Method for Classification of Objects with Fuzzy Values of Features

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Abstract

This article proposes a classification method for the case when the features of the classified objects are represented by the terms of linguistic variables, or when values with measurement errors are formalized by fuzzy sets. The method is based on the logical approach to the inference in MIMO fuzzy systems using fuzzy truth value. The use of the fuzzy truth value makes it possible to reduce the computational complexity to polynomial and thus to classify objects with a large number of features. The rule base may be composed from the sample objects whose membership degrees to the given classes can be determined using the method of pairwise comparisons.

Keywords 1

Fuzzy inference, fuzzy classification, fuzzy features, fuzzy truth value, extension principle.

1. Introduction

Many of classification methods that have been proposed in recent decades (e.g. [1, 2]) are based on the technologies of soft computing, such as neural networks, fuzzy inference systems and evolutionary algorithms. Neural networks cannot utilize the linguistic information obtained from a subject area expert. At the same time, traditional fuzzy systems do not support machine learning. As a response to these drawbacks, so-called neuro-fuzzy systems that combine the advantages of neural networks and fuzzy systems were developed.

The methods considered above assume that the features of the classified objects can only take scalar values. This article proposes a classification method for the case when the features of the classified objects are represented by the terms of linguistic variables [3, 4], or when values with measurement errors are formalized by fuzzy sets.

The article is divided into several sections. The second section provides a formal definition of fuzzy inference in MIMO (Multiple Inputs – Multiple Outputs) systems. The third section presents an inference method based on the fuzzy truth value, which allows to perform fuzzy inference with a polynomial dependence of computational complexity on the number of inputs. The fourth section defines the inference result for the rule base using the center of gravity defuzzification method. The last section addresses the issue of multi-classification based on the described inference method.

2. Fuzzy inference in MIMO systems

MIMO fuzzy systems formalize the reasoning based on the fuzzy rule base defined as

$$\begin{aligned} &R_k\text{: If } x_1 \text{ is } A_{1k} \text{ and } x_2 \text{ is } A_{2k} \text{ and ... and } x_n \text{ is } A_{nk}, \\ &\text{then } y_1 \text{ is } B_{1k} \text{ and } y_2 \text{ is } B_{2k} \text{ and ... and } y_m \text{ is } B_{mk}, k=\overline{1,N}, \end{aligned} \tag{2.1}$$

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where N is the number of rules, $A_{ik} \subseteq X_i, i = \overline{1,n}, B_{jk} \subseteq Y_j, j = \overline{1,m}$ are fuzzy sets described by membership functions $\mu_{A_{ik}}(x_i)$ и $\mu_{B_{jk}}(y_j)$ respectively; $x_1, x_2, \dots, x_n$ and $y_1, y_2, \dots, y_m$ are input and output variables respectively, and

$$[x_1, x_2, \dots, x_n]^{\mathrm{T}} = \boldsymbol{x} \in X_1 \times X_2 \times \dots \times X_n = \boldsymbol{X},$$
$$[y_1, y_2, \dots, y_m]^{\mathrm{T}} = \boldsymbol{y} \in Y_1 \times Y_2 \times \dots \times Y_m.$$

Let us denote $\boldsymbol{A_k} = A_{1k} \times A_{2k} \times \dots \times A_{nk}$, while

$$\mu_{\boldsymbol{A_k}}(\boldsymbol{x}) = \underset{i=\overline{1,n}}{\mathrm{T}_1} \mu_{A_{ik}}(x_i),$$

where T_1 is an arbitrary t-norm. Hence, if the outputs y_j, $j = \overline{1,m}$ are independent, (2.1) can be represented as a set of fuzzy implications

$$R_{jk}: \boldsymbol{A_k} \to B_{jk}, \qquad j = \overline{1,m}. \tag{2.2}$$

A given rule R_{jk} is formalized as a fuzzy relation defined on the set $\boldsymbol{X} \times Y_j$, i.e. $R_{jk} \subseteq \boldsymbol{X} \times Y_j$ is a fuzzy set with the following membership function:

$$\mu_{R_{jk}}(\boldsymbol{x}, y_j) = \mu_{\boldsymbol{A_k} \to B_{jk}}(\boldsymbol{x}, y_j), \qquad j = \overline{1,m}.$$

