Development of a Markov Model of Changes in Patients' Health Conditions in Medical Projects

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Abstract

In this article, there is discussed the components of the forecast model by changes in the condition of patients in the management of medical projects. We proposed the use of Markov chains to obtain quantitative results and take into account random processes in the management of medical projects. In recent years, the problem of health care of Ukrainian citizens has become a threat to national security. The main reason for the current situation lies in the imperfect health care management system, insufficient funding and irrational distribution of funds allocated by the state to the industry. Resolving the contradictions between the needs of the population of Ukraine in timely and quality medical care and the tasks of medical institutions in providing these services is possible only if the implementation in the medical field of project management methodology based on the use of mathematical modeling methods. During the implementation of medical services projects there is a need to assess the prospects for the use of new forms of services in different environments of the project. This estimate can be performed using mathematical models. At present, such models are based on the approximation of real data, which reflect the delay in the effectiveness of medical institutions with patients. In this case, the projects have already begun - they are implemented, and experimental models reflect a certain result. However, at the stage of preliminary project preparation it is necessary to determine the expected results in the "prediction" mode. Such properties are inherent in Markov models, which are built for specific states of the system, and can be used for cases of changes in the internal characteristics of the system. In the course of the research we built a model of realization of Markov processes using the construction of the process graph of the WHO model and the C6 model of medical project management.

Keywords 1

Medical services, Markov Model, medical project

1. Introduction

Current trends in project management are aimed at transforming projects into dynamic systems that are not only subject to market requirements, but also through the use of modern models are continuously improved on the basis of proactive approaches to change management. Existing health care project management systems do not always provide solutions to improve quality and accessibility, due to the lack of effective models, methods, tools for evaluating project results for the implementation of management mechanisms, including through feedback. Therefore, the development of models that reflect the state of the health care system and the formation on their basis of mechanisms for proactive management of projects in the field of health care will ensure the quality and accessibility of health care services.

The introduction of project management in the field of medical services in medical institutions operating in a turbulent competitive environment, necessitates the management of quality and cost of treatment of patients in medical projects with continuous improvement of the content and system of health

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care [19]. Under conditions of fierce competition for project-based medical institutions and institutions, together with the need to improve the mechanisms of product (service), process, development and business values, the issue of improving project management models and methods that are drivers of innovative development to expand the range of medical services.

The design of medical services is the most important condition for the successful implementation of medical activities, which is usually considered the art of the doctor. But the management of medical services also contains an organizational and technical component - planning, implementation of treatment projects, control, analysis and correction of results. When managing health care projects, one of the main tasks assesses the effectiveness of projects. In a general sense, measuring the effectiveness of treatment projects are expressed in the study of the health of the patient community. Due to the lack of models and methods for early evaluation of the effectiveness of treatment projects, as a rule, they are planned based on the results of best practice. Usually the evaluation of efficiency is carried out due to intuitive assumptions or methods of field observations. But this approach allows you to evaluate existing treatment projects, which by definition reduces the value of the service. Therefore, for proactive management of projects for the provision of medical services, the task of early assessment of the expected result is already relevant when planning.

In a general sense, measuring the effectiveness of treatment projects is expressed in the study of the health of the patient community.

2. Analysis of recent research and publications

Analysis of world experience in the organization of the health care system has shown the feasibility of using a project approach that allows you to most effectively solve the problem of achieving this goal in a limited time, financial, material, human, etc. types of resources. Therefore, it is time to create conditions for the transformation of medical projects in the direction of proactive management through the use of models that reflect the essential features of the studied system - the patient community.

Application of Markov processes to random processes in various branches of applied science swallowed in [1-7, 17-20]. [9] proposes new models of innovation management in projects based on the use of Markov processes. This approach can be used for further development of medical projects. [10] describes Markov models with changes that can be applied to medical projects. However, there are no methods of interaction between groups of patients.

In [11] a detailed description of the technological approach to the management of any project. At the same time [12] describes integrated methodologies of organizational management and management of products and services of medical projects. However, methods for evaluating the effectiveness of communicative interaction are insufficiently described. In [13, 14, 16] approaches to the management of complex IT projects based on predictive management using artificial intelligence methods, in particular, using artificial neural networks. However, studies based on the interaction of random variables have not been conducted.

