Time Series Models for the Colombian TRM Exchange Rate

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Abstract

The US Dollar and the Colombian Peso currency exchange rate (*Tasa Representativa del Mercado*, *TRM*) is a financial series characterized by periods of high volatility. In this paper it was applied three classes of time series: the *ARMA, ARMA-GARCH* and *Markov Switching* (*MS*) models to represent the log-returns of the *TRM* between 2013-01-03 and 2020-07-02. The best models among several fitted models for each class were determined based on the Akaike and the Bayesian information criteria (AIC and BIC, respectively) and one-step forecasts in the period 2020-07-03 to 2020-07-31. The *ARMA-GARCH* model allowed a more precise description of the conditional variance than the *ARMA* model. Furthermore, the *MS* model defined 3 regimes each one with its own *AR* process. The regime with highest variability showed sporadic occurrences, and can be associated to three important events at global scale: the oil crisis in 2014, the US-China trade war in 2018, and the COVID-19 pandemics in march 2020. The results demonstrate the robustness of the models for forecasting in one-step or longer time windows.

Keywords

Exchange rate, volatility, ARMA, ARMA-GARCH, Markov Switching, Forecast, COVID-19.

1. Introduction

The value of the US Dollar and its market value in Colombia, known as the *TRM*, has been in the general interest of the government, shareholders and other economic agents as it is the reference currency for conducting international transactions in the country. The presence of periods of high volatility is characteristic of the *TRM* series since its conversion to a floating exchange rate in September 1999 [7]. Therefore, it has been essential to have robust tools to describe its behavior and to forecast its future values and volatility.

Several studies based on time series models have been conducted to analyze and predict the *TRM*. In [9], Hernandez and Mesa evaluated the impact of the intervention of the Central Bank on factors that affected the average response and variance of the *TRM*. Lega in 2007 found that the volatility has high persistence and amplitude, specially in periods of devaluation [10]. Cepeda and Casas in 2008 used a *GARCH* model to describe the variance of the *TRM*, finding that the *GARCH(1,2)* best explained the performance of stock prices in the period of evaluation [11]. In 2017, González applied latent regime models such as *Markov Switching* (*MS*), *ARIMA*

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and *Random walk* models, finding that the *MS* model performed better in a forecast horizon of 25 days and *ARIMA* in one-step forecasting [14].

Modeling currencies exchange rates from other Latin-american countries has also been of interest in recent years. Ortiz et al. fitted the US Dollar and Mexican Peso (MXN-USD) currency exchange rate with two classes of *GARCH* models with different distributions of the residuals and compared their performances [12]. Espinosa Gonzáles et al. implemented a *MS* to describe the exchange rate of the Peruvian Real (PEN-USD) finding that the best description of the series was given by a three regime model [6]. Later, Rodríguez et al. characterized the volatility of daily stock markets returns in Argentina, Brazil, Chile, Mexico and Peru, by applying extended linear models contrasting with traditional time series models [13]. Meneses and Alvarado implemented back propagation artificial neural networks to describe and predict in one-step and longer time windows the MXN-USD exchange rate [15].

However, there is a need to implement more robust models for the *TRM* that provide better forecasts for longer time windows. This article applies *ARMA*, *GARCH*, and *MS* models for the daily value of the *TRM* by fitting in a longer time window than the preceding works for the *TRM*, specifically from January 3, 2013 to July 2, 2020. This period is characterized by several events of high volatility and abrupt changes in the mean level.

2. Time series models

In order to apply the time series models explained in this section, the data series must have a constant zero mean. Therefore, in the following it was assumed that y_t is the *log-returns* of the *TRM* series:

$$y_t = ln(TRM_t) - ln(TRM_{t-1}). \tag{1}$$

Section 2 shows the statistical tests that assure that y_t satisfies these minimal stationary conditions.

2.1. Autoregressive moving average models $ARMA(p, q)$

The process $ARMA(p, q)$ is constituted by the sum of an autoregressive process $AR(p)$ and a moving average process $MA(q)$ [1]:

$$y_t = \phi_0 + \sum_{i=1}^{p} \phi_i y_{t-i} + \sum_{j=1}^{q} \theta_j \epsilon_{t-j} + \epsilon_t, \tag{2}$$

where ϵ_t is a *white noise* process at time t with zero mean and variance σ^2, the *AR* components are the contribution of the previous values of y_t in the p previous times, and the *MA* components are the contribution of the white noise until q periods in the past. Under certain conditions of the parameters $\{\phi_i\}$ and $\{\theta_j\}$, *ARMA* processes are stationary, i.e., their mean and variance are constant in time.

