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Inference method for miso-structure systems with fuzzy inputs using parallel computing techniques

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Abstract

The paper is devoted to the problem of computational complexity of fuzzy inference in the case of systems with fuzzy inputs. The application of parallel computing is considered in order to accelerate these processes. The paper proposes an algorithm to efficiently perform fuzzy inference based on fuzzy truth values on a GPU. The relevance of this algorithm is shown, including a computational experiment that compares the time characteristics in a sequential operation mode and when performing calculations on a GPU in parallel mode.

Keywords 1

Fuzzy systems of MISO structure, fuzzy truth value, parallel computing, fuzzy inference

1. Introduction

In the absence of sufficiently accurate knowledge about the control object, traditional methods of solving control problems are ineffective or may not be applicable at all. In this case, you can build a fuzzy control system using the fuzzy sets theory and fuzzy logic. Fuzzy control based on fuzzy modeling, the types of which are determined by methods of fuzzy inference. The most popular in control is the inference method presented by E. Mamdani [1,2].

In modern packages of fuzzy modeling [3], the inference is computed only at crisp values of the input variables of the control system. However, in some applications, input data may contain either non-numeric (linguistic) values [4, 5], or noisy input signals [6]. As in the first and in the second case, they are formalized by membership functions and called fuzzy inputs.

Mamdani's approach is reduced to interpretation of expression “if X is A, then Y is B”, where X and Y are linguistic variables, and A and B linguistic values of X and Y respectively. The source of uncertainty is that “if X is A, then Y is B” can be interpreted in two different ways. The first, and most obvious way, is to treat such an expression as “if X is A, then Y is B” or as “(X, Y) is $A \times B$”, where $A \times B$ is a cartesian product of fuzzy sets A and B. So in this interpretation “if X is A, then Y is B” is a joint constraint on X and Y. An alternative way is understanding of “if X is A, then Y is B” as a conditional constraint or, equivalently, implication. This direction was investigated for systems with many fuzzy inputs into [7]. This paper is devoted to the development of the Mamdani approach.

In [6], inference is considered for such systems with fuzzy inputs, which are based on operations of $\max\_\min$ or $\max\_\mathrm{product}$ composition with linear complexity. Operator min (and product (arithmetic product) are t-norms [8] and correspond to Mamdani’s [1] and Larsen’s [9] inference rules. But for other t-norms, need to change of which occurs in the training of fuzzy systems, it is not possible to realize an inference with the polynomial computational complexity in systems with many fuzzy inputs. This article discusses methods that solve this problem regardless of the t-norms that are changed.

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The second one considers the inference for one rule based on the fuzzy truth value. The third section shows the inference for fuzzy rule base and construction of the corresponding network structures based on center of sums defuzzification method. The last section is considered parallel implementation of the above method

2. Statement of the problem

Define the linguistic model as a base of fuzzy rules R_k, $k = \overline{1,N}$:

$$R_k{:}\, If\ x_1\ is\ A_{1k} \# x_2\ is\ A_{2k} \# \dots \#\ x_n\ is\ A_{nk}, then\ y\ is\ B_k, \tag{1}$$

where N is a number of fuzzy rules, $A_{ik} \subseteq X_i$, $i = \overline{1,n}$, $B_k \subseteq Y$ are fuzzy sets that are described by membership functions $\mu_{A_{ik}}(x_i)$ and $\mu_{B_k}(y)$ respectively. $x_1, x_2, \dots, x_n$ are input variables of the linguistic model, and $[x_1, x_2, \dots, x_n]^T = \boldsymbol{x} \in X_1 \times X_2 \dots \times X_n$. Characters X_i, $i = \overline{1,N}$ and Y denoted respectively domain range of the input and output variables. In (1), linguistic bindings "AND" or "OR", denoted by «#».