The goal of the article is to substantiate and develop a model for predicting the effectiveness of medical projects based on random processes using data analysis of grouping processes in a medical project and the application of modeling of Markov processes of changes in patients.

3. Model development and use of modelling method

To build a Markov model of changing the health status of the community of consumers of medical services, it is necessary to decompose the system into specific conditions and build a scheme of transitions between these conditions. An important aspect of developing a Markov model is the method of determining the calculation of transition probabilities.

One of the most well-known models for qualitative reflection of system states is the generally accepted scheme of the World Health Organization (WHO), the concept of which is based on the obvious fact that there are certain transitions from one state to another. The scheme of reactions of the population according to WHO materials, is given in fig. 1, reflects the component of public health as a result of the impact on people of environmental factors. The area of each of the zones is proportional to the share of the population with the corresponding signs of influence.

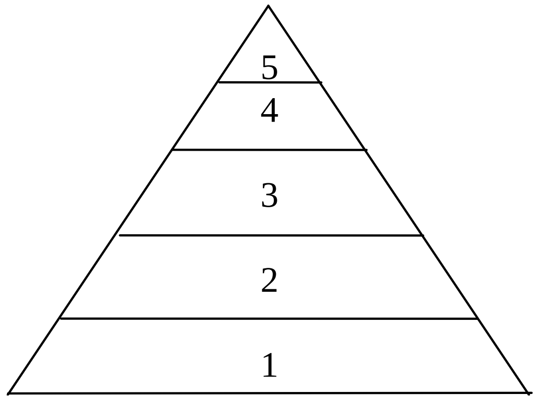


Figure 1: The reaction of the population to the effects of harmful environmental factors:
1 - accumulation of chemicals in human organs and tissues; 2 - physiological and other shifts of unknown origin; 3 - signs of the disease; 4 - morbidity; 5 - mortality.

The WHO model assumes that in order to go from the 1st to the 2nd state, one must be in the 1st state. At the same time, there are always more patients of a lower level than those who have moved to a higher level. All these transitions are calculated from the total number of patients in the market segment. This model [5] allows you to estimate the approximate volume of health services, and on the other hand allows you to set the right goals for the management of health care projects. The transition from each condition will be due to different features in terms of medical institutions. This approach is not entirely correct, because the transition from the 1st state is possible in a turbulent environment to any other state. Moreover, as a result of medical measures it is possible to move to a state with a lower serial number.

The WHO model allows only a qualitative assessment of the effectiveness of various health care projects and to develop the most effective strategy for promoting a particular medical service on the market. Many processes in medical activity develop as random processes. The WHO model does not allow to obtain quantitative results of project effectiveness. The obvious contradiction is the need to develop a strategy for providing quality medical services not on the basis of modeling these processes, but based on the method of trial and error in management decisions [8].

To model the change in the state of health of the population, it is proposed to allocate 6 states, in one of which with a certain probability can be each agent of the system (Fig. 2). Denote by S_i $\{i=1, 2,\ldots, 6\}$ possible conditions of some community of patients of consumers of medical services caused by carrying out projects: S_1 - practically healthy; S_2 - able to work; S_3 - temporarily incapacitated; S_4 - chronic disease; S_5 - critical condition; S_6- exit (death, emigration, etc.).

These conditions of the S_1-S_6 system form a complete list of health conditions of a certain group of the population. In the general case, the system of states S_1-S_6 can be represented by an oriented graph (Fig. 2).