Let us denote the input values of a system described by expression (2.2) as $\boldsymbol{A'} = A'_1 \times A'_2 \times \dots \times A'_n$ or as "x_1 is A'_1 and x_2 is A'_2 and ... and x_n is A'_n" with a corresponding membership function $\mu_{\boldsymbol{A'}}(\boldsymbol{x})$ defined as

$$\mu_{\boldsymbol{A'}}(\boldsymbol{x}) = \underset{i=\overline{1,n}}{\mathrm{T}_2} \mu_{A'_i}(x_i).$$

Hence, according to the *fuzzy modus ponens* rule, the output values $B'_{jk} \subseteq Y_j$, $j = \overline{1,m}$ are determined by the fuzzy composition of the fuzzy set $\boldsymbol{A'}$ and the relation R_{jk}, i.e.

$$B'_{jk} = \boldsymbol{A'} \circ (\boldsymbol{A_k} \to B_{jk}), \qquad j = \overline{1,m},$$

whence

$$\mu_{B'_{jk}}(y_j) = \sup_{\boldsymbol{x} \in \boldsymbol{X}} \left\{ \mu_{\boldsymbol{A'}}(\boldsymbol{x}) \overset{\mathrm{T}_3}{*} I\left(\mu_{\boldsymbol{A_k}}(\boldsymbol{x}), \mu_{B_{jk}}(y_j)\right) \right\}, \qquad j = \overline{1,m}, \tag{2.3}$$

where T_3 is an arbitrary t-norm, $I(\cdot, \cdot)$ is a fuzzy implication. The computational complexity of (2.3) equals $\mathrm{O}(|X_1| \cdot |X_2| \cdot \ldots \cdot |X_n| \cdot |Y|)$, i.e. it depends on the number of inputs exponentially.

Fuzzy implications which are commonly used in considered logical approach [5] can be divided into the following types [1]:

- S-implications:

$$I(a, b) = S\{N\{a\}, b\}, \tag{2.4}$$

e.g. the implications of Łukasiewicz, Reichenbach, Kleene-Dienes, Fodor and Dubois-Prade;

- R-implications:

$$I(a, b) = \sup_{z \in [0;1]} \{z \mid T\{a, z\} \le b\}, \tag{2.5}$$

e.g. the implications of Rescher, Goguen and Gödel;

- Q-implications:

$$I(a, b) = S\{N\{a\}, T\{a, b\}\}, \tag{2.6}$$

e.g. Zadeh's implication.

In (2.4) – (2.6), T, S and N stand for an arbitrary t-norm, s-norm, and fuzzy negation respectively, $a, b \in [0; 1]$.

3. Inference method based on fuzzy truth value

The method utilizes the truth modification rule [3]

$$\mu_{\boldsymbol{A}'}(\boldsymbol{x}) = \tau_{\boldsymbol{A_k}|\boldsymbol{A}'}\left(\mu_{\boldsymbol{A_k}}(\boldsymbol{x})\right),$$

where $\tau_{\boldsymbol{A_k}|\boldsymbol{A}'}(\,\cdot\,)$ is the fuzzy truth value of the fuzzy set $\boldsymbol{A_k}$ relative to $\boldsymbol{A}'$, which represents the compatibility $CP(\boldsymbol{A_k}, \boldsymbol{A}')$ of $\boldsymbol{A_k}$ with respect to $\boldsymbol{A}'$, where $\boldsymbol{A}'$ is considered reliable [6]:

$$\tau_{\boldsymbol{A_k}|\boldsymbol{A}'}(v) = \mu_{CP(\boldsymbol{A_k},\boldsymbol{A}')}(v) = \sup_{\substack{\mu_{A_k}(\boldsymbol{x})=v \\ \boldsymbol{x}\in\boldsymbol{X}}} \{\mu_{\boldsymbol{A}'}(\boldsymbol{x})\}, \qquad v \in [0;1].$$

Let us denote $\mu_{\boldsymbol{A_k}}(\boldsymbol{x}) = v$. Then:

$$\mu_{\boldsymbol{A}'}(\boldsymbol{x}) = \tau_{\boldsymbol{A_k}|\boldsymbol{A}'}\left(\mu_{\boldsymbol{A_k}}(\boldsymbol{x})\right) = \tau_{\boldsymbol{A_k}|\boldsymbol{A}'}(v). \tag{3.1}$$

Using (3.1) we can transform the *fuzzy modus ponens* rule (2.3) into

$$\mu_{B'_{jk}}(y_j) = \sup_{v\in[0;1]} \left\{\tau_{\boldsymbol{A_k}|\boldsymbol{A}'}(v) \overset{\mathrm{T}_3}{*} I\left(v, \mu_{B_{jk}}(y_j)\right)\right\}, \qquad j = \overline{1,m}. \tag{3.2}$$