In following notation $\boldsymbol{X} = X_1 \times X_2 \dots \times X_n$ and $\boldsymbol{A_k} = A_{1k} \times A_{2k} \times \dots \times A_{nk}$, the rule (1) represented as a fuzzy implication

$$R_k{:}\ \boldsymbol{A_k} \longrightarrow B_k,\ k = \overline{1,N}.$$

R_k can be formalized as a fuzzy relation defined on a set $\boldsymbol{X} \times Y$, that is $R_k \subseteq \boldsymbol{X} \times Y$ is fuzzy set with membership function:

$$\mu_{R_k}(\boldsymbol{x}, y) = \mu_{\boldsymbol{A_k} \longrightarrow B_k}(\boldsymbol{x}, y).$$

The Mamdani model defines the function assignment like $\mu_{\boldsymbol{A_k} \longrightarrow B_k}(\boldsymbol{x}, y)$ based on known membership functions $\mu_{A_k}(x)$ and $\mu_{B_k}(y)$ in the following way [2, 6]

$$\mu_{\boldsymbol{A_k} \longrightarrow B_k}(\boldsymbol{x}, y) = T_1\left(\mu_{A_k}(x), \mu_{B_k}(y)\right) = \mu_{\boldsymbol{A_k}}(x) \overset{T_1}{*} \mu_{B_k}(y) \tag{2}$$

where $\overset{T_1}{*}$ is an arbitrary *t*-norm that is used as a parameter.

The task is to determine the fuzzy inference $B_k' \subseteq Y$ for the system represented as (1), if the inputs are fuzzy sets $\boldsymbol{A'} = A'_1 \times A'_2 \times \dots \times A'_n \subseteq \boldsymbol{X}$ or $x_1\ is\ A'_1\ and\ x_2\ is\ A'_2\ and\ \dots\ and\ x_n\ is\ A'_n$ with the corresponding membership function $\mu_{\boldsymbol{A'}}(x)$. In accordance with the generalized fuzzy rule modus ponens [4], fuzzy set B'_k determined by the composition of a fuzzy set $\boldsymbol{A'}$ and relation R_k, such

$$B'_k = \boldsymbol{A'} \circ (\boldsymbol{A_k} \to B_k)$$

or using the membership functions:

$$\mu_{B'_k}(y) = \sup_{x \in X} \{\mu_{\boldsymbol{A'}}(x) \overset{T_2}{*} (\mu_{\boldsymbol{A_k}}(x) \overset{T_1}{*} \mu_{B_k}(y))\} \tag{3}$$

where $\overset{T_2}{*}$ can be any *t*-norm. Complexity of the expression (2) is $O(|X|^{\mathrm{n}} \cdot |Y|)$.

3. Inference method based on fuzzy truth value

For fuzzy systems with one input, following (2), the inference is described by the relation:

$$\mu_{B'_k}(y) = \sup_{x \in X} \{\mu_{\boldsymbol{A'}}(x) \overset{T_2}{*} (\mu_{\boldsymbol{A_k}}(x) \overset{T_1}{*} \mu_{B_k}(y))\} \tag{3}$$

Applying the rule of truth modification [5]:

$$\mu_{A'}(x) = \tau_{A_k/A'}(\mu_{A_k}(x)),$$

where $\tau_{A_k/A'}(\cdot)$ is a fuzzy truth value of a fuzzy set A_k in relation to A', representing the compatibility membership function $CP(A_k, A')$ A_k towards A', and A' is considered reliable [9]:

$$\tau_{A_k/A'}(v) = \mu_{CP(A_k, A')}(v) = \sup_{\substack{\mu_{A'}(x) = v \\ x \in X}} \{\mu_{A_k}(x)\}, \qquad v \in [0,1]$$

Moving from variable x to variable v, denoting $\mu_{A_k}(x) = v$:

$$\mu_{A'}(x) = \tau_{\frac{A_k}{A'}}(\mu_{A_k}(x)) = \tau_{\frac{A_k}{A'}}(v) \tag{4}$$

Then the generalized modus ponens rule for systems with one input (3) can be written as follows:

$$\mu_{B'_k}(y) = \sup_{v\in[0,1]} \left\{ \tau_{\frac{A_k}{A'}}(v) \overset{T_2}{*} \left(v \overset{T_1}{*} \mu_{B_k}(y) \right) \right\} \tag{5}$$