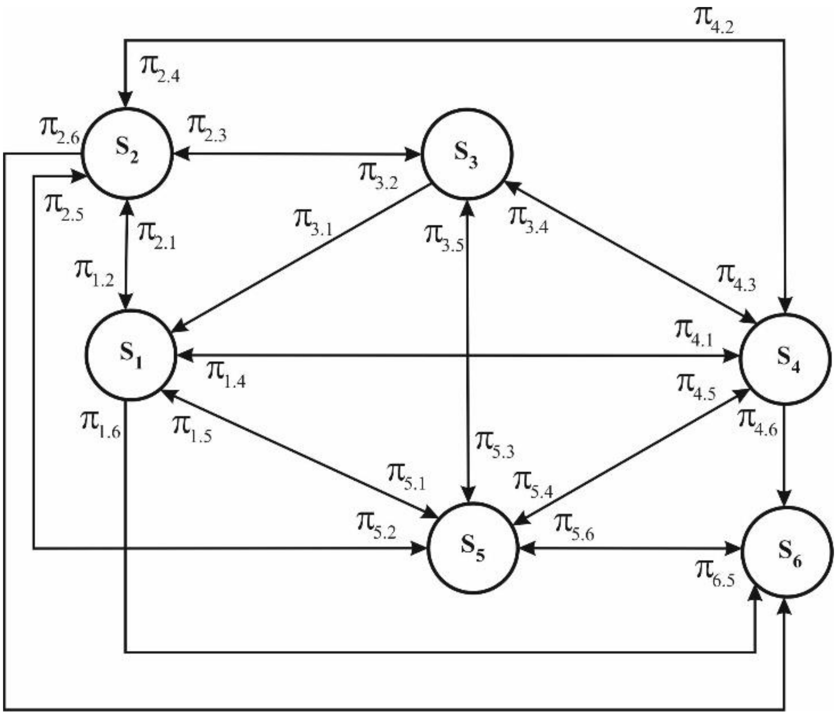


Figure 2: The oriented graph of the WHO model

Existing transitions between different states are determined by expert assessments. A matrix containing all the transition probabilities of the Markov chain shown in Fig. 2, has the form:

$$\left\|\pi_{ij}\right\| = \begin{Vmatrix} \pi_{1.1} & \pi_{1.2} & \pi_{1.3} & \pi_{1.4} & \pi_{1.5} & \pi_{1.6} \\ \pi_{2.1} & \pi_{2.2} & \pi_{2.3} & \pi_{2.4} & \pi_{2.5} & \pi_{2.6} \\ \pi_{3.1} & \pi_{3.2} & \pi_{3.3} & \pi_{3.4} & \pi_{3.5} & \pi_{3.6} \\ \pi_{4.1} & \pi_{4.2} & \pi_{4.3} & \pi_{4.4} & \pi_{4.5} & \pi_{4.6} \\ \pi_{5.1} & \pi_{5.2} & \pi_{5.3} & \pi_{5.4} & \pi_{5.5} & \pi_{5.6} \\ \pi_{6.1} & \pi_{6.2} & \pi_{6.3} & \pi_{6.4} & \pi_{6.5} & \pi_{6.6} \end{Vmatrix}. \quad (1)$$

The Markov Model nature of medical service projects is confirmed by the fact that in both medical projects and Markov chains there are transitions between states of the system step by step, there are transitional probabilities between individual states, the sum of transitional probabilities of a state is equal to one, the sum of probabilities of all states place similarity of topological structure of transitions.

Compare the properties of treatment service projects and the obtained model to prove that the system can be described using Markov chains [13].

The properties of medical service projects that correspond to the Markov chains include:

- operational actions in projects: a) random process; b) for the community of consumers of services there is a certain set of conditions; c) it is not possible to take into account the prehistory of the transition of the community of service consumers to a certain state; d) therapeutic measures carried out at time tk, transfer the system to a new state;
- therapeutic measures correspond to the steps of the process;
- the result of treatment projects forms the distribution of probabilities of the states of the community of service consumers, and it is possible to indicate the possible transitions of the system from each state to another in one step;

- the probability of transitions to other states depends on the properties of the system in which random processes operate [15];
- since the states of the community of consumers of medical services are a complete group, the sum of these probabilities is equal to one;
- transitions from any state of the system to other states constitute a complete group of events, one of which must take place ;.
- the states of the system are displayed in a graph, indicating possible transitions from one state to another in one step.

The analysis of the properties of the object and the model allow us to draw a conclusion about the validity of the use of Markov chains for modeling projects for the provision of medical services [16].

Various approaches can be used to determine the values π_{ij}. An effective way to find the transition probabilities is the method of directly measuring the time spent on certain operations of a certain state based on the “photograph” of the operations. It is assumed that the transition probabilities are calculated as the ratio of the time allotted for each transition to the total operation time. This method can be used in the construction of Markov chains for production operations and processes, which, as a rule, are normalized, and the time of their execution is regulated by the relevant regulatory documents (instructions, rules, regulations, etc). Transient probabilities can be determined using statistical data, for example, the conditions of patients receiving medical services are clearly recorded in medical records. In fact, such a method is identical to the direct measurement of the number of patients in certain groups (conditions).

