

Comparison of Two Approaches to Calculate Orthoscopic Interference Pictures*

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Abstract. In this article a computer model of the polariscope is regarded as a 3D scene. In this case, the interference pictures are the result of rendering. The light rays pass through several well-specified polariscope blocks. When developing a suitable renderer, algorithms are selected and estimated for calculating the behavior of the beams based on their physical correctness, speed, etc. A plane parallel plate of an anisotropic crystal is the main block of the scene that affects the resulting image. This article discusses the calculation of the interaction of light with this plate only. Two approaches to calculate orthoscopic interference pictures of optically anisotropic transparent crystals are considered. One is described in many well-known books and bases on definite simplifications. The other is a direct physically based modeling of a light ray path through a plane parallel plate made of a uniaxial crystal taking into account all losses of intensity while passing boundaries between media. The purpose of this paper is to estimate a difference between values obtained via different approaches

Keywords: Polarized Light, Anisotropic Crystal, Orthoscopic Pictures.

1 Introduction

Mineralogists watch orthoscopic and conoscopic interference pictures (patterns, figures) while look at anisotropic transparent minerals through a polarizing microscope. A polariscope is a simplest optical device that allows to see such pictures. Mineralogists [1] note that the polariscope may be one of the most underestimated tools in gemology. Most gemologists use it to quickly determine if the stone at hand is isotropic or

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anisotropic or, at best, to determine the optic character of gemstones. With some small additions, one can determine both optic character and the optic sign of a gemstone. It is also the preferred tool – next to the microscope – for separating synthetic amethyst from its natural counterparts (although with recent synthetics that may prove difficult).

A virtual polariscope (VP) is our development of a computer model of a polariscope [2]. In order to compute orthoscopic and conosopic pictures the VP is represented as a 3D scene. A process of calculation of a picture is considered as rendering of this scene.

Specialists make decisions on minerals taking into account colors they see. There are standard color charts that support this process. Such charts are called *Michel-Levy Color Charts (or nomograms)* [3], prepared for each light source used in polarizing microscopes, and help to determine a mineral type, specimen thickness, birefringent power, etc. Note that cameras and printing equipment for printing charts also must comply with certain requirements for correct color reproduction.

In modern conditions, many charts are digitized and available on the Internet, see [3]. Their use requires calibrated monitors. There are also programs that allow you to calculate and build a chart directly on the computer for the selected mineral (orthoscopic wedge pattern) and the selected standard light source, for example, the program SICC [4]. The development of such programs is relevant, since new light sources are emerging, which are characterized by specific spectra. Obviously, the resulting interference colors will not match the Michel-Levy nomograms corresponding to other sources. Fig. 1 shows two examples of color charts calculated using the SICC program for different standard lighting sources CIE D65 and CIE D33.

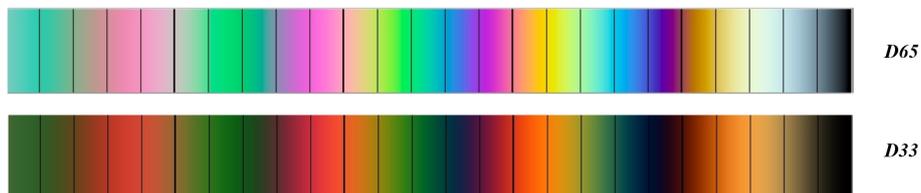


Fig. 1. Different views of color chart for different light sources.

The polariscope consists of a small number of blocks, each of which performs a well-specified task of processing polarized light rays. Every microscope includes a lens. We have already had to develop a computer model of a lens made of an isotropic transparent material that physically correctly processes rays of linear polarized light, because the developers did not pay attention to this issue or paid very little attention, see [5]. A mineral sample is also a VP unit that processes light rays. We implemented its separate computer model to examine it alone. We count that the existing applications of computer analogs of microscopes were quite suitable for energy calculations only, and the corresponding programs provided acceptable accuracy of the results. For a polariscope, it is necessary to consider polarized light, and for calculating interference patterns, the exact phase of the incoming light wave is important.

The idea of this article arose when reading the work [6], in which the author used a common method for calculating interference colors. This approach (let us call it *qualitative*) is well described in [7] and a number of other books aimed at mineralogists, for example, [8]. The same approach is used in the mentioned computer programs. This calculation does not take into account *the energy loss caused by light reflections* from the boundaries between media.

The module for calculating the physically correct passage of light in the studied mineral sample is one of the main ones in forming the final image. In this article, based on a direct physically correct simulation of the process of passing a light beam in a sample, the effect of reflections on the result is estimated based on the algorithm from [9].

