3 | 23 | 20 | 13 | 21 | 18 | 6 | 18 | 13 | 21 | 12 | 19 |
| r_21 | 23 | 11 | 10 | 14 | 13 | 14 | 17 | 7 | 8 | 15 | 17 | 13 |
| r_22 | 15 | 19 | 3 | 12 | 12 | 13 | 20 | 8 | 11 | 11 | 7 | 14 |
| r_23 | 18 | 21 | 11 | 23 | 9 | 8 | 23 | 22 | 7 | 18 | 13 | 6 |
| r_24 | 12 | 14 | 17 | 1 | 15 | 17 | 1 | 20 | 16 | 12 | 18 | 18 |

Given the origin of the data, as initial estimates of the objects' differences we take the measure of the Hamming's distance of (the city-block distance), calculated in the space of the original features:

$$\delta_{ij} = \sum_{k=1}^{n} |x_{ik} - u_{jk}|. \quad (7)$$

The algorithm of NMMDS is quite a time consuming from a computational point of view. An additional complication is the large dimension of the matrices that will be processed, as they will contain information about 24 regions of Ukraine. To automate the calculations, we will use special software that allows statistical data processing. One such tool is the Statistica software (Russian Localization), developed by StatSoft Inc, which has built-in tools for multidimensional scaling. The dimension of the new space is chosen to be 2. The starting configuration of the studied objects in the program is determined automatically.

The fragment of the Statistica's window with the entered values of the differences' matrix, calculated according to 2019 data, is shown in figure 1.


Данные: 2019 (24v * 28c)