Transition probabilities, determined on the basis of expert assessments of specialists of the INTO-SANA medical institution, are given below:

$$\left\|\pi_{ij}\right\| = \begin{Vmatrix} 0{,}75 & 0{,}15 & 0 & 0{,}05 & 0{,}05 & 0 \\ 0{,}2 & 0{,}599 & 0{,}12 & 0{,}06 & 0{,}02 & 0{,}001 \\ 0{,}03 & 0{,}2 & 0{,}519 & 0{,}2 & 0{,}05 & 0{,}001 \\ 0{,}07 & 0{,}15 & 0{,}3 & 0{,}278 & 0{,}2 & 0{,}002 \\ 0{,}09 & 0 & 0{,}2 & 0{,}4 & 0{,}308 & 0{,}002 \\ 0 & 0 & 0 & 0 & 0{,}01 & 0{,}99 \end{Vmatrix}. \quad (2)$$

A property of the developed model is the dependence of the random process of change of S_i states in time $t[0, T]$. The value of s is a possible state of a random process $S_i(t)$, if in the interval $[0, T]$ there is such a time t that the probability $P\{s\text{-}z<S(t)<s+z\} \geq 0$ for any $z>0$. Time t runs a discrete series of values $t_0, t_1, t_2, \ldots, t_N$: $\{t_n, n=0, \ldots, N\}$and the random variable $S_i(t_n)=S_i|_n$ can take a discrete set of values $s_1, s_2, \ldots, s_k$ або $\{s_k, k= 1, \ldots, K\}$.

It is known that when the initial state of the system is determined and for the matrix of transition probabilities it is possible to find the probability of each of the states $p_1(k), p_2(k), \ldots p_6(k)$ after any k step.

$$p_i(k) = \sum_{j=1}^{m} [p_j(k-1) \cdot \pi_{ji}]\Big|_{m=6}; \quad i = 1, 2, \cdots, n \quad (3)$$

The obtained probabilities of conditions as a result of the performed medical measures allow to predict and evaluate the efficiency of medical institutions. The results of changes in the probabilities of the state of the community of consumers of medical services in steps are shown in Fig. 3. These results reflect the current level of medical services, which is now characterized (in the quasi-stationary position in step k = 40) by the following distribution of probabilities of conditions: : $p1$ (40) = 0,30; $p2$ (40) = 0,26; $p3$ (40) = 0,19; $p4$ (40) = 0,14; $p5$ (40) = 0,08; $p6$ (40) = 0,03.

With the help of the developed model it is possible to estimate how the condition of consumers of medical services on various influences and projects, including in the conditions of the organization of insurance medicine will change.

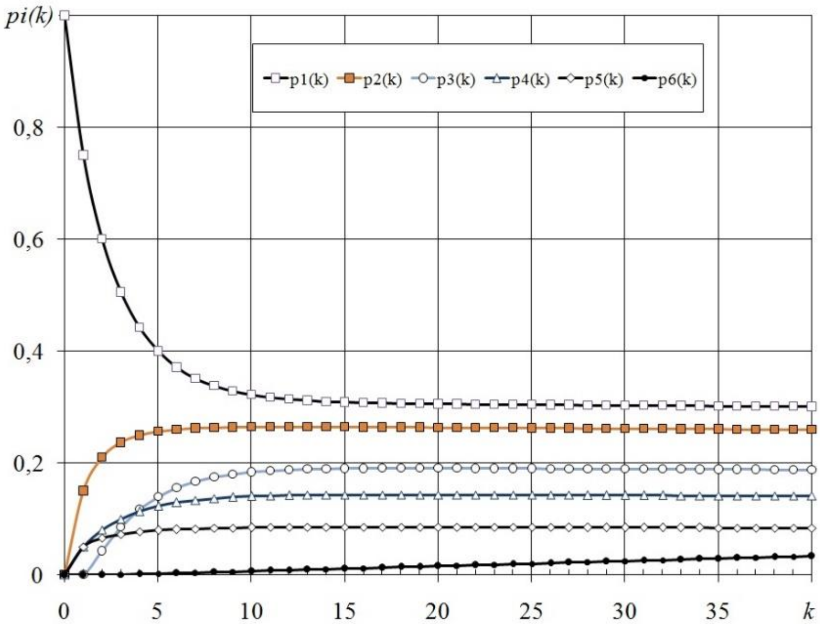


Figure 3: Changing the probabilities of the state of the health care system: *p1 (k)* - almost healthy; *p2 (k)* - able-bodied; *p3 (k)* - temporarily incapacitated; *p4 (k)* - chronic disease; *p5 (k)* - critical state; *p6 (k)* - death.