2.2. Generalized autoregressive model of conditional heteroscedasticity *GARCH*(p, q)

One of the main problems in time series modeling is that the conditional variance $\sigma^2_{t|t-1}$ of the series upon its past values is not stationary in time (i.e., the series is heterocedastic). By defining the white noise ϵ_t with a conditional variance as an *ARMA*(p, q), it is obtained a *GARCH* model. Specifically:

$$\epsilon_t = \sigma_{t|t-1} \upsilon_t, \tag{3}$$

$$\sigma^2_{t|t-1} = \omega + \alpha_1 \epsilon^2_{t-1} + \dots + \alpha_p \epsilon^2_{t-p} + \beta_1 \sigma^2_{t-1|t-2} + \dots + \beta_q \sigma^2_{t-q|t-q-1}, \tag{4}$$

where υ_t is a white noise process with zero mean and variance equal to one, and $\{\alpha_i\}, \{\beta_j\}, \omega$ are parameters that need to be estimated from the data. For stability, stationarity and positive values in the series, it is necessary that $\alpha_i, \beta_j \geq 0$, $\sum_{i=1}^{p} \alpha_i + \sum_{j=1}^{q} \beta_j < 1$, and $\omega > 0$ [2, 3, 4].

2.3. Regime change models: Markov Switching

Many financial series show abrupt changes caused by macroeconomic factors, government policies, or other circumstances [8]. This is materialized in the series through the appearance of sudden changes in their usual behavior, generating breaks in the mean and variance of the process. For instance, a relative long period of low volatility can abruptly change to a high volatility one, which in some cases can last for long periods, generating *clusters* in the series. This clearly implies a problem when trying to apply the assumptions of stationary.

One of the approaches is to assume that the time series parameters depend on an external environment that evolves following a *markovian process*, for example a *discrete Markov chain*. In this context, the state of the Markov chain s_t is called the *regime*, takes values in a countable set ($s_t = 1, 2, \dots, n$), and the transition between regimes k and l in one-step has probability P_{kl} which depends only on the present state (the *markovian property*):

$$P_{kl} = P\left(s_{t+1} = l | s_t = k\right) = P\left(s_{t+1} = l | s_t = k, s_{t-1} = k_1, \dots\right). \tag{5}$$

A Markov Switching (*MS*) model for y_t with n regimes and *AR*(p) processes, denoted as *MS*(n)*AR*(p), has the form:

$$y_t = \phi_{0,s_t} + \sum_{i=1}^{p} \phi_{i,s_t} y_{t-i} + \epsilon_t. \tag{6}$$

That is, the *AR* process followed by y_t has parameters ϕ_{i,s_t} and white noise with variance $\sigma^2_{s_t}$ that depends on the regime s_t at time t [5]. When fitting an *MS*(n)*AR*(p) to a time series, the *AR* parameters ϕ_{i,s_t} for each regime $s_t = 1, \dots, n$ and the one-step transition probabilities P_{kl} of the Markov chain must be determined.

3. Modeling of the TRM series

This section presents a descriptive analysis of the series of the *TRM* and its log-returns, a methodology for the fitting and selection of the models, and the results of the fitted *ARMA, GARCH,* and *MS* models to the log-return series.