Expression (3.2) has the computational complexity of $\mathrm{O}(|v| \cdot |Y|)$.
As it follows from [7],

$$\mu_{CP(\boldsymbol{A_k},\,\boldsymbol{A}')}(v) = \underset{i=\overline{1,n}}{\widetilde{\mathrm{T}}_1}\, \mu_{CP(A_{ik},\,A'_i)}(v_i) =$$

$$= \left(\mu_{CP(A_{1k},\,A'_1)}(v_1)\ \widetilde{\mathrm{T}}_1\ \mu_{CP(A_{2k},\,A'_2)}(v_2)\right) \widetilde{\mathrm{T}}_1\ \mu_{CP(A_{3k},\,A'_3)}(v_3)\ \widetilde{\mathrm{T}}_1 \ldots\ \widetilde{\mathrm{T}}_1\ \mu_{CP(A_{nk},\,A'_n)}(v_n),$$

where $\widetilde{\mathrm{T}}_1$ is an extended using the extension principle n-ary t-norm [6], and

$$\mu_{CP(A_{ik},\,A'_i)}(v_i) = \sup_{\substack{\mu_{A_{ik}}(x_i)=v_i \\ x_i\in X_i}} \left\{\mu_{A'_i}(x_i)\right\}.$$

For example, binary t-norm is defined by the expression

$$\mu_{CP(\boldsymbol{A_k},\,\boldsymbol{A}')}(v) = \underset{i=\overline{1,2}}{\widetilde{\mathrm{T}}_1}\, \mu_{CP(A_{ik},\,A'_i)}(v_i) = \sup_{\substack{v_1\ \mathrm{T}_1\ v_2\ =\ v \\ (v_1,\,v_2)\in[0;1]^2}} \left\{\mu_{CP(A_{1k},\,A'_1)}(v_1)\ \mathrm{T}_2\ \mu_{CP(A_{2k},\,A'_2)}(v_2)\right\}.$$

The latter expression has the computational complexity of $\mathrm{O}(|v|^2)$.

4. Inference result for a rule base

Let us derive the output value for a base of N rules of the form (2.1) using the center-of-gravity defuzzification method [8]:

$$\bar{y}_j = \frac{\sum_{k=\overline{1,N}} \bar{y}_{jk} \cdot \mu_{B'_j}(\bar{y}_{jk})}{\sum_{k=\overline{1,N}} \mu_{B'_j}(\bar{y}_{jk})}, \qquad j = \overline{1,m},$$

where $\bar{y}_j$ is the crisp value of j-th output, $\bar{y}_{jk}$ are the points of membership functions $\mu_{B_{jk}}(y_j)$, for which the following is true:

$$\mu_{B_{jk}}(\bar{y}_{jk}) = \max_{y_j\in Y_j} \left\{\mu_{B_{jk}}(y_j)\right\} = 1.$$

Logical approach defines the fuzzy set B_j' as follows [8]:

$$B_j' = \bigcap_{l=\overline{1,N}} B_{jl}',$$

or as

$$\mu_{B_j'}(y_j) = \underset{l=\overline{1,N}}{\mathrm{T}} \left\{\mu_{B_{jl}'}(y_j)\right\}. \tag{4.1}$$

From (3.2) and (4.1) follows:

$$\bar{y}_j = \frac{\sum_{k=\overline{1,N}} \bar{y}_{jk} \cdot \underset{l=\overline{1,N}}{\mathrm{T}} \left\{ \sup_{v\in[0;1]} \left\{ \tau_{A_l|A'}(v) \overset{\mathrm{T}_3}{*} I\left(v, \mu_{B_{jl}}(y_j)\right)\right\}\right\}}{\sum_{k=\overline{1,N}} \underset{l=\overline{1,N}}{\mathrm{T}} \left\{ \sup_{v\in[0;1]} \left\{ \tau_{A_l|A'}(v) \overset{\mathrm{T}_3}{*} I\left(v, \mu_{B_{jl}}(y_j)\right)\right\}\right\}}, \qquad j = \overline{1,m}. \tag{4.2}$$

5. Fuzzy classification method based on fuzzy inference

The rules (2.1) together with the inference method (4.2) are used to solve modelling problems. However, they can also be used to solve multi-classification problems [9].