4. Calculation of an output value for the rule base based on center of sums defuzzification method

When the condition of the Decomposition theorem of the multidimensional membership function (2) is satisfied and using the linguistic binding "AND", (3) takes the form:

$$\mu_{\mathrm{B}'_k}(y) = \min_{i=\overline{1,n}} \left\{ \sup_{x_i\in X_i} \left\{ \mu_{A'_i}(x_i) \overset{T_2}{*} T_1 \left(\mu_{A_{ik}}(x_i),\, \mu_{B_k}(y) \right) \right\} \right\}, k = \overline{1,\mathrm{N}} \tag{6}$$

An expression (6) can be written through the fuzzy truth value as follows from (5), and (6) takes the form:

$$\mu_{\mathrm{B}'_k}(y) = \min_{i=\overline{1,n}} \left\{ \sup_{v_i\in[0,1]} \left\{ \tau_{A_{ik}/A'_i}(v_i) \overset{T_2}{*} \left(v_i \overset{T_1}{*} \mu_{B_k}(y) \right) \right\} \right\},\ k = \overline{1,\mathrm{N}} \tag{7}$$

An expression (7) characterized by complexity of order $O(|v_i| \cdot |Y| \cdot n)$ and corresponds to a polynomial.

If $T_1 = T_2 = T$, then considering the *t*-norm property of associativity, (7) can be converted to:

$$\begin{aligned} \mu_{\mathrm{B}'_k}(y) &= \min_{i=\overline{1,n}} \left\{ \sup_{v_i\in[0,1]} \left\{ \tau_{A_{ik}/A'_i}(v_i) \overset{T}{*} \left(v_i \overset{T}{*} \mu_{B_k}(y) \right) \right\} \right\} = \\ &= \min_{i=\overline{1,n}} \left\{ \sup_{v_i\in[0,1]} \left\{ (\tau_{A_{ik}/A'_i}(v_i) \overset{T}{*} v_i) \overset{T}{*} \mu_{B_k}(y) \right\} \right\} = \\ &= \min_{i=\overline{1,n}} \left\{ \sup_{v_i\in[0,1]} \left\{ \tau_{A_{ik}/A'_i}(v_i) \overset{T}{*} v_i \right\} \overset{T}{*} \mu_{B_k}(y) \right\} = \min_{i=\overline{1,n}} \left\{ \prod\nolimits_{A_{ik}/A'_i} \overset{T}{*} \mu_{B_k}(y) \right\}, \\ & k = \overline{1,n} \end{aligned} \tag{8}$$

$$\prod\nolimits_{A_{ik}/A'_i} = \sup_{v_i\in[0,1]} \left\{ \tau_{A_{ik}/A'_i}(v_i) \overset{T}{*} v_i \right\} \tag{9}$$

$\prod_{A_{ik}/A'_i}$ is a scalar value, according to the definition [11] is a possibility measure of A_{ik} corresponds to the input A'_i [10].

Consider the fuzzy systems introduced in section 2, and taking into account the above transformations, obtain a crisp output value using the center of sums defuzzification method [6]. In this case, the output value can be calculated as:

$$\bar{y} = \frac{\sum_{k=\overline{1,N}} \bar{y}_k \cdot \mu_{B'_k}(\bar{y}_k)}{\sum_{k=\overline{1,N}} \mu_{B'_k}(\bar{y}_k)} \tag{10}$$

where $\bar{y}$ is a crisp output value of system consisting of *N* rules (1); $\bar{y}_k$ are centers of membership functions $\mu_{B_k}(y)$, $k=\overline{1,\mathrm{N}}$:

$$\mu_{B_k}(\bar{y}_k)=\underset{y\in Y}{sup}\{\mu_{B_k}(y)\}=1 \tag{11}$$

Following to (7) and (10):

$$\bar{y}=\frac{\sum_{k=\overline{1,N}}\bar{y}_k\underset{i=\overline{1,n}}{min}\{\underset{v_i\in[0,1]}{sup}\{\tau_{A_{ik}/A_i'}(v_i)\overset{T_2}{*}(v_i\overset{T_1}{*}\ \mu_{B_k}(\bar{y}_k))\}\}}{\sum_{k=\overline{1,N}}\underset{i=\overline{1,n}}{min}\{\underset{v_i\in[0,1]}{sup}\{\tau_{A_{ik}/A_i'}(v_i)\overset{T_2}{*}(v_i\overset{T_1}{*}\ \mu_{B_k}(\bar{y}_k))\}\}} \tag{12}$$

