Pitch-Class Distributions in the Music of Anton Webern

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Abstract
This paper uses pitch-class distributions as an analytical tool with which to consider chromaticism in Webern’s 31 published works. Previous research shows that tonal music typically has a varied distribution; intuition suggests that the distributions of dodecaphonic music will be much more even. The range of values in the distribution is used as an indication of variety, and Variability-Based Neighbour Clustering is used to group the works. The results of the analysis draw attention to Op. 10: three movements have idiosyncratic distributions, and the highest-level partition of the corpus is within Op. 10. Expectations regarding distributions for dodecaphonic music are confirmed, but the tonal music diverges from typical tonal distributions.

Keywords
Webern, pitch-class distributions, clustering, corpus study, harmony, computational musicology

1. Introduction
Webern’s music has benefited from a wide variety of analytical interest; nonetheless, these enquiries have typically isolated individual works or movements for detailed consideration. The most wide-ranging studies are Allen Forte and Kathryn Bailey’s monographs, which cover, respectively, the atonal music up to and using dodecaphony [9, 3]. Both have overriding theories; nevertheless, the focus is on explaining individual pieces. This paper, by contrast, is a corpus study. Using digital tools to gather and assess data, this project cuts across preconceived boundaries to consider Webern’s wider compositional practice.

The premise of this paper is distinct from many typical corpus studies [19, 2, 29]; rather than using a sample of works to stand for a larger population, and in so doing assess a broader style, this study considers the complete oeuvre of one composer. Through the statistical tools that underpin this paper, it is possible to identify what is normative in Webern’s practice and what is unusual—his defaults and deformations, to adopt the *Formenlehre* language [10]. Of course, this binary characterisation is reductive: Webern’s style changed radically, and an important part of this paper is identifying when and how this took place.

This paper deploys pitch-class1 distributions, a technique with a significant pedigree in the analysis of tonal [19, 1, 2] and neo-tonal music [23], to consider macroharmonic effects in Webern’s music.2 Pc distributions might seem to be relevant only to tonal and neo-tonal repertoire, particularly as they are often used in key-finding. The contention of this paper is

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1Henceforth, pitch-class will be abbreviated to ‘pc’.
2The terminology is from Dmitri Tymoczko, who differentiates statistical distributions from subjective profiles [23, p. 175].
that in fact they provide new information about atonal music. They are an effective way of considering chromaticism, and differentiating so-called ‘free atonal’ from dodecaphonic music. To achieve this, having gathered the data, descriptive statistical analysis is used to consider Webern’s practice, with particular concern for when and how the style changes, and the impact of pre-conceived boundaries such as the onset of atonality and dodecaphonicism.

There has been some statistical analysis of Webern’s music previously: Herbert Eimert considered interval content in Op. 28/i [8] and Roland Jackson assessed harmonic features of extracts from Opp. 5 & 28 [12]. More recently, Tymoczko included a smattering of examples [23, pp. 154–186], and Jenine Brown has considered intervallic patterns in Op. 24/iii, with supporting analyses of four other movements [6]. Nonetheless, these have suffered from a limited sample and little extended enquiry. Precedents for this paper that are more methodologically alike come from stylometry, ‘the statistical analysis of literary style’ [11, p. 111].

2. Methodology

2.1. The Corpus

In this paper, the corpus is restricted to the 31 works with opus numbers. To what extent we can understand this as a single coherent corpus is open to challenge, Webern’s compositions change radically from the Passacaglia to the Cantatas, but they are clearly unified by his creative mind. Movements of multi-movement works are considered individually: each movement is sufficiently harmonically self-dependent that it is more meaningfully analysed on its own terms, much as Bailey understands dodecaphonic movements as having their own prime row-forms [3, p. 12]. In justifying this, she draws on a tonal analogy, which is also instructive with regard to pc distributions. To conflate the pc distributions of two movements in different keys might provide some interesting large-scale perspective, but obscures so much detail as to be actively counterproductive, as the patterns will ‘cancel each other out’; the same is true in an atonal context.

