Light Reflection from Real Surfaces: Probabilistic Model of the Layer Radiance Factor*

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Abstract. The article is devoted to the modelling of light reflection from real surfaces. Most of the existing models are often created to solve a certain task and therefore can not be used in other realms. To create a model useful for different purposes, we propose a reflective surface representation as a scattering layer bounded by the diffuse bottom and a randomly rough Fresnel upper boundary. Such an approach allows one to account for light polarization in the scattering layer and slope correlation of the randomly rough boundary which paves the way for the observation of some physical effects that take place in nature (for instance, statistical lens emergence). The first results of the light reflection modelling were taken at such initial parameters as to compare to those obtained in other research. Former occurred to be qualitatively of the same form as the latter. The model needs to undergo further validation in numerous experiments to prove its serviceability for different types of reflective surfaces.

Keywords: Light Reflection, Polarization, Surface, Mathematical Model.

1 Introduction

While solving a large range of applied problems (3D-visualization, remote sensing, radiative transfer in turbid media, etc.), one inevitably faces the task of the radiance factor modelling, which has been remaining a problem of extremely high importance since the middle of the last century.

Despite the abundance of existing empirical, semi-empirical and analytical models, having been proposed through the decades (for example, [1–5]), each of them is rather "ad hoc". Even though they describe light reflection accurately enough in the realm of their validity, the models often make gross mistakes if out of the realm.

We reckon that the sole way to create a multipurpose comprehensive model of light reflection is to repeat and reproduce at the most possible precision all the physical processes occurring in nature when light interacting with a surface.

Though most of the existing models consider reflection from a randomly rough surface only, light does not reflect from a surface itself, but it does both from the upper

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material facets and the material volume. Light rays penetrate the near-surface layers and then they scatter on the material particles, after that the rays re-enter the outer medium, all the processes being strongly dependant on the state of light polarization.

It was shown, that account for light polarization may lead to more than 30% difference in results of multiple reflections modelling beside those obtained in the traditional (depolarized) way [6]. Light polarization being taken under consideration in scattering must bring about even more appreciable effect.

Another noteworthy feature is the slope correlation of the randomly rough Fresnel boundary formed by material facets. Since the Gemini V Program [7], there has been manifold evidence [8] of astronauts and cosmonauts being managed to observe from space the bottom of seas and oceans at the depth of hundreds of meters. That may be due to the water waves under certain circumstances forming a statistical lens — the state of a surface acting upon passing rays as though a real optical lens.

When considering a solid object surface one has to reckon in slope correlation as well, for it always appears after the surface being treated with a tool.

Based on the above considerations we represent a real reflective surface as a scattering layer bounded by a diffuse bottom and correlated randomly rough Fresnel upper boundary. The result of modelling will be wherein sought in the form of radiance factor curve.

Thus, in order to reach the aim one has to:

1. define a method for correlated randomly rough surface construction;
2. elaborate radiation transfer algorithm for the scattering layer;
3. realize the algorithm in a program and analyse obtained results.

The following sections of the article are devoted to solving these tasks.

2 Modelling of The Randomly Rough Fresnel Surface with The Slope Correlation Account

There are two ways to construct a randomly rough surface [9]. The essence of the first one (the method mathematical expectations) is that we model not the surface itself, but only intersection points of light rays and the surface. Such an approach leads to a quite significant decrease in computational time consumption and therefore is fairly attractive. However, the degree of its development appears now to be insufficient. We could find no robust research on elucidating all its issues, the slope correlation account being the most important of them. So we passed to the method of spectral representation.

This method consists in [10] Fourier-transformation of the random field (two-dimensional in the current case) and a correlation function. Then one is capable to model a random surface with correlation account by means of Monte-Carlo Methods. Let us further consider the method in detail.

2.1 General Provisions

The main idea of the spectral models construction is [10] to use a certain approximation of integral for a numerical model of a probabilistic process $x(t)$ with correlation function $B(x)$:

$$x(t) = \int_0^{+\infty} \cos(t\lambda)\xi(d\lambda) + \int_0^{+\infty} \sin(t\lambda)\eta(d\lambda),$$

$$B(x) = \int_0^{+\infty} \cos(t\lambda)\nu(d\lambda),$$

where $\xi(d\lambda)$, $\eta(d\lambda)$ are real orthogonal stochastic measures on the semi-axis $[0, +\infty)$; $\nu(d\lambda) = 2\mu(d\lambda)$; $\xi(d\lambda) = 2\mathrm{Re}\zeta(d\lambda)$; $\eta(d\lambda) = -2\mathrm{Im}\zeta(d\lambda)$; $\zeta(d\lambda)$ is a stochastic spectral measure of the process $x(t)$.