Considering (11):

$$\underset{v_i\in[0,1]}{sup}\left\{\tau_{A_{ik}/A_i'}(v_i)\overset{T_2}{*}\left(v_i\overset{T_1}{*}1\right)\right\}=\underset{v_i\in[0,1]}{sup}\{\tau_{A_{ik}/A_i'}(v_i)\overset{T_2}{*}v_i\} \tag{13}$$

Since *t*-norms by definition satisfies the boundary condition $T(a;1)=a$, then substituting (13) into (12):

$$\bar{y}=\frac{\sum_{k=\overline{1,N}}\bar{y}_k\ \cdot\underset{i=\overline{1,n}}{min}\left\{\prod_{A_{ik}/A_i'}\right\}}{\sum_{k=\overline{1,N}}\underset{i=\overline{1,n}}{min}\left\{\prod_{A_{ik}/A_i'}\right\}} \tag{14}$$

So the result $\bar{y}$ obtained using the center of sums defuzzification method with fuzzy inputs does not depend on *t*-norm T_1.

Consider inference when the input data is crisp [12]:

$$\tau_{A_{ik}/A_i'}(v_i)=\delta(v_i)=\begin{cases}1\ if\ v_i=v_{ik}\\0\ if\ v_i\neq v_{ik}\end{cases},$$

where $v_{ik}=\mu_{A_{ik}}(\bar{x}_i)$, $k=\overline{1,\mathrm{N}}$; $i=\overline{1,\mathrm{n}}$; $\bar{x}_i$, $i=\overline{1,n}$ are crisp input values, then

$$\Pi_{A_{ik}/A_k'}=\underset{v_i\in[0,1]}{sup}\left\{\delta(v_i)\overset{T_2}{*}v_i\right\}=v_{ik}$$

taking into account that $T_2\ (1;v_{ik})=v_{ik}$.

As a result an output value is calculated as

$$\bar{y}=\frac{\sum_{k=\overline{1,N}}\bar{y}_k\cdot\underset{i=\overline{1,n}}{\min}\{v_{ik}\}}{\sum_{k=\overline{1,N}}\underset{i=\overline{1,n}}{\min}\{v_{ik}\}} \tag{15}$$

Thus, with crisp input data and the center of sums defuzzification method, the system output does not depend on *t*-norms T_1 and T_2. Obtained result (15) is consistent with well-known results. [13]. The structure of the fuzzy system, which is described by the relation (14), is shown in Figure 1.

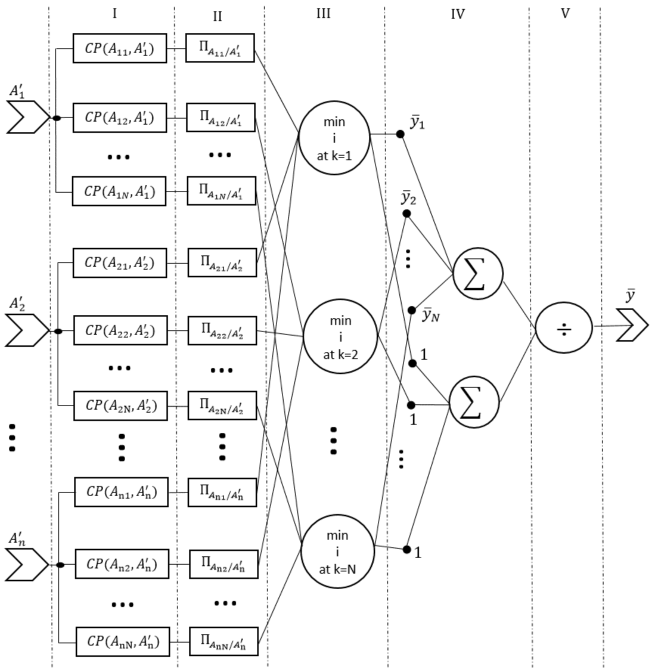

Figure 1: The network structure of a fuzzy system described by expression (14)

5. Parallel implementation

The above algorithm was implemented using the OpenCL parallel programming standard. It includes a program interface (API) for coordinating parallel computing in a heterogeneous processor environment and a cross-platform language used in a specific computing environment [14].