This corpus omits the Werke ohne Opus, some of which are now accepted into the canon [26]. The 31 works with opus numbers present a meaningful corpus and such restriction avoids more controversial adjudication that would accompany the inclusion of other works. This corpus also only includes the revised, published versions of the early works. Felix Meyer and Anne C. Shreffler have convincingly shown that Webern’s revisions between 1915 and the mid-1920s sought to ‘update’ his works to his new, classicist aesthetic [15, 16, 14]. Broadening the corpus with these presently-excluded works could certainly benefit further research: it would particularly expand the tonal subset of the corpus, presently comprising only Opp. 1 & 2, whilst the unrevised versions could lend an insightful perspective and follow Meyer & Shreffler’s exhortation to ‘[broaden] the object of analysis’ [16, p. 376]. For now, we must be aware that the corpus is privileged towards Webern’s later aesthetic, even though his revisions had limited impact on the pitch and durational matters discussed in this paper.

The movements of the corpus have been encoded in conventional Western music notation using Sibelius software and exported as XML files, before data collection using music21 [7]. In each case the published score has been replicated as closely as possible. Overall, this required little editorial intervention, particularly concerning pitch and duration notation. The greatest

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3*Pace Forte [9, p. x] I will use this term to refer to the music from Op. 3 to Op. 16, those works that are accepted as atonal, but are not dodecaphonic.

4All code and data is available at https://github.com/joshua-ballance/Pitch-Class-Distribution. Where copyright allows, scores can be found at musescore.com & imslp.org.
ambiguities surrounded the exact placement of dynamic and tempo markings, and whilst the
first has little bearing on this project, in practice the second could usually be inferred from
proximate musical gestures. At the heart of this project lies the methodological assumption
that scores can be viably used as proxies for sounds and so, in order to minimise this difference,
various notational alterations had to be made. Obvious examples of this include spelling
out artificial harmonics or accounting for piano sustaining pedal, but ornaments are more
contentious. The approach adopted here has been to view trills as fundamentally ‘ornamental’,
so the G♯ trill in Op. 10/i is seen as a single pitch, rather than a prolonged G♯–A dyad.

An important part of empirical musicology is interrogating pre-conceived notions, however
implicitly expressed. I will therefore present some reasonable hypotheses suggested by the liter-
ature, which will frame the data analysis. As with all quasi-scientific approaches, it is important
to acknowledge all outcomes, whether they confirm consensus or provide new information. It is
easy to discount results that support pre-conceived ideas; in fact, particularly given the dearth
of statistical musicology, presenting reproductions of earlier results or extending more limited
ideas to a larger corpus is highly significant for the field of knowledge.

2.2. The Pitch-Class Distribution

Turning to the analytical method, a pc distribution is, as might be expected, the distribution
of pc’s in a given piece (tonal distributions are often expressed in terms of scale degrees).
These proportions may be calculated in terms of pc instances, or, as in this project, durations.
These durations are measured in seconds (or fractions thereof) according to the metronome
marks given in the score; gradual changes in tempo (rits. and accels.) are modelled according
to Sibelius playback. In summary, therefore, this algorithm counts all of the individual notes
in a given movement, records their pc value and duration, and summarises the cumulative
proportion of each pc as a proportion of the duration of all the notes. The duration of all
the notes is not the duration of the piece: rests are not counted, and each of the notes in a
simultaneity is counted individually, therefore providing a greater duration than in the original.
For the purposes of the distribution, all pc’s are deemed to be of equal significance, irrespective
of their textural or orchestral presentation. As an analytical method, this is therefore
conceptually simple and easy to deploy, and by considering proportions it normalises for the
length of works [5, pp. 263–264].