In practice, when assessing the effectiveness of medical services, there are six health conditions, one of which is likely to be every patient. Therefore, the proposed Markov model (6S) includes the following generally accepted states: S_1 - almost healthy; S_2 - able to work; S_3 - temporarily incapacitated; S_4 - chronic disease; S_5 - critical condition; S_6 - output (lethal outcome). There is a certain system of connections between these states - transitions (Fig. 4).

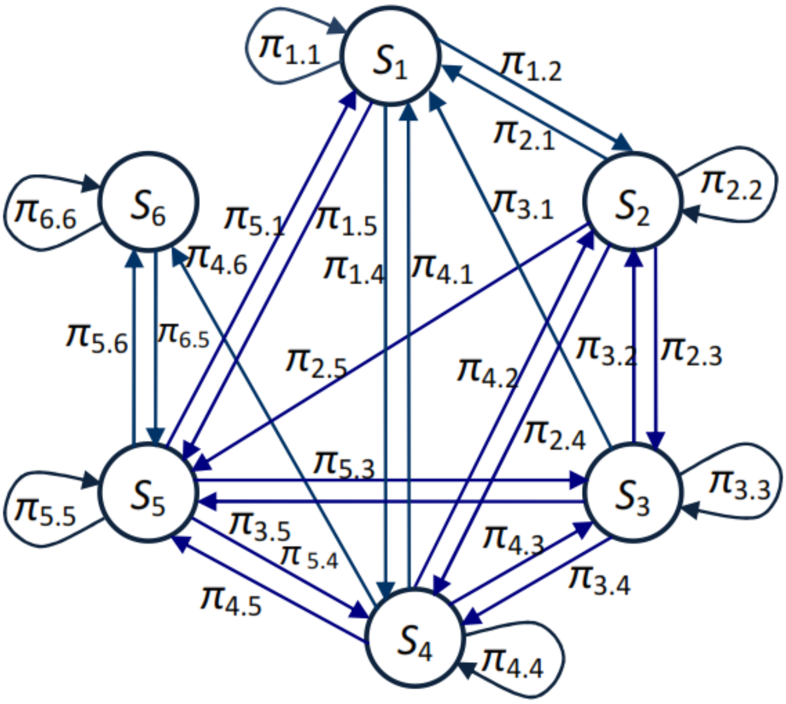


Figure 4: The oriented graph of states of 6S's model

For example, if the patient is in state S_2, then in the case of the disease it is possible to transition not only to state S_3, but also to other states such as S_4 or S_5. Such a division of systems into certain states with given transitions between them is inherent in Markov chains.

A matrix containing all the transition probabilities π_{ij} of the Markov chain shown in Fig. 4, has the form:

$$\|\pi_{ij}\| = \begin{Vmatrix} \pi_{1.1} & \pi_{1.2} & \pi_{1.3} & \pi_{1.4} & \pi_{1.5} & \pi_{1.6} \\ \pi_{2.1} & \pi_{2.2} & \pi_{2.3} & \pi_{2.4} & \pi_{2.5} & \pi_{2.6} \\ \pi_{3.1} & \pi_{3.2} & \pi_{3.3} & \pi_{3.4} & \pi_{3.5} & \pi_{3.6} \\ \pi_{4.1} & \pi_{4.2} & \pi_{4.3} & \pi_{4.4} & \pi_{4.5} & \pi_{4.6} \\ \pi_{5.1} & \pi_{5.2} & \pi_{5.3} & \pi_{5.4} & \pi_{5.5} & \pi_{5.6} \\ \pi_{6.1} & \pi_{6.2} & \pi_{6.3} & \pi_{6.4} & \pi_{6.5} & \pi_{6.6} \end{Vmatrix}. \quad (4)$$

The analysis of the properties of the object and the model allow us to draw a conclusion about the validity of the use of Markov chains for modeling projects for the provision of medical services (Table 1).