| | 1 r_1 | 2 r_2 | 3 r_3 | 4 r_4 | 5 r_5 | 6 r_6 | 7 r_7 | 8 r_8 | 9 r_9 | 10 r_10 | 11 r_11 | 12 r_12 | 13 r_13 | 14 r_14 | 15 r_15 | 16 r_16 | 17 r_17 | 18 r_18 | 19 r_19 | 20 r_20 | 21 r_21 | 22 r_22 | 23 r_23 | 24 r_24 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| r_1 | 0 | 50 | 16 | 22 | 17 | 40 | 31 | 15 | 20 | 32 | 25 | 8 | 19 | 16 | 22 | 25 | 25 | 22 | 10 | 45 | 17 | 18 | 23 | 24 |
| r_2 | 50 | 0 | 46 | 36 | 39 | 10 | 19 | 43 | 48 | 18 | 43 | 42 | 35 | 36 | 28 | 41 | 25 | 38 | 42 | 5 | 47 | 38 | 31 | 26 |
| r_3 | 16 | 46 | 0 | 30 | 33 | 40 | 33 | 31 | 4 | 28 | 39 | 14 | 11 | 32 | 20 | 41 | 35 | 38 | 12 | 41 | 33 | 26 | 39 | 32 |
| r_4 | 22 | 36 | 30 | 0 | 25 | 28 | 31 | 31 | 32 | 34 | 11 | 24 | 37 | 34 | 16 | 35 | 23 | 18 | 28 | 35 | 31 | 38 | 35 | 20 |
| r_5 | 17 | 39 | 33 | 25 | 0 | 29 | 20 | 6 | 37 | 21 | 22 | 19 | 26 | 13 | 23 | 10 | 14 | 11 | 21 | 34 | 8 | 21 | 12 | 13 |
| r_6 | 40 | 10 | 40 | 28 | 29 | 0 | 9 | 33 | 44 | 20 | 35 | 32 | 33 | 26 | 20 | 31 | 15 | 28 | 32 | 7 | 37 | 28 | 21 | 16 |
| r_7 | 31 | 19 | 33 | 31 | 20 | 9 | 0 | 24 | 37 | 13 | 38 | 23 | 26 | 17 | 19 | 22 | 8 | 27 | 23 | 14 | 28 | 19 | 14 | 11 |
| r_8 | 15 | 43 | 31 | 31 | 6 | 33 | 24 | 0 | 35 | 25 | 26 | 17 | 24 | 9 | 29 | 10 | 18 | 15 | 19 | 38 | 6 | 17 | 12 | 17 |
| r_9 | 20 | 48 | 4 | 32 | 37 | 44 | 37 | 35 | 0 | 30 | 43 | 18 | 13 | 36 | 24 | 45 | 39 | 42 | 16 | 43 | 37 | 30 | 43 | 36 |
| r_10 | 32 | 18 | 28 | 34 | 21 | 20 | 13 | 25 | 30 | 0 | 41 | 24 | 17 | 24 | 18 | 25 | 15 | 30 | 24 | 15 | 29 | 28 | 25 | 14 |
| r_11 | 25 | 43 | 39 | 11 | 22 | 35 | 38 | 26 | 43 | 41 | 0 | 31 | 44 | 35 | 23 | 26 | 30 | 11 | 35 | 42 | 22 | 43 | 34 | 27 |
| r_12 | 8 | 42 | 14 | 24 | 19 | 32 | 23 | 17 | 18 | 24 | 31 | 0 | 13 | 18 | 16 | 27 | 21 | 24 | 4 | 37 | 19 | 20 | 25 | 18 |
| r_13 | 19 | 35 | 11 | 37 | 26 | 33 | 26 | 24 | 13 | 17 | 44 | 13 | 0 | 25 | 21 | 34 | 28 | 33 | 9 | 30 | 26 | 21 | 32 | 25 |
| r_14 | 16 | 36 | 32 | 34 | 13 | 26 | 17 | 9 | 36 | 24 | 35 | 18 | 25 | 0 | 32 | 13 | 13 | 24 | 20 | 31 | 15 | 8 | 7 | 18 |
| r_15 | 22 | 28 | 20 | 16 | 23 | 20 | 19 | 29 | 24 | 18 | 23 | 16 | 21 | 32 | 0 | 33 | 19 | 18 | 18 | 23 | 29 | 36 | 33 | 14 |
| r_16 | 25 | 41 | 41 | 35 | 10 | 31 | 22 | 10 | 45 | 25 | 26 | 27 | 34 | 13 | 33 | 0 | 16 | 17 | 29 | 36 | 8 | 21 | 12 | 19 |
| r_17 | 25 | 25 | 35 | 23 | 14 | 15 | 8 | 18 | 39 | 15 | 30 | 21 | 28 | 13 | 19 | 16 | 0 | 19 | 23 | 20 | 22 | 17 | 14 | 5 |
| r_18 | 22 | 38 | 38 | 18 | 11 | 28 | 27 | 15 | 42 | 30 | 11 | 24 | 33 | 24 | 18 | 17 | 19 | 0 | 26 | 33 | 13 | 32 | 23 | 16 |
| r_19 | 10 | 42 | 12 | 28 | 21 | 32 | 23 | 19 | 16 | 24 | 35 | 4 | 9 | 20 | 18 | 29 | 23 | 26 | 0 | 37 | 21 | 18 | 27 | 20 |
| r_20 | 45 | 5 | 41 | 35 | 34 | 7 | 14 | 38 | 43 | 15 | 42 | 37 | 30 | 31 | 23 | 36 | 20 | 33 | 37 | 0 | 42 | 33 | 26 | 21 |
| r_21 | 17 | 47 | 33 | 31 | 8 | 37 | 28 | 6 | 37 | 29 | 22 | 19 | 26 | 15 | 29 | 8 | 22 | 13 | 21 | 42 | 0 | 23 | 16 | 21 |
| r_22 | 18 | 38 | 26 | 38 | 21 | 28 | 19 | 17 | 30 | 28 | 43 | 20 | 21 | 8 | 36 | 21 | 17 | 32 | 18 | 33 | 23 | 0 | 13 | 22 |
| r_23 | 23 | 31 | 39 | 35 | 12 | 21 | 14 | 12 | 43 | 25 | 34 | 25 | 32 | 7 | 33 | 12 | 14 | 23 | 27 | 26 | 16 | 13 | 0 | 19 |
| r_24 | 24 | 26 | 32 | 20 | 13 | 16 | 11 | 17 | 36 | 14 | 27 | 18 | 25 | 18 | 14 | 19 | 5 | 16 | 20 | 21 | 21 | 22 | 19 | 0 |
| Means | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| Std.Dev. | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| No.Cases | 1 | 2 | | | | | | | | | | | | | | | | | | | | | | |
| Ìàòðèöà | 2 | 1 | | | | | | | | | | | | | | | | | | | | | | |

**Fig. 1.** Matrix of differences for data 2019.

As a result of the calculations performed according to the above non-metric scaling algorithm, implemented in Statistica, we obtain the coordinates of point-objects (regions) in the new scale space (table 3). Note that the calculations were performed separately for each year's data.

A graphical representation of the regions in the space of latent scales is shown in figures 2-5. Analysis of the charts presented allows us to conclude that there are no trends for the sustainability of economic development for Ukraine's regions during the period under study. The location of regions in the built scale-space doesn't meet certain patterns or dependencies.