Paper Structure The second section of the article describes the polariscope and sets forth the preparation of interference orthoscopic pictures based on the qualitative method. The third section is devoted to the calculation of pictures by direct modeling. The calculation results are also presented here, and the influence of the error due to ignoring reflections is estimated. The calculations were performed for two transparent non-absorbing optically uniaxial crystals of calcite and quartz. Finally, conclusions are made and further plans considered.

2 Interference Orthoscopic Pictures

Consider a monochrome light source with a wavelength λ . For a real source, analogous calculations should be performed for all wavelengths of the spectrum. The qualitative algorithm is described as in the books [7, 8]. Let us look at the process of getting a picture, based on Fig. 2.

2.1 Polariscope

In Fig. 2 left, the following components are marked:

1. Source of parallel unpolarized light (inside the case).
2. Polarizer P . The polarizer has a preferred direction d_P .
3. Plane-parallel plate PP from an optically uniaxial crystal with the following characteristics: h is a thickness; d_O is the direction vector of the optical axis; refraction indices for each wavelength λ : $n_o(\lambda)$ is for an ordinary ray, $n_e(\lambda)$ is for an extraordinary one.
4. Analyzer A (second polarizer). It has a privileged direction d_A .
5. Screen (camera, eye) for fixing the image. It should be noted that converting the spectrum of light that reaches the screen to color is not the task of this article.

The case of crossed polarizer and analyzer is considered, i.e. the vectors d_P and d_A are perpendicular. The optical axis of the studied crystal d_O is directed along the bisector of the angle between the vectors d_P and d_A . The light rays fall perpendicular to the sample. With these parameters, ordinary and extraordinary rays move in the same direction. Similarly the book [7] only transparent non-absorbing crystals are considered.

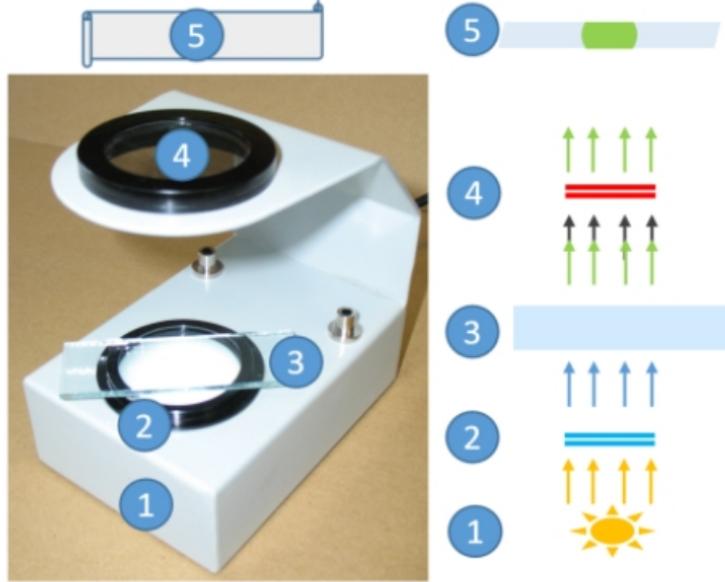


Fig. 2. Left: a photo of polariscope. Right: a model of polariscope.

2.2 Qualitative Approach. Transformations of Light Rays

Recall that a ray of monochrome light with a wavelength λ is considered, therefore, we will assume that the corresponding quantities depend on the parameter λ , which is omitted. Note that no reflected rays are considered. The following stages of light transformations are marked in the Fig. 2 right.

- Rays(1 \rightarrow 2): arrows indicating parallel beams of unpolarized monochrome light before the Polarizer.
- Rays(2 \rightarrow 3): arrows indicating parallel rays of linear polarized light from the Polarizer to the plate. Let R_i denote the ray incident on the plate. Oscillations of the electric vector \mathbf{E} are carried out in the direction d_P , and its amplitude is equal to E .
- Rays(3 \rightarrow 3): the rays inside the plate. At the entrance to the plate, two families of linearly polarized rays are formed, polarized in mutually perpendicular planes and corresponding to two refractive indices: the ordinary ray R_{ot} and the extraordinary ray R_{et} with equal amplitudes

$$\frac{\sqrt{2}}{2}E. \quad (1)$$

- Rays(3 \rightarrow 4). Inside the plate, the R_{ot} and R_{et} rays pass through the optical paths $n_o h$ and $n_e h$, respectively. At the entrance to the plate, both types of rays had the same phase, and then the phase difference appears at the output and is equal