Comparison of the properties of the original - the community of patients and the Markov model allows us to conclude that the identity of the characteristic properties that are important for solving problems of modeling the state of the patient community in the provision of medical services. Model identification indicates the possibility of applying Markov chain theory to model medical projects.

4. Experimentation

Table 1

Identification of properties of Markov chains of medical projects for the provision of medical services

Properties	Markov chain	Medical projects
1. Transitions from the real state to the future	A random process has a feature: for each moment of time t0 the probability of any state of the system in the future (at $t > t_0$) depends only on its current state and does not depend on when and how the system came to this state.	Service delivery process: a) random process; b) the community of patients can be divided into certain conditions; c) it is impossible to take into account the prehistory of transitions between states; d) services provided at time t0, transfer the system to a new state.
2. Correspondence of steps	In a Markov chain with discrete time the transition from some state to other states is carried out by process steps.	The medical services provided correspond to the process steps that bring the system to a new state.
3. The presence of a transition probability	Transition probabilities depend only on the state from which and to which the transition is made	Medical services change the probabilities of patient communities, and the probabilities of transition to other conditions depend on the quality of service delivery.
4. Condition for the sum of probabilities of states of the system	The sum of probabilities of all states of the Markov chain at each step is equal to one.	The conditions of the patient community are a complete group, so the sum of the

		probabilities of these conditions is one.
5. Condition for the sum of transient probabilities from any state	The sum of transient probabilities from some state of the Markov chain to other states is equal to one.	Transitions from any state of the system to other states constitute a complete group of events.
6. Topological similarity	The states of the Markov chain are represented by a graph, which indicates the possible transitions between states in one step.	The states of the system are represented by a graph, where the edges are possible transitions from one state to another in one step.

The transition probabilities π_{ij}, determined on the basis of expert assessments of specialists are given below:

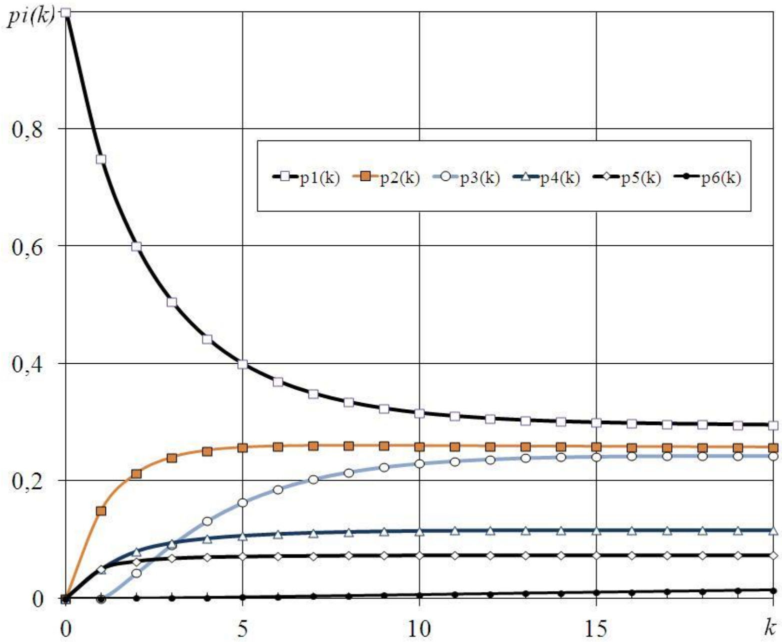
$$\left\|\pi_{ij}\right\| = \left\|\begin{matrix} 0,75 & 0,15 & 0 & 0,05 & 0,05 & 0 \\ 0,2 & 0,599 & 0,12 & 0,06 & 0,02 & 0,001 \\ 0,03 & 0,2 & 0,519 & 0,2 & 0,05 & 0,001 \\ 0,07 & 0,15 & 0,3 & 0,278 & 0,2 & 0,002 \\ 0,09 & 0 & 0,2 & 0,4 & 0,308 & 0,002 \\ 0 & 0 & 0 & 0 & 0,01 & 0,99 \end{matrix}\right\|. \quad (5)$$

The obtained probabilities of conditions as a result of the performed medical measures allow to predict and evaluate the efficiency of medical institutions. The matrix of transition probabilities π_{ij} reflects the existing level of medical services, which is now characterized for $k = 40$ by the following distribution of probabilities of states of consumers of medical services: $p1(40) = 0,30$; $p2(40) = 0,26$; $p3(40) = 0,19$; $p4(40) = 0,14$; $p5(40) = 0,08$; $p6(40) = 0,03$.