Let $\Psi(\vec{x})$ is a random real field with correlation function $B(\vec{x})$, and its cosine transform is

$$B(\vec{x}) = \int_0^{+\infty} \cos(\vec{\lambda}\vec{x})p(\vec{\lambda})d\vec{\lambda},$$

where $p(\vec{\lambda})$ is spectral density; $\vec{x}, \vec{\lambda} \in R_n$. In order to simplify further equations, we assume

$$M\,\Psi(\vec{x}) = 0, \quad D\,\Psi(\vec{x}) = 1,$$

M and D being averaging and variance operators respectively.

In case of finite spectre one may construct [11] the required field $\Psi(\vec{x})$ through the formula

$$\Psi(\vec{x}) = \sum_{k=1}^{N} \sqrt{p_k}[\xi_k \sin(\vec{\lambda}_k\vec{x}) + \eta_k \cos(\vec{\lambda}_k\vec{x})], \tag{1}$$

where (ξ_k, η_k) are collectively independent standard Gaussian random variables.

Representation (1) can be used for construction of a field with continuous spectre on the basis of a certain quadrature formula

$$B(\vec{x}) = \int_0^{+\infty} \cos(\vec{\lambda}\vec{x})p(\vec{\lambda})d\vec{\lambda} \approx \sum_{k=1}^{N} p_k \cos(\vec{\lambda}_k\vec{x}).$$

Let us split the space R_n into parts $\mathcal{D}_1, ..., \mathcal{D}_N$. And let the random points $\vec{\lambda}_1, ..., \vec{\lambda}_N$ are distributed within the parts in accordance with the densities

$$p_k(\vec{\lambda}) = p(\vec{\lambda}) / \int_{\mathcal{D}_k} p(\vec{\lambda})d\vec{\lambda}, \quad \vec{\lambda} \in \mathcal{D}_k.$$

Then we have

$$B(\vec{x}) = M \sum_{k=1}^{N} p_k \cos(\vec{\lambda}_k\vec{x}), \quad p_k = \int_{\mathcal{D}_k} p(\vec{x})d\vec{x}.$$

In practical tasks, the properties of the solution being investigated are often defined with needed certainty by the correlation function of the initiate field [11]. Then one may assume $N = 1$, $\mathcal{D}_1 = R_n$.

It is known [12] that the values of ξ_k and η_k may be obtained using the following formulae

$$\xi = \sqrt{-2\ln\alpha'}\cos 2\pi\alpha'', \quad \eta = \sqrt{-2\ln\alpha'}\sin 2\pi\alpha'',$$

α' and α'' being independent and uniformly distributed in $(0, 1)$. Then equation (1) takes form

$$\Psi(\vec{x}) = \sum_{k=1}^{N} \sqrt{-2p_k \ln\alpha'}\cos(\vec{\lambda}_k\vec{x} - 2\pi\alpha''), \tag{2}$$

where $\vec{\lambda}_k$ are random and distributed as shown above.

2.2 Modelling Formula for The Case of A Random Surface

Let us further consider a specific example of a randomly rough surface modelling with correlation function $B(r) = \exp(-k_0 r)$, k_0 being constant and $r^2 = x^2 + y^2$.

Taking into account the assumption of the isotropic surface, we pass from cosine-transform to Hankel transform

$$B(r) = \int_0^\infty p(\lambda)\lambda J_0(r\lambda)d\lambda,$$

where J_0 is Bessel function of the first kind. Then spectral density $p(\lambda)$ is

$$p(\lambda) = \mathcal{K}_0^{-1}[B(r)] = \mathcal{K}_0^{-1}[\exp(-k_0 r)] = \frac{k_0}{(\lambda^2 + k_0^2)^{3/2}}, \quad \lambda^2 = \mu^2 + \nu^2.$$

From the equation

$$C\int_0^\infty p(\lambda)d\lambda = 1,$$

one can find $C = k_0$. Further, solving the equation

$$k_0\int_0^\lambda p(\tau)d\tau = \beta,$$

with respect to λ, we achieve the expression for modelling of the value:

$$\lambda = k_0\sqrt{\frac{\beta^2}{1-\beta^2}},$$

where β is uniformly distributed in $(0,1)$.