$$\delta = \frac{2\pi}{\lambda}h(n_o - n_e). \quad (2)$$

- Amplitudes and polarizations of the rays R_{oti} and R_{eti} are preserved.
- The Analyzer converts the oscillation directions of the rays R_{oti} and R_{eti} to the direction d_A and outputs the rays R_{otia} and R_{etia} , respectively, see Fig. 3. Intensity losses in the Analyzer are not taken into account.
 - Rays(4 → 5) are obtained as a result of interference of R_{otia} and R_{etia} rays, since they go in the same direction and their polarization planes coincide and we apply classical formula [7]

$$I_{res} = I(R_{otia}) + I(R_{etia}) + 2\sqrt{I(R_{otia})I(R_{etia})} \cos \delta. \quad (3)$$

The resulting intensity is calculated using the formula [7]

$$I_{res} = E^2 \sin^2\left(\frac{\delta}{2}\right). \quad (4)$$

Let $E = 1$, then $I_{res} = \sin^2\left(\frac{\delta}{2}\right)$. In such a way, we simplify farther calculations.

3 Direct Modeling of Light Propagation

We proceed to direct modeling of the passage of rays of linearly polarized light. Also consider a monochrome light source with a wavelength of λ . Fig. 3 shows the rays that occur when light falls on the plate. For numerical experiments, let us select two minerals: calcite with huge birefringence equal to 0.187635 and quartz with a weak difference between indices of refraction of ordinary and extraordinary rays, not more than 0.009657. Indices of refraction are computed using Sellmeier approximation [10].

Calculations are as follows.

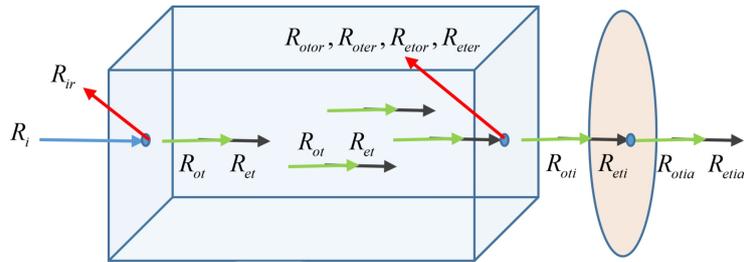


Fig. 3. Interaction of light ray with a plate and Analyzer. Red arrows denote the fact of reflection only, the directions are wrong.

Front Plane Incident ray R_i is isotropic linear polarized. An amplitude of its electrical component is equal to 1. Intensity $I(R_i) = 1$ also. The only reflected ray R_{ir} is isotropic linear polarized too. An ordinary ray R_{ot} and an extraordinary ray R_{et} are transmitted into a plate.

Inside Plate Both rays coincide. They are linear polarized in perpendicular planes.

Back Plane R_{ot} falls on a back plane and generates up to three linear polarized rays: reflected ordinary ray R_{otor} , reflected extraordinary ray R_{oter} , and transmitted isotropic linear polarized ray R_{oti} that preserves the polarization of the ray R_{ot} . Analogously R_{et} generates up to three linear polarized rays: reflected ordinary ray R_{etor} , reflected extraordinary ray R_{eter} , and transmitted isotropic linear polarized ray R_{eti} that preserves the polarization of the ray R_{et} . The rays R_{oti} and R_{eti} coincide but polarized in different planes. Note that intensities $I(R_{oter})$ and $I(R_{etor})$ are equal to zero because of selected directions d_P , d_A , and d_O . Table 1 presents the results of calculating the intensities by both methods: column 1 contains the name of the quantity; columns 2 and 3 contain values for calcite and quartz calculated by the direct modeling method; columns 4 and 5 are values calculated according to the qualitative approach taken from the books [7, 8]. The sign ‘-’ means that the corresponding value is not calculated by corresponding method. I_{out} is the total intensity of both rays after exiting the plate before entering the Analyzer. I_{res} is the intensity taking into account interference after exiting the Analyzer.

Table 1. Calculated values for $\lambda=380$ nm, plate thickness is 0,01 mm.