Let us denote the features of an object q as $[x_1, x_2, \dots, x_n]$, which have terms of linguistic variables as their values [3]. A_{ik}, A_i', $i = \overline{1,n}$, $k = \overline{1,N}$ are fuzzy sets which formalize these terms. Let us also denote the set of classes as $\Omega = \{\omega_1, \omega_2, \dots, \omega_M\}$. The knowledge is represented as N rules of the form

$$\begin{aligned} &R_k\text{: If } x_1 \text{ is } A_{1k} \text{ and } x_2 \text{ is } A_{2k} \text{ and} \dots \text{and } x_n \text{ is } A_{nk},\\ &\text{then } q_k \in \omega_1(\bar{z}_{1k}), q_k \in \omega_2(\bar{z}_{2k}) \dots, q_k \in \omega_M(\bar{z}_{Mk}), \qquad k = \overline{1,N}, \end{aligned} \tag{5.1}$$

where $\bar{z}_{jk}$ is a membership degree of the object q in a class ω_j according to rule R_k. The values of $\bar{z}_{jk}$ can be determined by means of pairwise comparison method developed by Thomas L. Saaty [10] on a set of sample objects $Q = \{q_k\}$, $k = \overline{1,N}$.

Therefore, $\{A_{ik}\}_{i=\overline{1,n}}$ is a set of fuzzy sets that describe the features of an object q_k, $k = \overline{1,N}$, which is used to construct the knowledge base, and $\{A_i'\}_{i=\overline{1,n}}$ are the values of features of the object that is being classified. The values $\bar{z}_{jk}$ correspond to values $\bar{y}_{jk}$ in section 4, which allows to combine (2.1) and (5.1) into

$$\begin{aligned} &R_k\text{: If } x_1 \text{ is } A_{1k} \text{ and } x_2 \text{ is } A_{2k} \text{ and} \dots \text{and } x_n \text{ is } A_{nk},\\ &\text{then } z_1 \text{ is } B_{1k} \text{ and } z_2 \text{ is } B_{2k} \text{ and} \dots \text{and } z_m \text{ is } B_{mk}, k = \overline{1,N}, \end{aligned}$$

where fuzzy sets $B_{1k}, B_{2k}, \dots, B_{mk}$ are singletons:

$$\mu_{B_{jk}}(z_j) = \begin{cases} 1, & \text{if } z_j = \bar{z}_{jk},\\ 0, & \text{if } z_j \neq \bar{z}_{jk}, \end{cases}$$

and (4.2) can be transformed into

$$\bar{z}_j = \frac{\sum_{k=\overline{1,N}} \bar{z}_{jk} \cdot \underset{l=\overline{1,N}}{\mathrm{T}} \left\{ \sup_{v\in[0;1]} \left\{ \tau_{A_l|A'}(v) \overset{\mathrm{T}_3}{*} I\left(v, \mu_{B_{jl}}(\bar{z}_{jk})\right)\right\}\right\}}{\sum_{k=\overline{1,N}} \underset{l=\overline{1,N}}{\mathrm{T}} \left\{ \sup_{v\in[0;1]} \left\{ \tau_{A_l|A'}(v) \overset{\mathrm{T}_3}{*} I\left(v, \mu_{B_{jl}}(\bar{z}_{jk})\right)\right\}\right\}}, \qquad j = \overline{1,m}. \tag{5.2}$$

Let us consider the form of (5.2) depending on type of implication.

- S-implications:

$$I\left(v, \mu_{B_{jl}}(z_j)\right) = \begin{cases} 1, & \text{if } z_j = \bar{z}_{jk},\\ N(v), & \text{if } z_j \neq \bar{z}_{jk}, \end{cases}$$

then (5.2) can be simplified into

$$\bar{z}_j = \frac{\sum_{k=\overline{1,N}} \bar{z}_{jk} \cdot \underset{\substack{l=\overline{1,N} \\ l\neq k}}{\mathrm{T}} \left\{ \sup_{v\in[0;1]} \left\{ \tau_{\boldsymbol{A}_l|\boldsymbol{A}'}(v) \overset{\mathrm{T}_3}{*} N(v) \right\} \right\}}{\sum_{k=\overline{1,N}} \underset{\substack{l=\overline{1,N} \\ l\neq k}}{\mathrm{T}} \left\{ \sup_{v\in[0;1]} \left\{ \tau_{\boldsymbol{A}_l|\boldsymbol{A}'}(v) \overset{\mathrm{T}_3}{*} N(v) \right\} \right\}}, \qquad j = \overline{1,m}.$$