Regarding preconceptions about pc distributions, the literature is clear that tonal works are
characterised by variety. Figure 1 is a distribution using data from Joshua Albrecht & Daniel
Shanahan’s corpus study of 625 major mode and 357 minor mode works [1]. Although the
corpus is weighted towards earlier music, it does include more chromatic music by Brahms,
Chopin, and Kabalevsky. The distribution clearly outlines the diatonic scale, particularly
emphasising the tonic and dominant. A historical analysis by Albrecht & David Huron has
shown that, despite detailed changes in distributions, these overarching features persist across
the period 1400–1750 [2]. Whether they will define music as chromatic as Webern’s Opp. 1
and 2 is less clear, but a fairly strong correlation might reasonably be expected. Conversely, a
dodecaphonic work would be expected to have a significantly less varied distribution. Whilst
a composer could write a classically dodecaphonic piece that replicated the distribution of
Figure 1, as Tymoczko points out, dodecaphonicism was in fact ‘explicitly designed to promote
flat pitch-class profiles’ [23, p. 183]. As for the freely atonal music, expectations are much
less clear: it is possible that the advent of atonality encouraged a swift move to pc equality,
but it is equally feasible that there was a gradual change over time; clustering the corpus will
help with this enquiry. Much of this analysis assumes a relationship between duration and significance. This is not wholly determinative; nonetheless, patterns in the overall distribution certainly play an important part in shaping the musical surface.

There is some debate in the literature about what to sample to consider pc distributions. In most cases, scholars have sought to find typical distributions of keys. Modulatory passages can cause issues with the count, and so scholars often sample only beginnings or endings of works, arguing that these will present more apt distributions [19, 2]. Jason Yust’s work has interrogated this further, suggesting that, in tonal music, entire pieces tend to have a flatter distribution than a combination of beginnings and endings, but that whilst endings behave independently, beginnings and whole pieces are similar. In the present paper, the focus is not on creating archetypal distributions, but instead on comparing the specific distributions of these works and so these are measured across movements. Yust’s finding that whole piece distributions approximate those of beginnings is interesting in this regard: his speculation is that ‘the typical choices of contrasting key areas tend to reinforce the distributional properties of the home key in some respects and cancel one another out in others’ [29, p. 13]. Whether the same is true of atonal music is beyond the scope of this paper, but this legitimates comparisons between these distributions and archetypal tonal ones.

As a final methodological point, I offer a note of caution about the implications of the results. It is easy to view quantitative results as unquestionable truths; as Caroline Bassett puts it, Big Data makes claims to provide ‘solutions beyond dispute’ [4, p. 550]. Within defined parameters this may be so, but it is crucial to maintain perspective on what this does not tell us. An important such qualification here is the reliance on texts as source material. Post-structuralist thought has successfully destabilised analysts’ default tendencies to use a score as a proxy for, representation of, or even incarnation of the hallowed ‘music itself’. It is certainly the case that the approach here could analyse recordings or performances; nonetheless, Webern is specific enough with regard to the parameters under consideration (primarily pitch and duration) that scores are acceptable. Even so, significant hesitancy must be applied to temporal modelling. Webern’s specificity is helpful, but any performance will diverge from

Figure 1: The Relative Frequency of Scale Degrees [data from 1, p. 67].
the score’s temporal indications; indeed, testimony regarding his own performances describes exactly this phenomenon [28, p. 75]. As such, claims about proportions are only indicative, not prescriptive; the values should be seen as suggestive rather than absolute.

3. Results

The following summary outlines the key results. In Figures 2 & 4 the corpus is ordered chronologically, by date of composition as recorded in the Moldenhauers’ biography [17, pp. 700–705]. This is largely equivalent to the ordering of opus numbers, though not universally so. The only major uncertainty concerning ordering is for Opp. 3 & 4 for which the dates of composition are unclear. These have therefore been sorted by their eventual position in the published works.

3.1. Spread

The data gathered for pc distributions can be shown by a bar chart for each piece. In considering the corpus as a whole, however, the most important metric is the distribution spread. This is calculated as the range of values: the difference between the proportions of the most and least common pitch classes. The greater the range, the more variation there is. As mentioned, tonal music is expected to have a high range, corresponding to pc proportion discrepancies; dodecaphonic music, by contrast, would likely have a small range. Figure 2 is a scatter plot of the ranges. Clusters will be discussed below, and derive from Figure 4. The general pattern is clear: despite some noise, distribution spreads decline across the corpus; indeed, the correlation between corpus position and range is a moderately strong inverse-correlation of -0.65. In summary, therefore, the results are largely as expected.