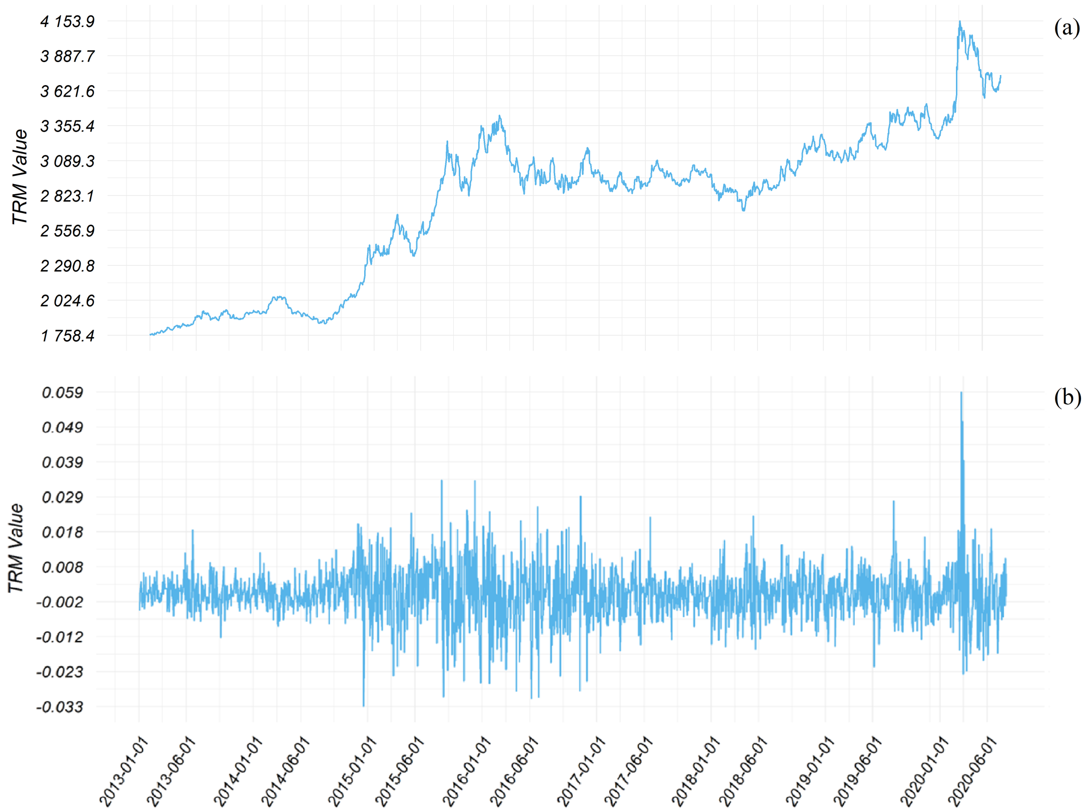

Figure 1: (a) *TRM* series and (b) log-return of the *TRM* series from 2013-01-02 to 2020-07-02.

3.1. The TRM and log-return series

It is clear from Figure 1(a) that the *TRM* series is not stationary. In fact, the *Dickey-Fuller test* suggests that the hypothesis of an stationary series must be rejected with a p-value of 0.9093. With the log-return series y_t defined by eq. (1) the hypothesis of a constant mean of this series cannot be rejected, according to the Dickey-Fuller test with a p-value of 0.01 (see Figure 1(b)). Therefore, it is justified to apply the models introduced in section 2 to this series.

It is important to note in Figure 1(a) that the mean of the *TRM* series has at least two periods of regular increase, from 2015-01 to 2016-01 and from 2018-06 to 2020-07. Besides, Figure 1(b) shows clusters of volatility in the period 2015-01 to 2016-01, from 2018-01 to 2018-06 and the period between 2020-03 an 2020-07. It is well known that three exogenous events influenced these abrupt changes in the *TRM* structure: the *oil crisis* 2014-10 that came with a quick fall of the hydrocarbons prices [16], the *US-China trade war* 2018-07 with duties imposed for both sides, and the global expansion of *COVID-19* in 2020 [18].

The autocorrelograms of y_t in Figure 2 suggest that the series could be an $AR(1)$ process as the *PACF* and *ACF* present sharp falls after lags 1 and 2, respectively. After these lags the autocorrelations seem to be not significantly different to 0.

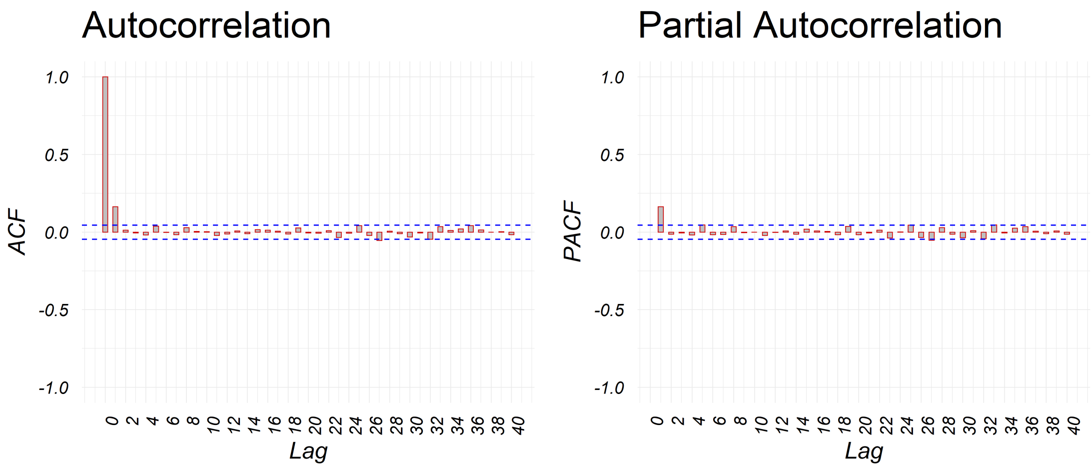

Figure 2: Autocorrelograms of the Log-returns series y_t.

Table 1
BIC, AIC and MAE values of the best five *ARMA* models for y_t.

Model	BIC	AIC
ARMA(1,0)	-12689.42	-12678.39
ARMA(0,1)	-12689.23	-12678.2
ARMA(2,0)	-12687.69	-12671.15
ARMA(0,2)	-12687.68	-12671.13
ARMA(1,1)	-12687.68	-12671.13

Table 2
ARMA(1,0) model parameters and log-likelihood values.

Model	ϕ_1	σ	Log-likelihood
ARMA(1,0)	0.165	0.00766	6347

3.2. Methodology for the fitting and selection of the time series models

It was used the **R** software for the fitting, selection and forecast of the time series models. An iterative methodology was implemented in order to select the best *ARMA, ARMA-GARCH* and *MS-AR* models. First, several number of models for each class were adjusted via *maximum-likelihood estimation* and the *BIC* and *AIC* indexes were calculated. Then, with the best five models according to the lowest values of *BIC* and *AIC*, one-step forecasts were generated at a horizon of 21 days. Finally, the errors *MAPE*, *MAE* and *RMSE* were calculated and the model with the lowest *MAPE* is chosen. This methodology assures the simplicity and precision of the selected model.