**Table 3.** Coordinates' values of point-objects (regions) in the new scale space.

| Code | Values | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | 2016 | | 2017 | | 2018 | | 2019 | |
| | Axis1 | Axis2 | Axis1 | Axis2 | Axis1 | Axis2 | Axis1 | Axis2 |
| r_1 | -0.55 | 0.15 | -0.81 | 0.64 | -1.00 | 0.58 | 0.57 | 0.84 |
| r_2 | -0.32 | 0.30 | 0.16 | 0.52 | 0.84 | 0.71 | 0.44 | -0.09 |
| r_3 | -0.34 | -0.36 | 0.14 | -0.18 | 0.32 | -0.86 | -0.11 | 0.50 |
| r_4 | 0.12 | -0.19 | -0.34 | 0.41 | -0.70 | 0.00 | 0.55 | 0.27 |
| r_5 | -0.40 | 1.07 | 1.11 | -0.89 | -1.26 | 0.09 | -1.28 | -0.04 |
| r_6 | 0.74 | 0.49 | 0.74 | 1.05 | 0.03 | 0.07 | -0.01 | -0.94 |
| r_7 | -0.49 | 0.80 | 1.00 | 0.58 | -0.47 | -0.95 | 0.28 | -0.36 |
| r_8 | 0.13 | 0.81 | -0.58 | -0.94 | -0.29 | 0.70 | -0.66 | -0.54 |
| r_9 | 0.99 | -0.03 | -0.46 | -0.20 | -0.24 | 0.37 | -0.01 | 0.25 |
| r_10 | 0.86 | -0.44 | 0.27 | -0.88 | -0.20 | -1.25 | 0.94 | -0.58 |
| r_11 | -0.49 | -0.14 | 0.08 | -0.13 | 0.21 | -0.37 | 0.58 | -0.61 |
| r_12 | -0.10 | -1.12 | -1.18 | -0.27 | 0.56 | 0.75 | -0.14 | 0.78 |
| r_13 | -1.11 | -0.39 | 1.05 | 0.10 | -1.13 | -0.38 | -0.52 | 0.85 |
| r_14 | -1.23 | 0.15 | 0.66 | 0.19 | 0.89 | 0.05 | -1.18 | 0.37 |
| r_15 | 0.27 | -0.66 | -1.25 | 0.28 | -0.22 | 1.10 | 1.15 | 0.25 |
| r_16 | 1.09 | 0.27 | -0.09 | -1.26 | -0.87 | -0.82 | -0.72 | 0.01 |
| r_17 | 0.29 | 1.18 | 0.31 | 1.11 | 0.28 | 1.19 | 0.18 | -0.45 |
| r_18 | 0.60 | -1.14 | 1.01 | -0.39 | 0.78 | -0.47 | 0.09 | -1.07 |
| r_19 | 0.35 | 0.25 | 0.71 | -0.64 | -0.30 | -0.51 | -0.32 | 1.21 |
| r_20 | 1.02 | -0.79 | -0.57 | 1.08 | -0.74 | 0.96 | 1.31 | -0.47 |
| r_21 | 1.04 | 0.92 | -0.11 | 1.14 | 0.75 | -1.05 | -0.37 | -0.80 |
| r_22 | -1.09 | 0.73 | -0.99 | -0.67 | 1.31 | 0.29 | -1.16 | 0.50 |
| r_23 | -0.85 | -0.72 | -0.64 | 0.08 | 1.01 | -0.42 | -0.95 | -0.72 |
| r_24 | -0.51 | -1.14 | -0.24 | -0.74 | 0.47 | 0.21 | 1.35 | 0.84 |

This result can be explained by the properties of the indicators that were selected for the calculations: according to the data, there is also no clear trend in the ranking of regions of Ukraine during the study period. On the other hand, the country's economy during this period was exposed to various destabilizing internal and external influences, which also affected the disparities in regional development.

Conclusions obtained by the results of calculations coincides with the conclusion made in paper [9], where the sustainable economic development of Ukraine's regions is based on a comprehensive index according to the State Statistics Service of Ukraine [6] also has been assessed.

The presented location of regions also complicates the interpretation of axes (new scales). To solve this problem, a well-known effective procedure is the rotation of the constructed scale space, which is described by the formula:

$$U^{(1)} = U^{(0)}W, \quad (8)$$

where $U^{(1)}$ – new scale space of latent characteristics;

$U^{(0)}$ – initial scale space of latent characteristics ;

W – rotation matrix, which sets the rotation of the axes of the initial scale space by some angle φ counterclockwise.