The host organizes a queue of command execution. Finding the correspondence of each input to the terms of the corresponding linguistic variables is placed in the queue as well as necessary supporting calculations. Then, according to the structure described in Figure 1, the obtained data are processed and an output value of the system is calculated.

For each input, it is necessary to find a measure of the possibility that this input corresponds to the terms of a linguistic variable. To do this, for a given membership function of the term on the interval, its numerical values are calculated with a given step. The obtained values are stored in a joint array for each variable on the GPU (Figure 2).

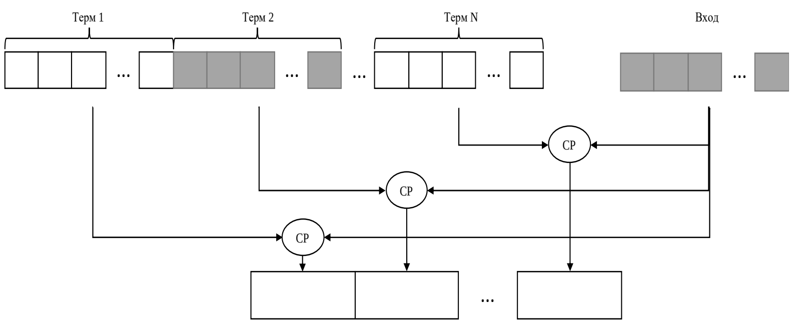

Figure 2: The distribution of data in the memory of the GPU

The procedure of calculating the correspondence of an input variable to a given term is described by the flowchart shown in Fig. 3. Analytically, this procedure is described by expressions (4) and (9). The input of the procedure receives an array with the values of the membership function *membershipArray*, indices *inInd* and *ind* of the first elements of the input and given terms, respectively, step *h*, and *n* that is the size of the array that defines the membership function of one term. Initially, an empty array *resArray* is created in the procedure to save the appropriate results. Then a loop is created with the value *i*, which varies from 0 to 1 (inclusive) with a step *h.* The loop body begins with a call of the kernel function *kernalCP* with the size of the computational lattice equal to *n.* his kernel function returns an array of length n with nonzero values of the membership function of a given term if the value of the membership function of the input term at the same point with a given accuracy equal to *i*. From the obtained values, the maximum is selected using the *kernelFindMax* kernel function, which is transmited to the input of the *tNorma* function as the first parameter, and the second parameter is *i.* The result is stored in a *resArray* array. After that, the value of the variable *i* changes and the next iteration begins. At the end of the procedure, the maximum value is selected from the *resArray*, which is the result of the procedure.

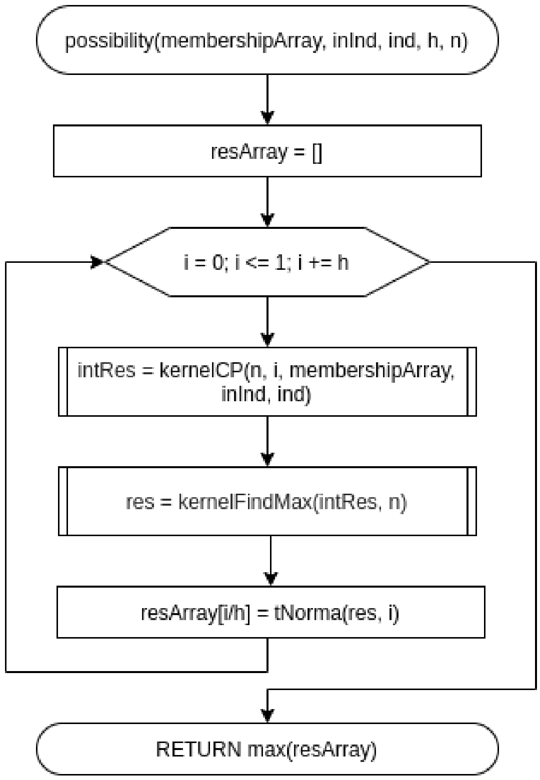

Figure 3: The flowchart of the algorithm for calculating the correspondence of the input term to a given

To determine the effectiveness of the program, an algorithm was also implemented without the use of parallel computing technology. In addition, a testing system was developed that generates test data for given parameters, calculates the output value by programs with parallel implementation and without. The results are compared and the percentage of error in the results is determined. Comparison of the calculation time of the output value depending on the number of inputs is shown in Figure 4. An error in obtained results was about 1%.