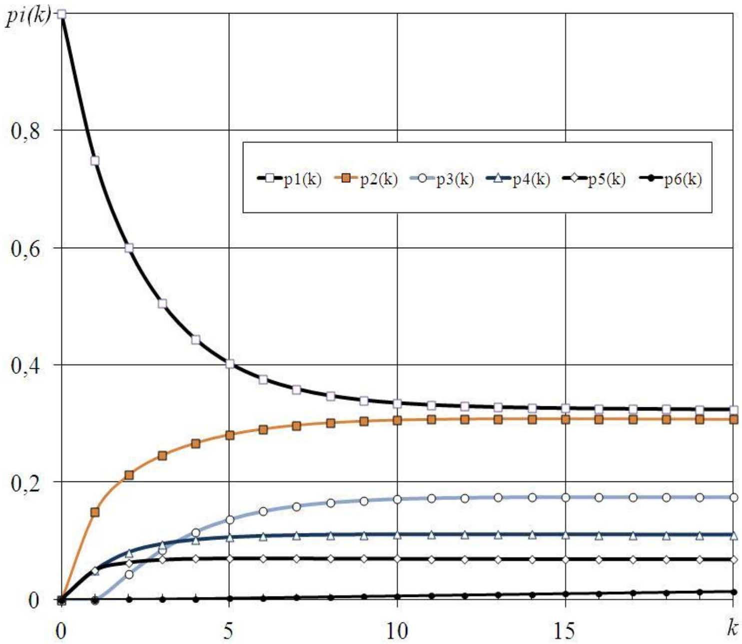
The quality of medical care affects the conditional probabilities of transitions from S_3 to states S_2 and S_4 (Fig.4). Possible intervals of change $\pi_{32} = 0 ... 0,6$. This means that in the case of high quality treatment for temporary disability up to 60% of patients can go into S_2. In the current situation, this value is 20%. At the same time, with high quality treatment, the value of π43 should decrease. Therefore, for the study, we take $\pi_{43} = 0.1$ and determine the probability distribution of states when π_{32} changes (Fig. 5). The introduction of quality medical care will lead to a change in some transitional probabilities, which will significantly change the general picture of the state of consumers of medical services.

In fig. 5 presents the results on the change of probabilities of the states of the system of medical services at different values of the transition probability π_{32}, which characterizes the level of quality of medical services. An increase in π_{32} from 0.15 to 0.6 leads to a significant decrease in the probability of p3 (40), which characterizes the temporary incapacity of health care consumers. This increases the probabilities of *p1 (40)* and *p2 (40)*, which correspond to the share of practically healthy and able-bodied consumers of medical services. It is these consumers of services who are solvent and can form the financial basis for the provision of medical services.

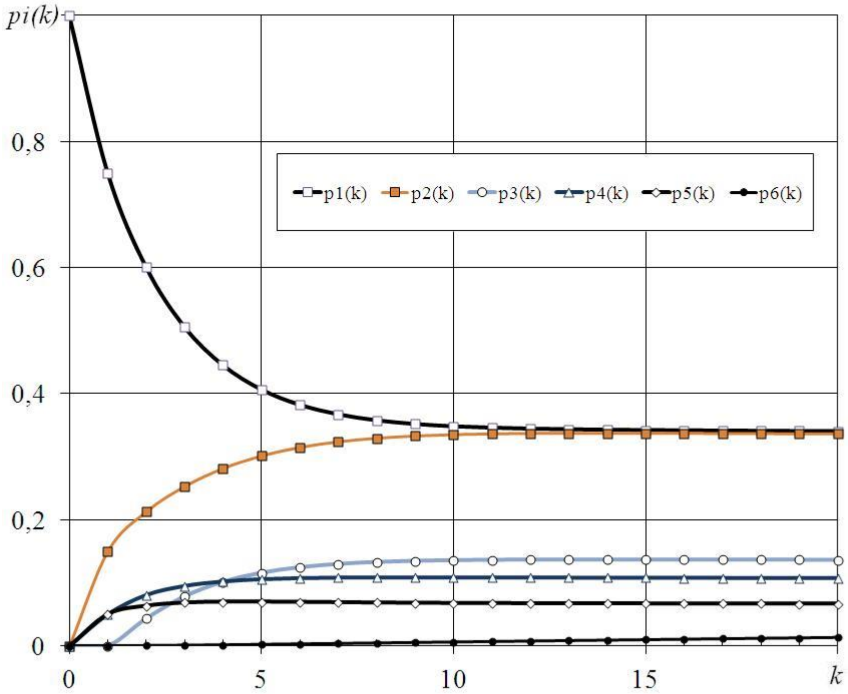
Forecasting the effectiveness of projects being developed is rationally performed using probabilistic models that reflect the specifics of random processes. The random nature of the demand for medical services is obvious, which allows us to present the activities of medical institutions using the Markov model.