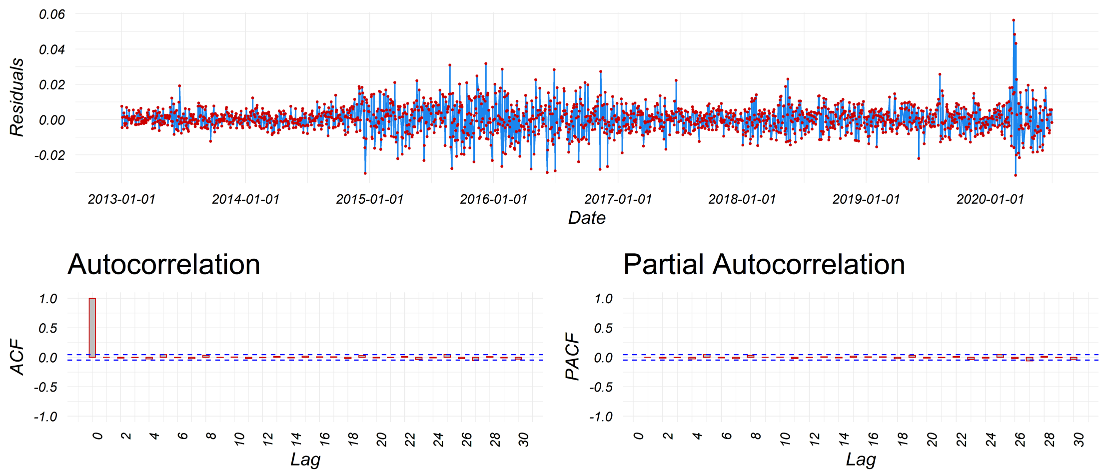

Figure 3: Autocorrelograms of residuals of the *ARMA(1,0)* model for y_t.

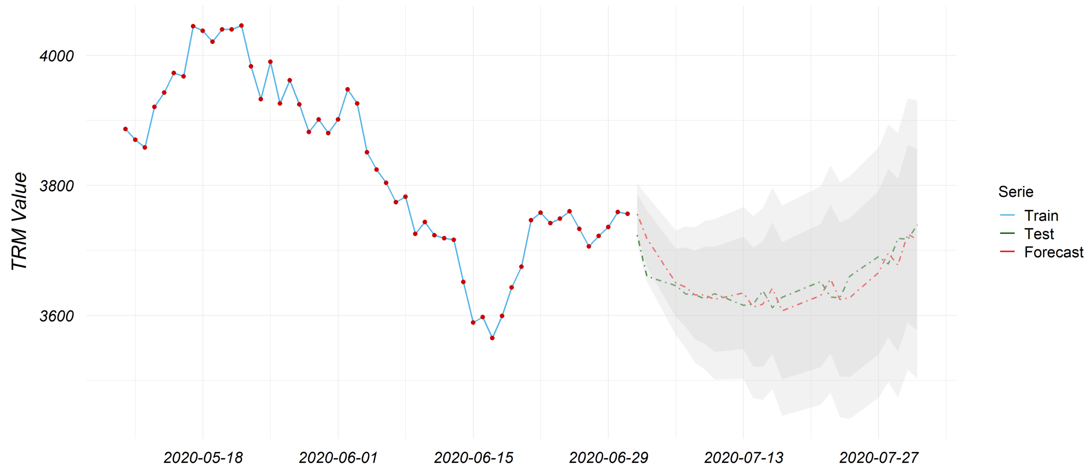

Figure 4: One-step *ARMA(1,0)* forecast.

3.3. ARMA models for the log-return series

Following the previous methodology, 900 *ARMA(p,q)* models (with different orders p, q) were evaluated. The five models with the lowest BIC and AIC values are presented in Table 1. The model with the best forecast is $ARMA(1, 0)$ (i.e., $AR(1)$); the parameters are shown in Table 2. Observe that ϕ_1 is less than 1, which assures stationary, and ϕ_0 is fixed to zero to assure a zero mean for the log-return series. The value for ϕ_1 shows that the log-returns of the TRM at $t + 1$ are related positively with the returns at t in around 16%. Regarding independence of the residuals, the $Ljung - box$ test shows that the residuals of the model are uncorrelated, which also can be verified graphically in the autocorrelogram of the series of the residuals (Figure 3).

Finally, the one-step forecast was performed with the function `forecast` in **R**. The projected

Table 3
BIC and *AIC* values of the best five *ARMA-GARCH* models for y_t.

Model	BIC	AIC
ARMA(1,0) GARCH(1,1)	-7.1915	-7.1765
ARMA(0,1) GARCH(1,1)	-7.1898	-7.1748
ARMA(1,1) GARCH(1,1)	-7.1904	-7.1724
ARMA(0,2) GARCH(1,1)	-7.1902	-7.1722
ARMA(2,0) GARCH(1,1)	-7.1901	-7.1721

Table 4
ARMA(1,0)GARCH(1,1) model parameters values.