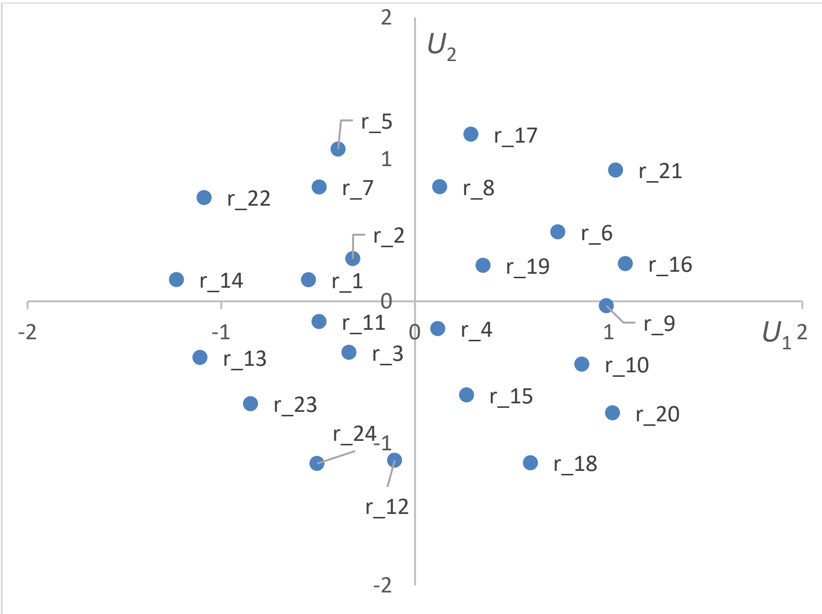



**Fig. 2.** Imaging regions in the new scale space for data of 2016.

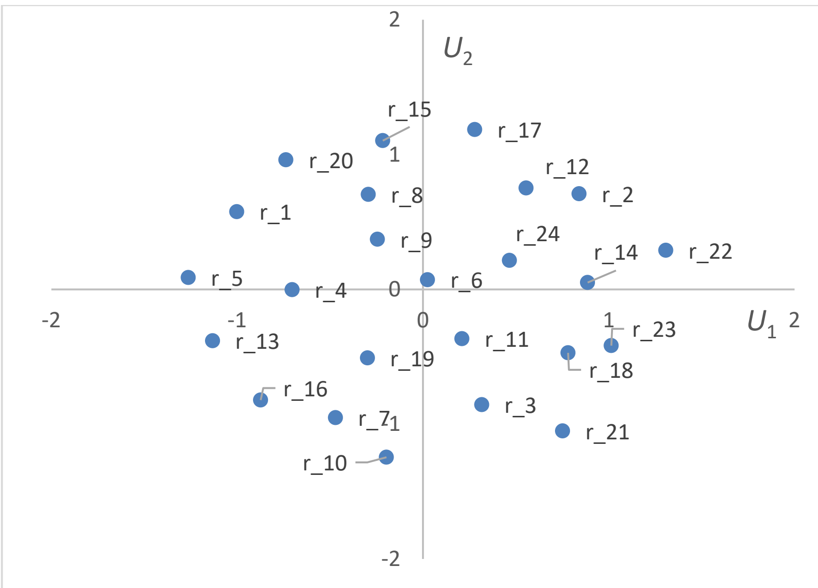



**Fig. 3.** Imaging regions in the new scale space for data of 2017.

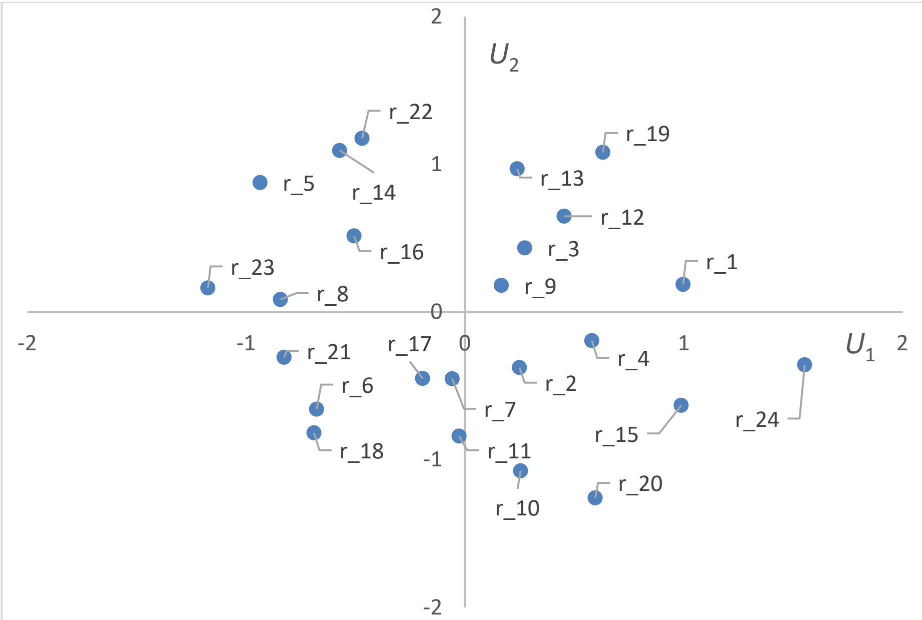



**Fig. 4.** Imaging regions in the new scale space for data of 2018.

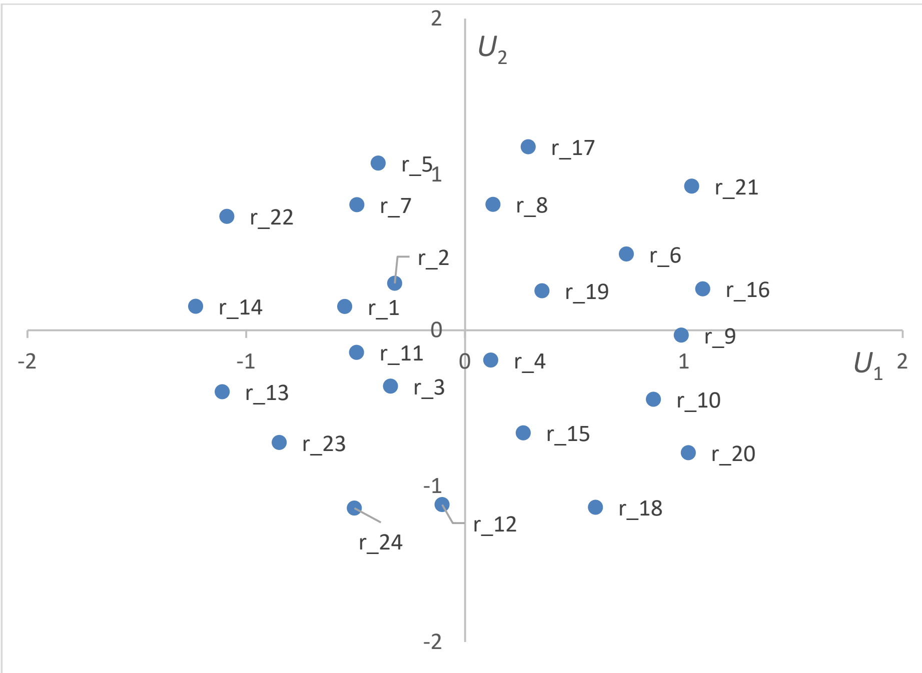



**Fig. 5.** Imaging regions in the new scale space for data of 2019.

The rotation matrix has a form:

$$W = \begin{bmatrix} \cos\varphi & -sin\varphi \\ sin\varphi & \cos\varphi \end{bmatrix}. \quad (9)$$

Currently, there are no formal rules that would allow analytically to justify the value of the angle of rotation φ for the best interpretation of the axes. This is usually done experimentally.

Let us illustrate the rotation of the scale-space for the graph constructed according to the data of 2019. In our study, it was found that an acceptable result is the rotation of the scale-space by the angle φ=45. The corresponding rotation matrix has the form:

$$W = \begin{bmatrix} 0.71 & -0.71 \\ 0.71 & 0.71 \end{bmatrix} \quad (10)$$

and the matrix $U_{2019}^{(1)}$, which reflects the coordinates of the points in the new space, consist of elements, presented in table 4:

**Table 4.** Coordinates of points in the new space for data 2019.

| Code | Values | |
|---|---|---|
| | 2019 | |
| | Axis1 | Axis2 |
| r_1 | 1.00 | 0.19 |
| r_2 | 0.25 | -0.38 |
| r_3 | 0.27 | 0.43 |
| r_4 | 0.58 | -0.19 |
| r_5 | -0.94 | 0.88 |
| r_6 | -0.68 | -0.66 |
| r_7 | -0.06 | -0.45 |
| r_8 | -0.84 | 0.08 |
| r_9 | 0.17 | 0.18 |
| r_10 | 0.25 | -1.07 |
| r_11 | -0.03 | -0.84 |
| r_12 | 0.45 | 0.65 |
| r_13 | 0.24 | 0.97 |
| r_14 | -0.57 | 1.09 |
| r_15 | 0.99 | -0.63 |
| r_16 | -0.51 | 0.52 |
| r_17 | -0.19 | -0.45 |
| r_18 | -0.69 | -0.82 |
| r_19 | 0.63 | 1.08 |
| r_20 | 0.60 | -1.26 |
| r_21 | -0.83 | -0.31 |
| r_22 | -0.47 | 1.18 |
| r_23 | -1.18 | 0.16 |
| r_24 | 1.55 | -0.36 |

Imaging regions in the scale space after the axis rotation for data of 2019 is shown in figure 6. Comparing the obtained configuration with the original data, we can conclude that the abscissa axis ($U_1$) can be interpreted as an axis that reflects the economic potential of the regions, and the $U_2$ axis – as the labor potential of the regions.

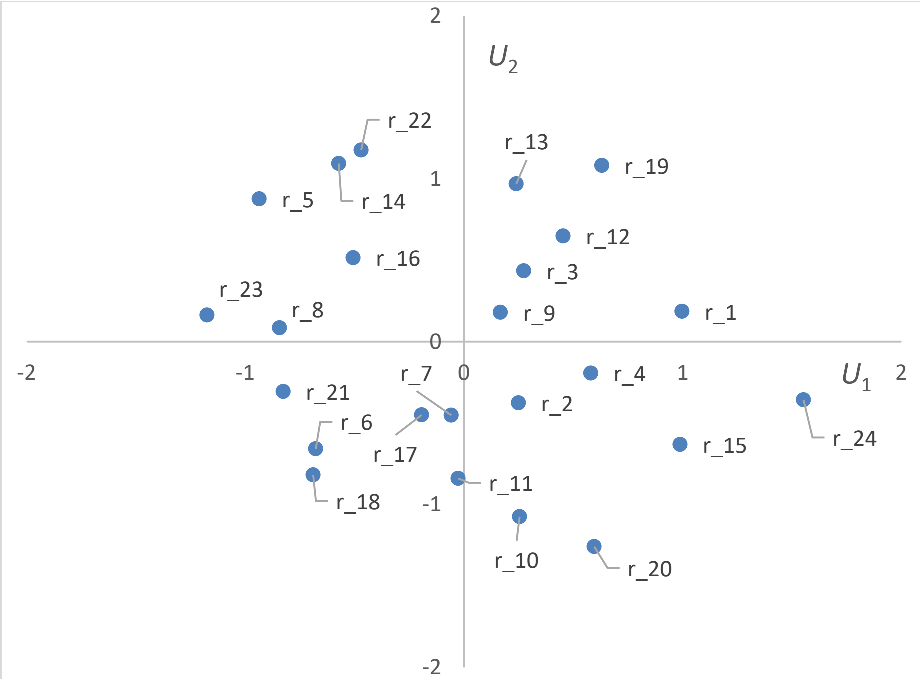



**Fig. 6.** Imaging regions in the scale space after the axis rotation for data of 2019.

Note that each data collection may have its rotation matrix. Moreover, there may be a situation where the rotation of space does not give the desired effect in terms of a reasonable interpretation of the axes.

A possible way out of this situation is to increase the number of initial indicators due to the condition of their close correlation.

## 4 Conclusions

Assessing the economies of regions in the context of ensuring their sustainable development remains an urgent problem. Studies have shown a wide range of methods for a comprehensive assessment of the economic development of regions. It is established that the multidimensionality of the description of economic development determines the use of methods of multidimensional statistics, in particular multidimensional scaling.

The scientific novelty of the proposed approach is the use of a set of non-metric indicators to assess the regions' economic development, which led to the further implementation of the methodology of multidimensional scaling in economic studies.

The approach for structuring regions in the context of sustainability of their economic development is considered in the paper. The results received a semantic interpretation, to improve which is proposed to rotate the scale space. The obtained results can be used as a basis for the formation of development strategies at both regional and national levels, as well as to assess the implementation of economic, social, and environmental aspects of sustainable development in the regions. The direction of further research is to expand the set of initial indicators for the application in multidimensional scaling, in particular, including the quantitative measured indicators into the initial data collection.

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