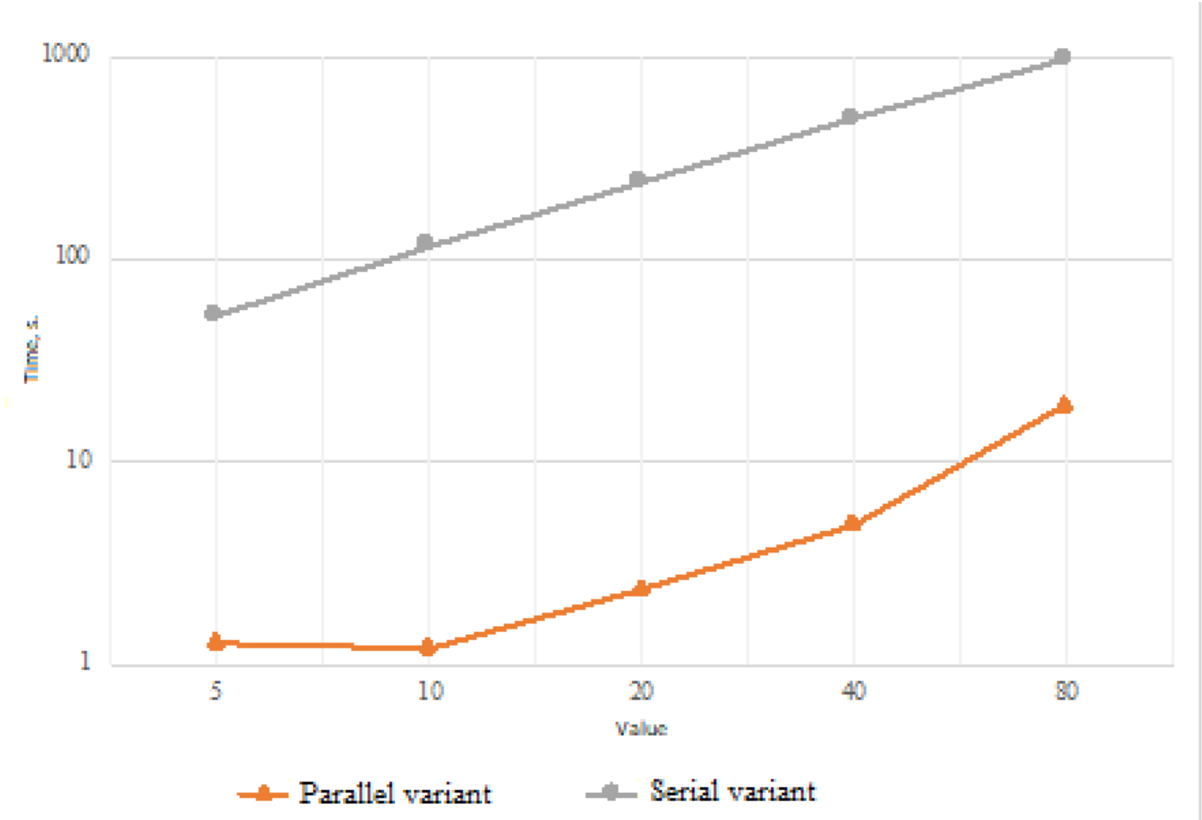

Figure 4: Graph comparing execution time serial and parallel variants

6. Conclusion

The inference based on the decomposition theorem makes it possible to extend the Mamdani approach for systems with many fuzzy inputs with polynomial computational complexity, regardless of the t-norms used, the change of which can be used in training of such systems. It must be taken into account that the decomposition theorem is satisfied when the t-norm MIN is applied in the antecedent of rule (1) when modeling the linguistic binding "AND", and the t-conorm MAX is applied to the linguistic binding "OR".

The output values for fuzzy systems of the Mamdani type are obtained using the middle center defuzzification method. These results are used to build a network structure. When implementing an algorithm for obtaining results using parallel computing and in developing algorithms for training parameters, this structure will be transformed into a neuro-fuzzy system, which is the task of future research.

7. Acknowledgements

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8. References

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