Development and Further Refinement of a Semi-empirical Wheel Traction Model

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Abstract. In this paper the theoretical basis, evolution and results of the field tests regarding the modelling of the agricultural tire-soil traction model are presented. The model is a reasonable compromise between the simpler empirical models, for which the range of applicability is limited to the cases having similar conditions to the ones from which the models were derived, and the analytical models, which require in-situ evaluation of a large number of soil properties. The model is based on the Mohr-Coulomb failure criteria, assuming that the maximum traction force is limited only by the soil shear strength. A computer program was developed in order to solve the system of equations introduced by the model, with the traction force and traction efficiency being evaluated. In the initial model the tire-soil contact patch was assumed to be an ellipse and no modifications of the tire cross-section were taken into account. Further developments took into account a super ellipse shape of the tire-ground contact surface, effect of tire slip over the contact patch area and deformation of the tire cross-section.

Keywords: traction model; Mohr-Coulomb failure criteria; goodness-of-fit; traction force; traction efficiency.

1 Introduction

Tire-soil interaction models are used in order to predict the wheel traction force and traction efficiency. They take into account the shape and area of contact patch between tire and soil, which is also used for the calculation of the surface pressure and for modelling stress propagation in soil in order to predict the compaction risk (Diserens et al., 2011).

Accurate prediction of traction performance of a tractor wheel depends largely on the model of the tire-terrain interaction. Hambleton & Drescher (2008) classified

wheel-soil interaction models into empirical, analytical and numerical models.

Empirical methods are mainly based on soil properties (cone index, plate sinkage, shear strength) using similitude and dimensional analysis.

The semi-empirical (analytical) models represent a physical-based approach, which considers the mechanics of the wheel-soil interaction and are suitable for practical applications (Battiato&Diserens, 2017). In the semi-empirical models, the shear deformation of soil is considered; the models are based on soil parameters obtained by the means of a bevameter technique (penetration and shear tests), assuming that the vertical deformation of soil is similar to the deformation under a sinking plate, while the shear deformation of soil under a traction device is similar to the shear action of a torsion device (Tiwari et al., 2010). The parameters involved in the equations are determined experimentally.

This paper presents the evolution of a semi-empirical tire-ground interaction model; while the basic elements of the model remained the same, different assumptions regarding the shape of the tire-ground contact area and the tire deformation were used in time.

2 Tire-ground interaction model

2.1 Initial model

We have chosen to use a Bekker type model, assuming that the circumferential force limits the value of the wheel net traction force.

In order to evaluate the dimensions of the contact area, the model assumes that, under the vertical load (G, Fig. 1), the wheel sinks into the soil, reaching depth (z_c) and the load induces tire deflection (z_p) (Rosca et al., 2004). As a result, the radius of the contact patch becomes r_d ($r_d > r_0$), and the circular length of the contact patch is:

$$l_c = 2 \cdot \beta \cdot r_d = 2 \cdot \alpha \cdot r_0 \,. \tag{1}$$

Using the Bekker equation (Bekker, 1969) and assuming the tire is perfectly elastic, we get:

$$k \cdot \int_0^{2\beta} r_d^{n+1} \cdot [\cos(\beta - \varphi) - \cos\beta]^n \cdot d\varphi + \frac{4}{3} q_p \cdot \beta^3 \cdot r_d^2 = \frac{4}{3} \cdot q_p \cdot \alpha^3 \cdot r_0^2 \,, \tag{2}$$

$$z_c = r_0 - z_p - r_0 \cdot \cos\beta \,, \tag{3}$$

$$z_p = r_0 \cdot (1 - \cos\alpha) - r_d \cdot (1 - \cos\beta) \,, \tag{4}$$

where q_p is the tire volume stiffness, z_p is the tire deformation due to the vertical load G ($G = q_p \cdot \Delta V_p$) and z_c is the soil deformation.

The tire change in volume due to deflection ΔV_p was evaluated considering that the tire radius increases from r_0 to r_d as the tire flattens in the contact area, while the tire width was considered constant, as shown in Fig. 2 (Ghiulai&Vasiliu, 1975).

Fig. 1. Schematics of the wheel-soil interaction model

r_0 – radius of the undeflected tire; r_d – radius of the contact patch under vertical load.

Fig. 2. Initial tire deformation model ΔV_p - tire change in volume due to deflection; b - tire width;

The shape of the tire-ground contact patch was assumed elliptical; the minor axis, l_w was calculated using z_p (Upadhyaya & Wulfsohn, 1990), while the major axis, l_c, results when solving the system of equations (1, 2, 3, 4).

A computer program, based on an iterative process, was used in order to solve the system of equations (1, 2, 3, 4) and find l_c, z_p, z_c and r_d (Rosca et al., 2014).

The maximum traction force was assumed to be limited only by the maximum shear strength of the soil, given by the Mohr-Coulomb criterion, based on soil cohesion and the internal friction angle of the soil.