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5For this list see https://github.com/joshua-ballance/Pitch-Class-Distribution.
6For an interactive plot in which each movement is labelled see https://bit.ly/FigureTwoInteractive.
7Correlations are Spearman correlations given to two decimal places.
A further point of interest is which movements are unusually varied. These movements are those with an anomalous range, calculated according to Tukey’s Rule. Interestingly, all of these come from Op. 10: movements i, iii, and iv, with spreads of 21.7%, 27.4%, and 17.8%, respectively. Figure 3 shows their distributions; for the first two movements, the distribution is characterised by one anomalously high value above an otherwise typical distribution; for Op. 10/iv, A is anomalously high, and A♯ is also noticeably elevated.

### 3.2. Clustering

Variability-Based Neighbour Clustering can be used to cluster the corpus empirically. It is a hierarchical agglomerative approach that preserves corpus position [first described in 22] [see 24, for an example of its application]. Figure 4 displays this clustering using complete pc distributions for each movement, rather than relying on spreads.

The implications of this clustering will be discussed below; at this stage, it is worth drawing out the structure of the various clusters. Unlike other clustering methods, the number of clusters does not have to be pre-determined; rather, this is left to the analyst to discern based on the results of the algorithm, as presented in Figure 4. In this case, the picture is somewhat messy, with several movements clearly dissimilar to their neighbours (e.g. Op. 10/iii & iv, Op. 16/i). Nonetheless, there are several principal clusters that can be discerned. On the highest level, the corpus is partitioned between Opp. 10/iii and 9/ii. Given the anomalous values of Op. 10/iii & iv, this is not hugely surprising. However, even if the three anomalous movements identified above are removed, the highest-level partition is between Op. 9/i & ii, and therefore in the same area. Subdividing these two principal regions further reveals other groupings: in the latter half, Op. 9/ii–Op. 16/i and Op. 17/iii–Op. 31/iii; the first half is less uniform.

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8An anomaly is thus defined as a value lying outside $1.5 \times IQR$ below the first or above the third quartile.

9This code is by Folgert Karsdorp (https://github.com/fbkarsdorp/diachronic-text-analysis).

10For legibility, a fully-labelled dendrogram is not presented here, but can be found at https://github.com/joshua-ballance/Pitch-Class-Distribution.

4. Discussion & Conclusions

The following discussion will consider the musical and historical implications of these results, seeking to identify trends and consider their more overt musical relevance. This research is fundamentally post-hoc analysis, but it is buttressed by Webern’s well-noted sensitivity to the structural functions of the total chromatic \[27, 18\]. The overall picture, that the later music was much less varied in its pc distributions, confirms the preconceptions laid about above.

Turning first to clustering, Figure 4 suggests that rather than the principal partition in the corpus lying between tonal and atonal music, or indeed the freely atonal and dodecaphonic music, the highest-level clustering partition suggested by this approach instead lies around Opp. 10 & 11, an observation supported by Figure 2. Shreffler has described the instrumental miniatures Opp. 9–11 as a ‘crisis’, inspiring a period of significant experimentation in Op. 12 and the ensuing Lieder in which Webern sought to establish a new style \[21, p. 11\]. This research certainly confirms her view that the miniatures were an extreme and required a significant break in the ensuing music, an assertion supported by the categorisation of Op. 10/i, iii, & iv as anomalous. In sum, then, this proposes that we can group Webern’s earlier and later practice apart from methods of construction: the first group encompassing tonal and some freely atonal music, leading up to the ‘crisis’; the second covering the new freely atonal style and dodecaphonicism. It is quite remarkable that the greatest change in the chromaticism of Webern’s music came not with the onset of systematic dodecaphony, but rather through his own aesthetic intuition.