ARMA	ϕ_0	ϕ_1	$v_t \sim$
(1,0)	0.18e-3	0.1924	Normal
GARCH	ω	α_1	β_1
(1,1)	5.560e-5	0.1018	0.8951

values show a fairly good fit that hardly departs from the actual value of the series in most periods. It is notable that in times of sudden changes, the level and trend correction took few steps to perform (Figure 4).

3.4. ARMA-GARCH models for the log-return series

Suppose an *ARMA*(p, q) model for y_t. If the white noise process ϵ_t follows the conditions for a *GARCH*(p', q') process (section 2.2), then y_t follows an *ARMA*(p, q)*GARCH*(p', q') model.

Around 3700 combinations of *ARMA-GARCH* models were evaluated by varying the orders p, q, p', q', and their parameters were estimated with the `rugarch` package in **R**. The model with the best performance was *ARMA(1, 0)GARCH (1, 1)* (Table 3). Recall that the *ARMA(1,0)* model for the mean was also the best *ARMA* model in section 3.3, although with slightly different parameters. However, they also fulfill the stationary conditions as well as the fitted parameters of the *GARCH*: $\phi_1 < 1$, $\alpha_1 > 0$, $\beta_1 > 0$, $\alpha_1 + \beta_1 \leq 1$ and $\omega > 0$ (Table 4). Finally, the *Ljung-box* test indicates that the residuals from the model fit are not correlated, which also can be evidenced in the autocorrelograms of the series of the residuals in Figure 5.

With respect to the one-step forecast of the selected model, the projected value shows a fairly good approximation that hardly departs from the actual value of the series. The confidence intervals, calculated based on the one-step predicted conditional variance from the *GARCH(1,1)* model, contain the real values of the series, although they are tighter compared with the confident intervals of the *ARIMA* model of section 3.3. Hence, the *ARMA(1,0)GARCH(1,1)* gives a better representation of the variability of the *TRM* series.

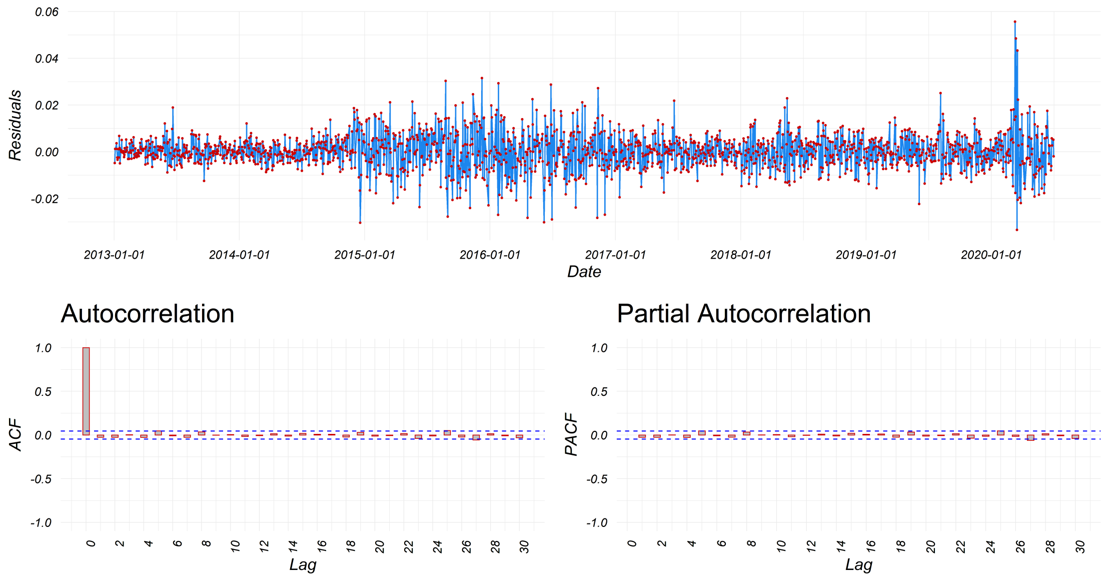

Figure 5: Autocorrelograms of residuals of the ARMA(1,0)GARCH(1,1) model for y_t.