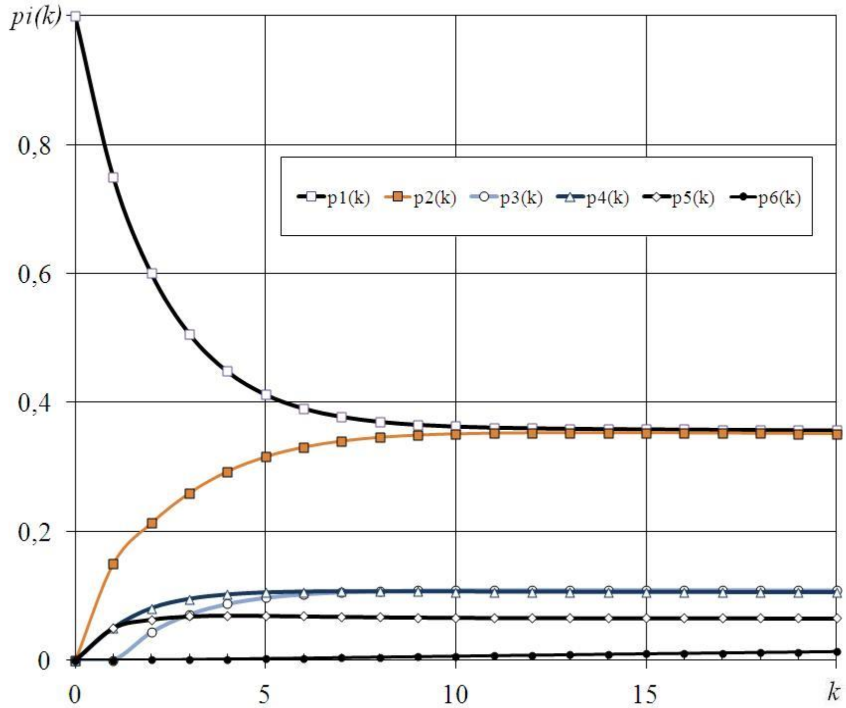
a) π_{32}=0,15



b) π_{32}=0,60



c) π_{32}=0,45



d) π_{32}=0,60

Figure 5: Changing the probabilities of the state of the health care system under conditions of improving the quality of treatment: *p1 (k)* practically healthy; *p2 (k)* able to work; *p3 (k)* temporarily incapacitated; *p4 (k)* chronic disease; *p5 (k)* critical state; *p6 (k)* death.

Conclusion

The design of medical services is the most important condition for the successful implementation of medical activities, which is usually considered the art of the doctor. But the management of medical services also contains an organizational and technical component - planning, implementation of treatment projects, control, analysis and correction of results. When managing health care projects, one of the main tasks is to assess the effectiveness of projects.

The analysis of the considered approaches to definition of efficiency of medical projects with reflection of specificity of casual processes allows to define use of modern mechanisms of modeling of Markov's model of changes. The current WHO model allows to evaluate only the qualitative results of a medical project to promote a specific medical service, but does not reflect quantitative results and random processes. To solve this problem, the authors used modeling of Markov processes.

This approach allowed to build a basis for predicting the effectiveness of medical projects, in particular in insurance activities.

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