Intensity	Calcite	Quartz	Books calcite	Books quartz
1	2	3	4	5
$I(R_i)$	1	1	1	1
$I(R_{ir})$	0.052688	0.048560	-	-
$I(R_{ot})$	0.467292	0.476043	-	-
$I(R_{et})$	0.480020	0.475397	-	-
$I(R_{otor})$	0.030568	0.022809	-	-
$I(R_{eter})$	0.019181	0.023393	-	-
$I(R_{oti})$	0.436724	0.453235	-	-
$I(R_{eti})$	0.460839	0.452004	-	-
I_{out}	0.897563	0.905239	-	-
$I(R_{otia})$	0.218362	0.226617	-	-
$I(R_{etia})$	0.230419	0.226002	-	-
$n_o - n_e$	0.187635	0.009657	0.187635	0.009657
δ	31.024889	-1.596758	31.024889	-1.596758
I_{res}	0.034026	0.464369	0.037743	0.512980

Thus at a wavelength of 380 nm, for calcite, total losses are 0.003717, and for quartz, 0.094762. Recall that after the Polarizer, the intensity was equal to 1. In Fig. 4 are graphs showing the dependence of the intensities on the wavelength in the visible range of 380–780 nm. Obviously, the calculated graphs repeat those obtained using a qualitative assessment. It is also seen that, due to energy losses during reflections, the calculated curves are located lower. Comparing the values $I(R_{otia})$ and $I(R_{etia})$ in Table 1, we can conclude that the discrepancies are greater, the greater the birefringence $n_o - n_e$.

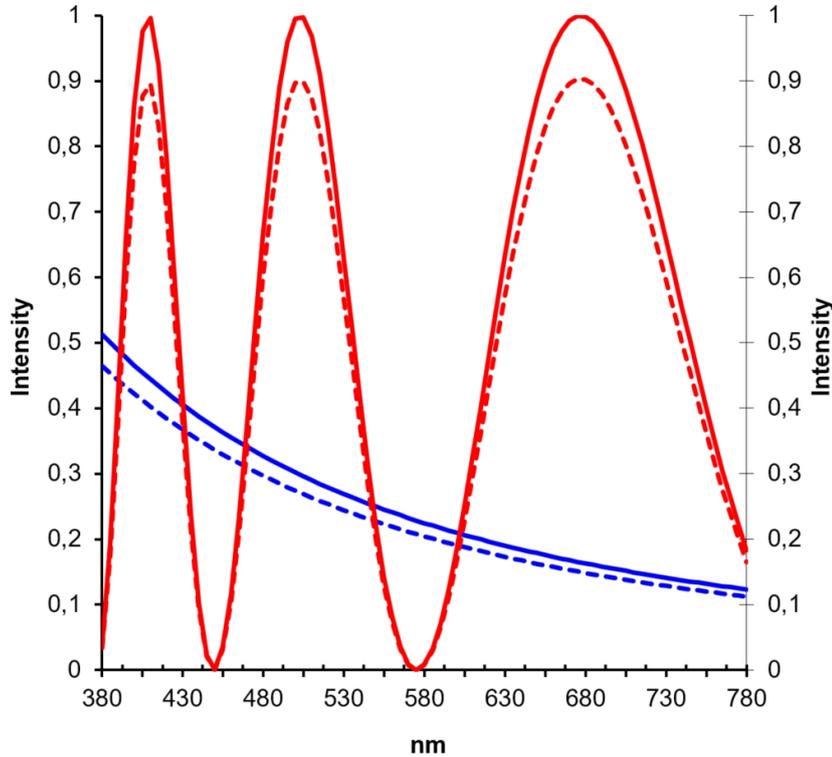


Fig. 4. Plots of $I_{res}(\lambda)$ for calcite (red), quartz (blue) calculated via qualitative method (solid) or direct modeling (dashed). Plate thickness is 0,01 mm.

4 Conclusions

In general, this work is part of a project to develop a computer model of a polariscope: the functions of a modeling block for the interaction of polarized light with transparent optically anisotropic crystals were investigated. At the same time, a comparison was made with an approximate qualitative calculation method widely used in the literature, and estimates of the errors that were obtained depending on the birefringence of the mineral were obtained. A polariscope is considered a 3D scene, and the visualization process is a scene rendering. It should be noted that the anisotropic object of the scene is encapsulated: linearly polarized rays fall on it, and it gives out linearly polarized rays. In general, rendering of scenes is carried out not by an ordinary isotropic ray, but by a linearly polarized ray. Such a renderer is able to calculate images of interference patterns [11], and its development serves as a guiding thread for our researches.

Note that in this paper we do not fulfill a tone reproduction, i.e. converting the light representation from the spectrum to RGB, since this is a separate task that we plan to investigate in the next stage of the development.

The practical aspect of the computer model of the polariscope in a modern environment that has arisen in connection with the coronavirus pandemic can be noted. The mode of distance work and study often becomes the main one. In such conditions, various computer models of devices can prepare people for work with them.

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