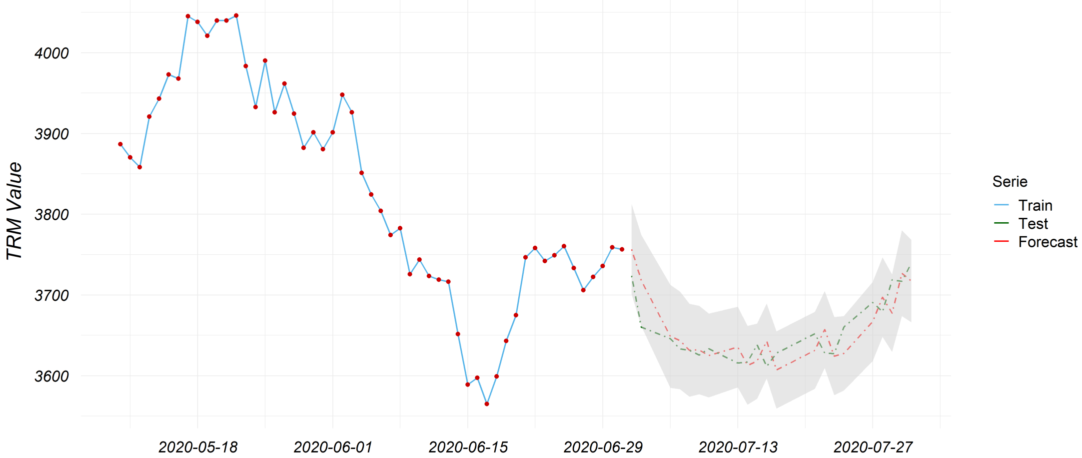

Figure 6: One-step ARMA(1,0)GARCH(1,1) model forecast for 21 days.

3.5. Markov Switching models for the log-return series

$MS(n)AR(p)$ models were adjusted using the `MSwM` package. The five models with the lowest BIC and AIC are shown in Table 5 from 63 models evaluated. The $MS(3)AR(0)$ was selected as it performed better in the forecast, that is, the best model consists in three regimes each one driving an $AR(0)$ process.

Table 6 presents the values of the intercepts ϕ_{0,s_t} and the standard deviation of the white noise σ_{s_t} for each regime s_t. Note that regime 2 has the higher volatility of the three regimes,

Table 5
Selection criteria of the best five MS-AR models.

Model	BIC	AIC
MS(3)AR(0)	-13191.16	-13127.99
MS(3)AR(1)	-13190.13	-13125.46
MS(2)AR(1)	-13164.99	-13116.86
MS(4)AR(0)	-13187.41	-13105.70
MS(2)AR(2)	-13154.44	-13102.81

Table 6
AR(0) model parameters for the three regimes $s_t = 1, 2, 3$ and transition probability matrix for s_t.

Regime	ϕ_{0,s_t}	σ_{s_t}	Transition probabilities		
s_t			1	2	3
1	0.0005	0.006	0.97	0.011	0.023
2	-0.0007	0.0114	0.008	0.98	0.00174e-6
3	0	0.0035	0.0177	0.002e-6	0.97

while regime 3 has the lowest by around half of the regime 1. Also, the coefficient ϕ_0 for regime 2 is the highest (in absolute terms) among the other regimes. This suggests that this regime predominates in the periods with rupture in the mean and with higher variance of the *TRM*. Moreover, Table 6 also shows the transition probability matrix of the Markov chain s_t of the regimes. It is possible to observe that once in any regime $i = 1, 2, 3$, s_t will stay there for around 98% of the times. However, the probability of transition from regime 1 to regime 2 is very low (0.011) and almost zero from regime 3 to regime 2. This reflects the fact that periods of high volatility of the *TRM* series are rare but can persist for long periods. And on the other hand, there exists a major probability of transitions between regime 1 and regime 3, and vice versa. This represents the normal dynamic of the *TRM* with long periods of low-mid variability through time.

The smoothed probabilities show excellent segmentation of the periods where the log-returns of the *TRM* series show marked fluctuations (Figure 7). It is observed that regime 2 periods start approximately at the same time when three critical worldwide situations initiated: the *oil crisis*, starting at 2015-01, the *US-China trade war* starting at 2018-06 and the *COVID-19* outbreak, which reached Colombia in 2020-03. These periods showed the fastest depreciation and volatility of the *TRM* in the last decade.

The one-step forecast of the selected model in Figure 8 shows acceptable results in the description of the average response, however the confidence intervals are wide. Figure 8 also shows the smoothed probabilities of each regime in the period 2020-05 to 2020-07. It is clear that the model develops a gradual transition from regime 2 (of high volatility) to regime 3 (of low-mid volatility), and that regime 1 loses preponderance. Regime 3 probability increases to almost 95% after an eight-day forecast. Finally, the residuals obtained from the model fit show