According to Wulfsohn & Upadhyaya (1992) and Lach (1996), the shear stress developed at the interface between the vehicle tire and the terrain is a function of shear displacement J:

$$\tau = \tau_{max} \cdot \left(1 - e^{-\frac{J}{K}} \right), \tag{5}$$

where K is the soil shear deformation modulus and J is the shear displacement, given by the relation presented by El-Gawwad et al. (1999).

The net traction force and traction efficiency were calculated with the formulae given by the ASAE S296 standard.

2.2 Model development

Variable shear area. The first improvement of the model took into account the fact that, according to some authors (Komandi, 1993; Abd El-Gawwad et al., 1999) the shear area varies during the traction as a function of slip:

$$A_{sh} = A_t \cdot \left[1 - (1 - s) \cdot e^{-Y} \right], \tag{6}$$

where $Y = c_1 \cdot l_c^{m_1} \cdot s^{m_2}$, with the values of the constants c_1, m_1 and m_2 depending upon the nature of the ground surface.

Shape of the contact patch. Another improvement of the tire-ground interaction model is to consider the shape of the contact patch to be a super ellipse, based on the results presented by Keller (2005), who also considered the contact patch as a super ellipse and made measurements of the vertical stress below tires using compression cells. The value of the super ellipse exponent was calculated with the formula presented by Keller (2005):

$$n = 2.1 \cdot (b \cdot d)^2 + 2 \tag{7}$$

where b is the tire width and d is the outer diameter.

Deformation of the tire cross-section. The next step consisted in approximating the shape of the tire cross-section with an ellipse (Koutný, 2007), as shown in Fig. 4a. Under the effect of vertical load (G, Fig. 1), the cross-section was deformed (Fig. 4b); thus, the minor semi-axis has decreased to $h-z_p$, while the major axis has increased from b to l_w.

a)   b)
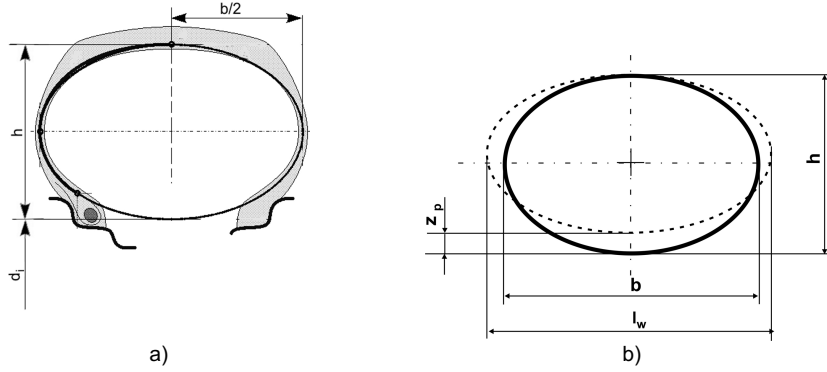
Fig. 4. Tire cross-section deformation
   a) tire section parameters; b) tire section deformation under load;
   d_i – rim diameter; h – tire section height; b-tire width (undeformed); l_w – tire width (under load); z_p - tire deflection under vertical load

The major axis of the ellipse was calculated assuming that its perimeter remained unchanged:

$$l_w = \sqrt{b^2 + 2 \cdot h \cdot z_p - z_p^2} \; . \tag{8}$$

2.3 Experimental tests

In order to validate the theoretical results, field tests were developed, using the U-650 tractor, equipped with the P2V plow; Table 1 presents the main features of the driving wheel and tire.
   During the experiments, drive wheel slip and net traction force were measured directly, for wheel slips up to 30%.

Table 1. Characteristics of the U-650 tractor and drive wheels.

Item	Value
Load on the driving wheel [kN]	11.75
Type of tire	$14.00 - 38$
Overall diameter of tire [m]	1.58
Tire width [m]	0.367
Transversal radius of the undertread [m]	0.3

2.4 Goodness-of-fit analysis

In order to evaluate the goodness-of-fit between model and experimental data the following criteria were considered (Schunn & Wallach, 2005):
- percentage of points within 95% confidence interval of data (Pw95CI);
- mean absolute deviation (MAD;
- root mean squared deviation (RMSD);
- mean scaled absolute deviation (MSAD);
- Pearson correlation coefficient r^2.

3 Results and discussion

Tables 2 and 3 summarize the results regarding the traction force and traction efficiency, for the initial model (which assumes that the area of the contact patch is not affected by wheel slip) and for the one that uses equation (13) in order to compute the shear area. Traction force data analysis showed that the use of variable shear area assumption has led to the improvement of the traction model, with smaller differences between experimental values and the calculated ones.

In the meantime, the differences between the values for the traction efficiency provided by the variable shear area model and the experimental ones have decreased for wheel slips lower than 16…17%.

Fig. 5 and 6 present the data for the traction force and traction efficiency based on the data provided by the variable shear area model.