After having passed to Cartesian coordinate system, it is easy to obtain

$$\mu = k_0\sqrt{\frac{\beta^2}{1-\beta^2}}\cos 2\pi\gamma, \quad \nu = k_0\sqrt{\frac{\beta^2}{1-\beta^2}}\sin 2\pi\gamma,$$

where β and γ are independent and uniformly distributed in $(0,1)$.

Thus, formula (2) takes its final form [11] if one substitute the expressions for μ and ν, as well as include averaging over realizations and foresee the case of $D\,\Psi(\vec{x}) \neq 1$:

$$\begin{aligned}\Psi(x,y) =& \frac{\sigma}{\sqrt{M}}\sum_{m=1}^{M}\sqrt{-2\ln\alpha'_m}\times \\ &\times\cos\left[k_0\sqrt{\frac{\beta_m^2}{1-\beta_m^2}}(x\cos 2\pi\gamma_m + y\sin 2\pi\gamma_m) - 2\pi\alpha''_m)\right],\end{aligned} \tag{3}$$

where σ is standard deviation.

3 Program Realization and Analysis of The Results

3.1 Realization of The Spectral Representation Method

The method of a correlated randomly rough surface construction described above was realized in MATLAB environment. However, in order to make the model more flexible we construct the final surface $\Psi_\Sigma(x,y)$ as a superposition of two surfaces:

$$\Psi_\Sigma(x,y) = \Psi(x,y|\vec{\omega}_1) + \Psi(x,y|\vec{\omega}_2),$$

where $\vec{\omega}$ is a point in the phase space $\Phi = \{M,\sigma,k_0\}$. Such a modification of the formula (3) allows us to construct surfaces of different kind, which, however, maintain their statistical properties.

We want to especially notice that due to the model and its flexibility we managed to observe the statistical lens effect yet mentioned above. For instance, a random surface realization with parameters:

$$\vec{\omega}_1 = \{500, 11.25, 0.025\}, \quad \vec{\omega}_2 = \{100, 0.2, 3\}$$

may look as shown at the Fig. 1. These parameters corresponds to an excited water surface with small ripples.

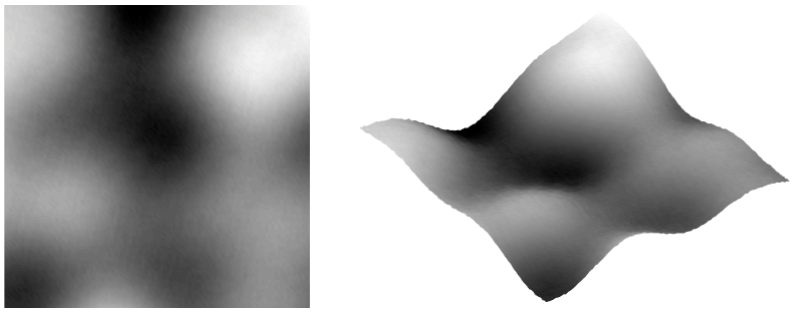

Fig. 1. Excited water surface with small ripples.

When we place a source of parallel rays in the form of the ring over the recess, we will see that the most of the rays converge together at a certain distance from the surface after passing through it. If one put a screen at the place of the rays' convergence, there will be an image of the luminous ring on the screen (Fig. 2).

Fig. 2. Rays passing through the water surface (left) and the image on the screen (right).

3.2 Program Realization of The Reflective Surface Model and Analysis of The Results

Since the base of the reflective surface model is a scattering layer, the main problem to solve is radiative transfer in the turbid medium. The general scheme of light distribution modelling has the following steps:

1. model the initial point in accordance to distribution density of the source;
2. model the free path ξ;
3. prove the ray remain within the scattering medium;
4. evaluate the next interaction point;
5. choose the type of interaction (absorption or scattering);

6. model the new ray direction.