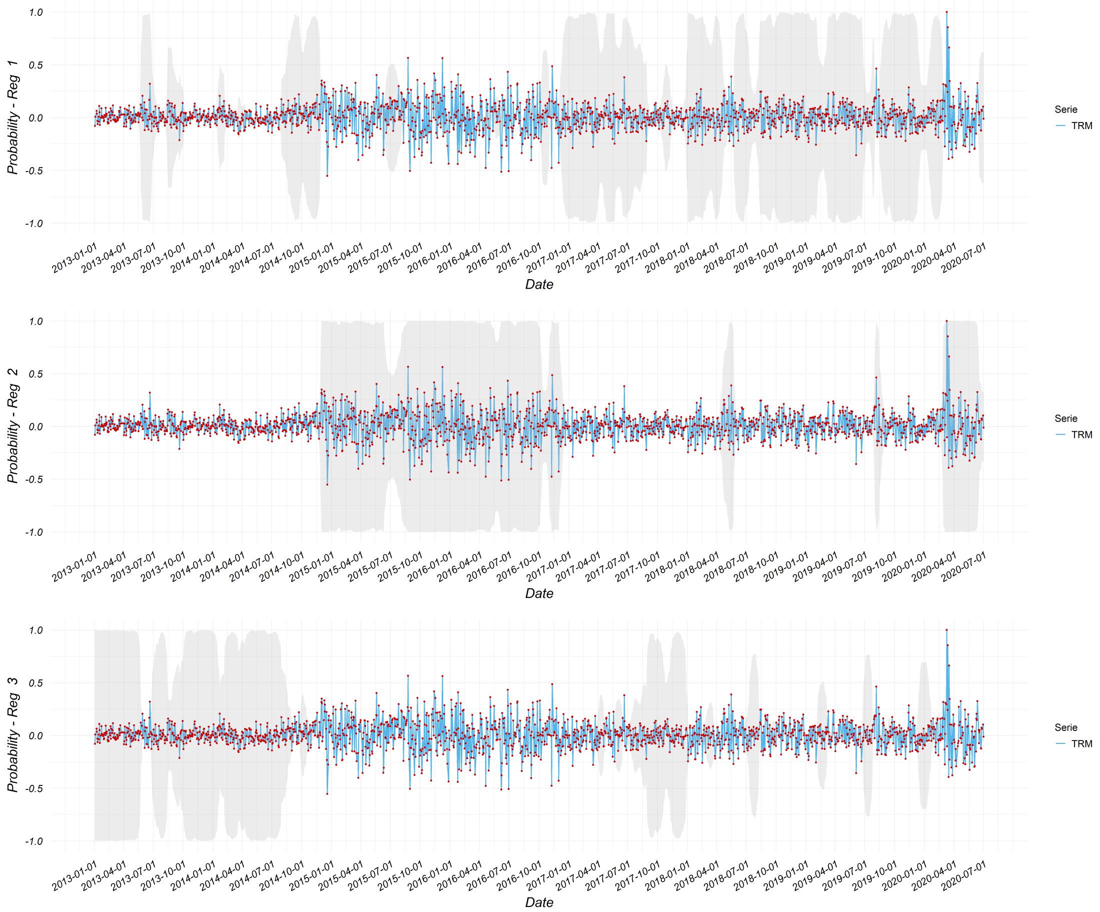

Figure 7: Smoothed probabilities of model MS(3)AR(1). Gray zones represent the smoothed probability of each regime.

no significant correlations after the first lag, which is verifiable from Figure 9.

3.6. Forecast comparison

When the results of the best *ARMA*, *ARMA-GARCH*, and *MS* models are compared, it was found that the *Markov Switching* model achieves the best one-step adjustment, getting lower error indicators (4% less) and *BIC* index as shown in Table 7. That is, by making the mean and variance depending on different regimes, it was possible a better forecast of the *TRM* series in the short term.

When evaluated in a longer time window, in this case a horizon of 21 days, the forecast obtained with the *ARMA* model shows a better performance, although similar to the *ARMA – GARCH* model (see Table 7). Then, for this situation an *ARMA* model is a simpler and a more easy-to-implement representation, but the *ARMA – GARCH* model provides a more accurate representation of the variance.

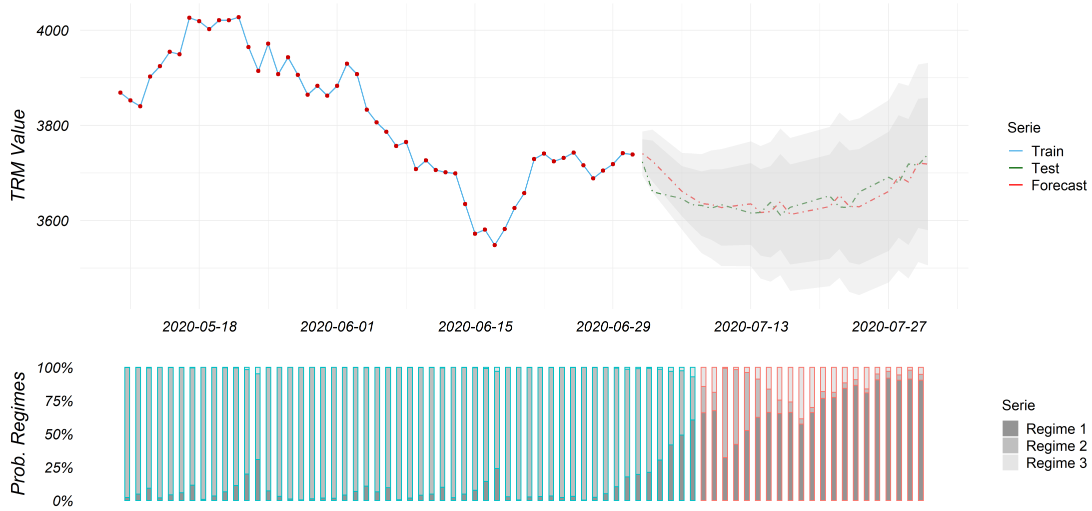

Figure 8: One-step MS(3)AR(1) model forecast for 21 days.