- Q-implications:

$$I\left(v, \mu_{B_{jl}}(z_j)\right) = \begin{cases} S\{N(v), v\}, & \text{if } z_j = \bar{z}_{jk}, \\ N(v), & \text{if } z_j \neq \bar{z}_{jk}, \end{cases}$$

then (5.2) can be written as:

$$\bar{z}_j = \frac{\sum_{k=\overline{1,N}} \bar{z}_{jk} \cdot \mathrm{T} \left\{ \sup_{v\in[0;1]} \left\{ \tau_{\boldsymbol{A}_k|\boldsymbol{A}'}(v) \overset{\mathrm{T}_3}{*} S\{N(v), v\} \right\}, \underset{\substack{l=\overline{1,N} \\ l\neq k}}{\mathrm{T}} \left\{ \sup_{v\in[0;1]} \left\{ \tau_{\boldsymbol{A}_l|\boldsymbol{A}'}(v) \overset{\mathrm{T}_3}{*} N(v) \right\} \right\} \right\}}{\sum_{k=\overline{1,N}} \mathrm{T} \left\{ \sup_{v\in[0;1]} \left\{ \tau_{\boldsymbol{A}_k|\boldsymbol{A}'}(v) \overset{\mathrm{T}_3}{*} S\{N(v), v\} \right\}, \underset{\substack{l=\overline{1,N} \\ l\neq k}}{\mathrm{T}} \left\{ \sup_{v\in[0;1]} \left\{ \tau_{\boldsymbol{A}_l|\boldsymbol{A}'}(v) \overset{\mathrm{T}_3}{*} N(v) \right\} \right\} \right\}},$$

$j = \overline{1,m}.$

- R-implications:

$$I\left(v, \mu_{B_{jl}}(z_j)\right) = \begin{cases} 1, & \text{if } z_j = \bar{z}_{jk} \text{ or } v = 0, \\ 0, & \text{if } z_j \neq \bar{z}_{jk} \text{ and } v > 0, \end{cases}$$

then (5.2) can be transformed into

$$\bar{z}_j = \frac{\sum_{k=\overline{1,N}} \bar{z}_{jk} \cdot \underset{\substack{l=\overline{1,N} \\ l\neq k}}{\mathrm{T}} \left\{ \sup_{v\in[0;1]} \left\{ \tau_{\boldsymbol{A}_l|\boldsymbol{A}'}(v) \overset{\mathrm{T}_3}{*} \delta(v) \right\} \right\}}{\sum_{k=\overline{1,N}} \underset{\substack{l=\overline{1,N} \\ l\neq k}}{\mathrm{T}} \left\{ \sup_{v\in[0;1]} \left\{ \tau_{\boldsymbol{A}_l|\boldsymbol{A}'}(v) \overset{\mathrm{T}_3}{*} \delta(v) \right\} \right\}} = \frac{\sum_{k=\overline{1,N}} \bar{z}_{jk} \cdot \underset{\substack{l=\overline{1,N} \\ l\neq k}}{\mathrm{T}} \left\{ \tau_{\boldsymbol{A}_l|\boldsymbol{A}'}(0) \right\}}{\sum_{k=\overline{1,N}} \underset{\substack{l=\overline{1,N} \\ l\neq k}}{\mathrm{T}} \left\{ \tau_{\boldsymbol{A}_l|\boldsymbol{A}'}(0) \right\}}, \qquad j = \overline{1,m},$$

where

$$\delta(v) = \begin{cases} 1, & \text{if } v = 0, \\ 0, & \text{if } v \neq 0. \end{cases}$$

The final solution can be obtained according to the following rule:

$$\begin{cases} q' \in \omega_j, & \text{if } \bar{z}_j > z_P, \\ q' \notin \omega_j, & \text{if } \bar{z}_j < z_N, \\ \text{not defined}, & \text{if } z_N \leq \bar{z}_j \leq z_P, \end{cases}$$

where z_P and z_N are fixed threshold values.

6. Conclusion

The article proposed a classification method for objects that have their features represented as terms of linguistic variables or just formalized by fuzzy sets. The method is based on application of the local approach and fuzzy truth values to the inference in MIMO systems. The use of fuzzy truth values and the extension principle enables us to perform the classification with a polynomial dependence of the computational complexity on the number of features.

A comparative analysis of performance of multi-classifiers based on S-, Q- and R-implications will be taken as a subject for further studies.

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8. References

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