The solution of the problem is sought for in the form of the radiance values in the points on a circle above the surface. In terms of the task, the use of local estimations of the Monte-Carlo Method [13] appears to be one of the most effective [14].

The basic principles used in the algorithm are the same as those in [6] with the exception that we needed to evaluate radiance (instead of illuminance) and account for scattering in the medium.

The light scattering on a particle is governed by the Henyey-Greenstein function in an easy and fairly certain way in the case of depolarized light:

$$\rho(\mu) = \frac{1-g^2}{(1+g^2-2g\mu)^{3/2}},$$

where μ is the cosine of the scattering angle, g is the average cosine of the scattering angle. The main advantage of the formula is its dependence on the only parameter — the average cosine of scattering angle. In [15] it was generalized on the case of polarized light as well.

The expression for the kernel of the local estimation given in [6] also changes its form and in the case of diffuse reflection is equal

$$\begin{aligned}\mathbf{k}(\vec{r},\vec{r}\,') =& \frac{\exp(-\sigma|\vec{r}-\vec{r}\,'|)}{(\vec{r}-\vec{r}\,')^2}|\hat{N}',\hat{l}'|\Theta(\vec{r},\vec{r}\,')\times \\ &\times\mathbf{R}(\hat{l}'\times\hat{l},\hat{N}\times\hat{l})\rho(\vec{r},\hat{l},\hat{l}')\mathbf{R}(\hat{l}'\times\hat{N}',\hat{l}'\times\hat{l}),\end{aligned} \tag{4}$$

in the case of the scattering on a particle

$$\begin{aligned}\mathbf{k}(\vec{r},\vec{r}\,') =& \frac{1-g^2}{2(1+g^2-2g\mu)^{3/2}}\frac{\exp(-\sigma|\vec{r}-\vec{r}\,'|)}{(\vec{r}-\vec{r}\,')^2}\Theta(\vec{r},\vec{r}\,')\times \\ &\times\mathbf{R}(\hat{l}'\times\hat{l},\hat{N}\times\hat{l})\rho(\vec{r},\hat{l},\hat{l}')\mathbf{R}(\hat{l}'\times\hat{N}',\hat{l}'\times\hat{l}),\end{aligned} \tag{5}$$

where σ is extinction coefficient; $\hat{l}'$ is unit vector of the incident ray direction; $\hat{l}$ is the same for the scattered ray; $\vec{r}$ is collision point; $\hat{N}$ is the surface outward normal; $\mathbf{R}$ is the reference plane rotation matrix; Θ is the visibility function.

The results of the light reflection modelling are shown at Fig. 3. The curve of reflected radiance was obtained for very rough Fresnel upper boundary with refractive index 1.52 (to repeat frosted glass). The curve is qualitatively of the same form as that gained in other research (for example, [9]).

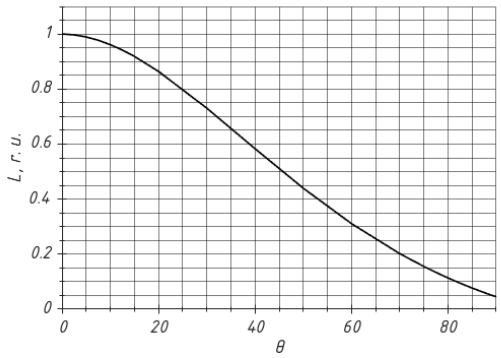

Fig. 3. Angular distribution of the reflected radiance.

4 Conclusion

In this research, we developed the model of a reflective surface based on its representation as a scattering layer bounded by the diffuse bottom and the correlated randomly rough Fresnel upper boundary. The influence of the slope correlation account was investigated as well. Basing on the results obtained one may conclude:

1. A comprehensive reflectance model must include both surface and volume reflection.
2. Account for the slope correlation of the randomly rough surface is necessary because of the possible emergence of effects observed in real life.
3. One of these effects is the statistical lens - the state of a waved water surface, when it acts upon passing rays as though a real optical lens.

Further development of the model may be carried out in the following directions:

- account for size and form of particles in the scattering medium;
- thorough validation of the model in experiments with various materials;
- creation of a list of parameters specific for materials of different kind.

The latter is especially important for practical application of the model.

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