As mentioned previously, the distributions of the three anomalous movements (Op. 10/i, iii, & iv) were characterised by a single particularly prominent note or pair of notes. In each case
this is musically apparent as a pedal: the Celesta G♯ trill in the first movement, the E shared variously between guitar, bells, celesta and harp in the third movement, and the A and B♭ in the viola and clarinet in the fourth movement. The effect of this is to provide a central timbral and harmonic focal point, a note with a high degree of salience, to use Lerdahl’s term [13], that anchors the otherwise aphoristic gestures that make up these movements. There are no similar pedals in either of the other movements of Op. 10, hence their smaller ranges. Forte writes that ‘long sustained single notes like this are not uncommon in Webern’s atonal works’ [9, p. 384]; this may be true, but the effect in these three works is clearly unique. Meyer & Shreffler describe how Op. 10 consists of movements from two previously separate orchestral pieces; these subcollections, however, are i & iv, and ii, iii, & v. They warn ‘against finding specific motivic connections between movements’ [16, p. 357], yet it is notable that this phenomenon of a focal pedal-point cuts across the compositional boundaries in this collection.

As for the start of the corpus, it is notable that the initial cluster splits Opp. 1–3/i and Opp. 3/ii–5/iii. This grouping suggests little difference between Webern’s tonal music and his early atonal music, a view reinforced by Figure 2. Indeed, the distributions of these two tonal works (Figure 5) are quite dissimilar from tonal distributions. Op. 1 has a correlation of 0.60 with Albrecht & Shanahan’s minor distribution [1]. At first glance this might seem quite high, and is bolstered by some tonic/dominant emphasis and diatonic patterning in the distribution. Nonetheless, in Albrecht & Huron’s study, the smallest correlation between the archetypal distribution and any of the analysed 50-year segments was 0.90, which described the difference between 1700–1750 and 1400–1450. As for Op. 2, its correlation with the major distribution is 0.39, and though there is some dominant emphasis, it is otherwise distinct from the archetypal distribution.\footnote{For these correlations, the distribution of Op. 1 was ordered according to a D tonic and Op. 2 a G tonic.} For context, the greatest correlation between Op. 3/i and either of the archetypal profiles is 0.45.\footnote{This is between the major profile and Op. 3/i with a ‘tonic’ (pc 0) of G♯. Interestingly, the highest correlation with the minor profile is 0.44, almost identical, and also assumes a ‘tonic’ of G♯.} Given the chromaticism of both tonal pieces this is understandable, and yet extreme. A tonal-atonal partition is the typical chronology in assessing his music, and this research shows it to be clearly insufficient with regard to chromaticism. That said, a sample size of two tonal works is hardly representative and would be helped by significant augmentation with some of the Werke ohne Opus.

Moving forward, the comparative uniformity of the latter period from Op. 9 onwards is notable, particularly from Op. 17/iii. For context, the range of ranges in Figure 2’s Clusters 1–2 is 17.9 (12.5 without the anomalous Op. 10/i) and in Clusters 3–4 is 10.0: the median range values, however, are 9.8 (9.5 without Op. 10/i) and 4.7, respectively. As such, whilst there is a similar amount of variety in the distribution of ranges, the later ranges tend to be much smaller, indicating significantly less variation in the pc distributions. The distributions of the dodecaphonic works themselves therefore confirm expectations: though there is inevitably some variation, this is minimal, and particularly in light of Figure 2 Webern’s claim that ‘All twelve notes have equal rights’ [25, p. 52] certainly seems apt. This rather complicates Bailey’s contention that through intersection and Ausfälle Webern limited certain pc’s. Without context from other composers’ dodecaphonic works it is impossible to discern whether her supposition that ‘it was not the equality of the twelve notes that was of primary concern to Webern’ [3, p. 146] is accurate. Indeed, she suggests that this feature emerged as part of Webern’s style; in fact, the correlation between range and corpus position within the dodecaphonic works is 0.00, indicating that there was no noticeable change. The final comment on
this music is to note that this shift does not come precisely with the onset of the dodecaphonic technique: there is no new cluster with Op. 17/i. This seems to confirm Shreffler’s picture of Webern experimenting with, and gradually adopting dodecaphonicism through Op. 15/iv and the canons of Op. 16 [20]. The outcome from the perspective of this research is quite clear: Webern achieved the distributional effect that dodecaphonicism produced independently of it.

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References