When the hypothesis of the super ellipse shape of the contact patch was considered, the results of the goodness-of-fit analysis, in terms of traction force, showed a better goodness-of-fit of the super ellipse type contact patch for almost all the criteria taken into account; in the meantime, in the case of super ellipse, 44.4% of the points predicted by the model are within the 95% confidence interval of each corresponding experimental data point, compared to only 33.3% when the elliptical shape was considered.

Finally, deformation of the tire cross-section was taken into account (Fig. 4), while maintaining the super ellipse shape of the tire-ground contact patch. Figures 7 and 8 present the predicted and experimental results concerning the traction force and traction efficiency for this case. The charts clearly show that the model predicted higher values of the traction force and traction efficiency when the deformation of the

tire cross section was considered, due to the increased value of the contact surface area.

Table 2. Net traction force [kN].

Wheel slip, %	Experiment	Constant area	Variable area
6.1	1.8	2.24	1.73
9.4	2.37	2.8	2.45
13.94	3.0	3.75	3.29
16.7	4.25	4.13	3.63
20.5	4.31	4.5	4.1
25.4	4.67	4.9	4.58
Average relative difference [%]		+ 10,25	- 2,6

Table 3. Traction efficiency.

Wheel slip, %	Experiment	Constant area	Variable area
6.1	0.6513	0.6824	0.6316
9.4	0.6784	0.6990	0.6680
13.94	0.6832	0.6900	0.6719
16.7	0.6997	0.6740	0.6610
20.5	0.6990	0.6500	0.6420
25.4	0.6919	0.6170	0.6110
Average relative difference		+ 3.1%	- 5.7%
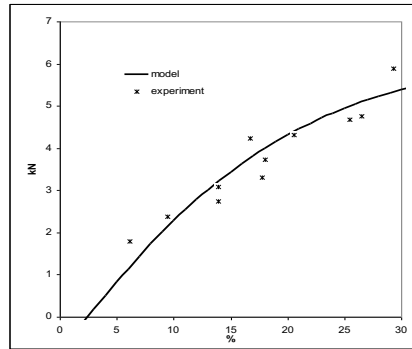
Fig. 5. Traction force (variable shear area)
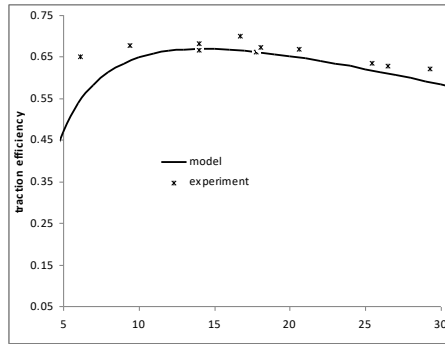
Fig. 6. Traction efficiency (variable shear area)

The goodness-of-fit analysis showed that, compared to the previous model, the most significant differences were recorded for the traction efficiency: the Pearson correlation coefficient r^2 increased from 0.186 to 0.216, the mean absolute deviation (MAD) decreased from 0.058 to 0.051, root mean squared deviation (RMSD) decreased from 0.0752 to 0.0686 and the mean scaled absolute deviation (MSAD) decreased from 5.225 to 4.557.

When referring to the values of the traction force, all the goodness-of-fit parameters recorded better values for the modified traction model.
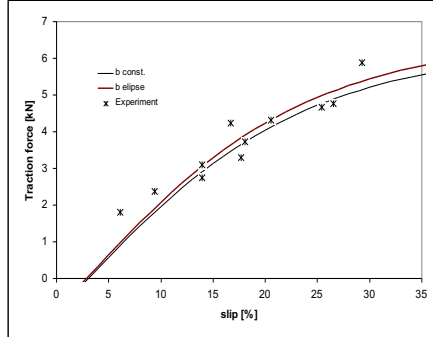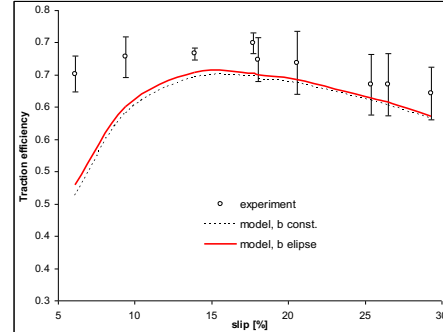
Fig. 7. Traction force

Fig. 8. Traction efficiency

4   Conclusions

The time evolution of a semi-empirical model for the prediction of traction performance of a tractor driving wheel is presented in this study.

The model was developed in several stages:

a) constant tire-soil shear area, elliptical shape of the contact patch and no deformation of the tire cross section;

b) variable tire-soil shear area (depending on wheel slip), elliptical shape of the contact patch and no deformation of the tire cross section;

c) variable tire-soil shear area, super ellipse shape of the contact patch and no deformation of the tire cross section;

d) variable tire-soil shear area, super ellipse shape of the contact patch and deformation of the tire cross section.

A goodness-of-fit analysis, based on several statistic criteria, was performed in order to validate the model; model predicted data and experimental data from ploughing tests were used in this analysis.

The successive development stages led to a better fit between theoretical and experimental data referring to traction force and traction efficiency.

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