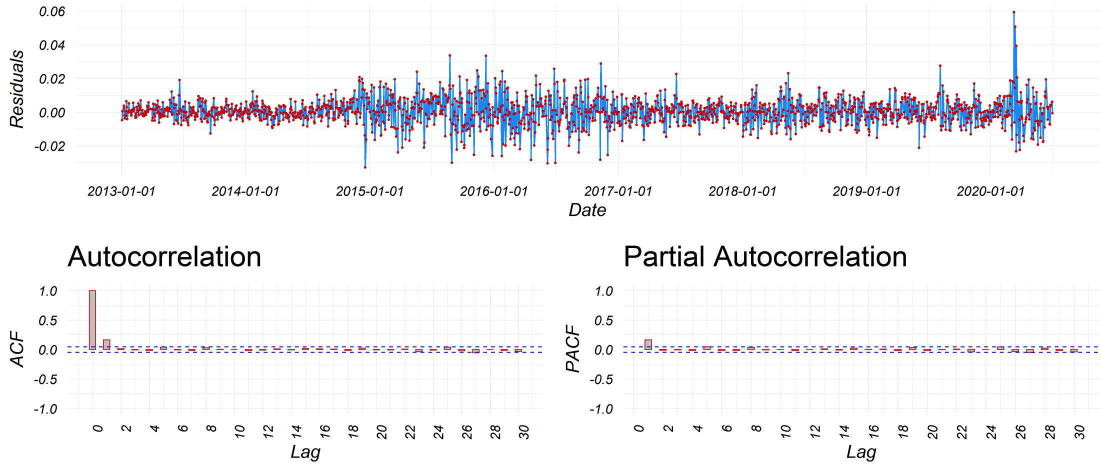

Figure 9: Analysis series of residuals MS(3)AR(1) model.

In the case of the modeling of the series over long periods of time, the fact of being able to characterize several regimes to represent specific realities of the financial series, has a positive effect when obtaining an approximation for the average value of the series. In the same way and as in applying the forecast to one step, there is a clear advantage in having the possibility of modeling the variance through time, which is at the same time a more realistic representation of the behavior of the *TRM*.

Table 7
Forecast comparison for the best models $MS(3)AR(1)$, $ARMA(1,0)$, $ARMA(1,0) - GARCH(1,1)$.

Model/Forecast	MAPE	MAE	RMSE	BIC
MS AR / One-Step	0.5186	18.991	23.8121	-13127.99
ARMA / One-Step	0.5375	19.7004	24.2352	-12678.39
ARMA-GARCH / One-Step	0.5389	19.7529	24.264	-7.1765
MS AR / Horiz.	2.989	108.88	115.048	-13127.99
ARMA / Horiz.	2.738	99.72	106.96	-12678.39
ARMA-GARCH / Horiz.	2.944	107.24	113.44	-7.1765

4. Conclusions

The *TRM* is one of the most important economic items for the Colombian economy. This financial series is influenced by various factors originated by national and international political and economic environment, adding stochastic behaviors to its structure and high volatility clusters, making its forecast quite difficult to carry out.

In this paper the *TRM* series was studied over a longer period of time (from 2013-01 to 2020-07) compared with previous works, with the aim to propose a more robust time series model. For this, an iterative methodology was proposed for the fitting, selection and forecast of the most adequate models from three classes of time series: *ARMA, ARMA-GARCH* and *Markov Switching (MS)*. The latter two with the property that they can model the non-stationary behavior of the *TRM* variance.

A *MS* model with 3 regimes and $AR(0)$ processes achieved the best fit over the *ARMA(1,0) GARCH(1,1)* and the *ARMA(1,0)* models, showing closer values to the real series for a one-step prediction horizon. It is noteworthy that the best *MS* model for the PEN-USD exchange rate in [6] had also three regimes, one of them representing the high volatility periods and the other two expressing the mid-low volatility periods in a "normal" economy context. In the present work, regime 2 represented these periods of high volatility, which in the time window at consideration started approximately simultaneously with three critical events at global scale: the *oil crisis* (2015-01), the *US-China trade war* (2018-06), and the *COVID-19* pandemics in Colombia (2020-03). Additionally, it is worthy to note the improvement in accuracy for the testing window compared with the *MS* model in [14]. Additionally, the *ARMA* model turned out to have fairly good records in the measurement criteria of the error for a longer time window forecast, with the advantage of being able to be implemented more easily. Despite this fact, the *GARCH* model forecast showed tighter confidence intervals.

In conclusion, the possibility to characterize several realities through the regimes of *MS* models gives the opportunity of obtaining a better description of the *TRM* series in a longer time window, and in the *GARCH* model to describe the stochastic nature of volatility. For future works it seems necessary to evaluate an hybrid *GARCH-MS* model that could forecast more accurately the abrupt changes in mean and variance of the *TRM* series.

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