Analysis of the multifactorial phenomena based on fuzzy Bayesian model

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Abstract

A method for analysis of the fuzzy Bayesian model and using it for the analysis of multifactorial phenomena in conditions of uncertainty is considered, based on the preliminary identification of stable patterns of the impact of phenomena on target indicators, as well as on setting unified types of indicators of the impact of phenomena on target indicators, depending on the events corresponding to these phenomena and intended to assess the frequency and magnitude of the impact of the corresponding phenomena.

Keywords 1

Fuzzy Bayesian model, multifactorial phenomena, fuzzy probability

1. Introduction

The tasks of analysis of the complex multifactorial phenomena and processes are becoming increasingly important in various fields. Such tasks are characterized by:

- complex structure and interconnection of many factors;
- uncertainty, incompleteness, variability and inaccuracy of data;
- the complexity of building and using traditional models and methods to analyze such phenomena and processes;
- the ability to type multifactorial phenomena and processes depending on the properties of the corresponding factors and their impact on the target indicators [1, 2, 3].

To analyze such phenomena and processes, it is advisable to use fuzzy Bayesian models, the advantages of which are:

- the visibility of presentation and interpretation of factors based on the cause-and-effect relations between them;
- the ability to work in a small amount of data;
- the accounting for the complexity and uncertainty of information by introducing fuzziness in the description and calculation of model parameters [4, 5, 6].

The introduction of fuzziness into Bayesian models is the result of supplementing the probabilistic uncertainty characteristic of the Bayesian approach with fuzzy estimates of the probabilities of the relationships of its vertices. However, this approach entails some difficulties, namely, the correct calculation of fuzzy probabilistic estimates and their interpretation.

There are several ways to introduce fuzziness into Bayesian networks [7]:

- supplementing the Bayesian rule with membership functions of the corresponding variable values;
- replacement of the probabilities of the values of network variables with fuzzy sets (terms of linguistic variables), and operations on crisp values with operations of S- and T-norms over fuzzy sets;

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- replacement of probabilities of values on variables with fuzzy numbers, and operations with extended operations on fuzzy numbers.

This paper proposes to use the third method of introducing fuzziness. The article presents a method for the analysis of fuzzy Bayesian models and its use for the analysis of multifactorial phenomena, considered on the example of modeling and analysis of the impact of climatic phenomena on the vulnerability of the urban environment of Moscow.

2. The way of analysis of the multifactorial phenomena based on fuzzy Bayesian model

The specificity of the analysis of multifactorial phenomena, which, as a rule, consists in studying the effect of a set of factors on target indicators, creates the basis for a typification of the approach to the construction of fuzzy Bayesian models intended for this.

The proposed approach to the construction of fuzzy Bayesian models for the analysis of multivariate phenomena is based on:

- firstly, the identification of stable patterns of the impact of phenomena on target indicators, which are ternary combinations of the type: "phenomenon", "type of impact", "event";
- secondly, the assignment of unified types of indicators of the impact of phenomena on target indicators depending on the events corresponding to these phenomena and designed to assess the frequency and scale of the impact of the corresponding phenomena.

A method for analysis of multivariate phenomena based on fuzzy Bayesian models using the example of the problem of analysis of the impact of climatic phenomena on the vulnerability of the urban environment of Moscow is to consider.

Stage 1. Determination of climatic phenomena that make a significant contribution to the vulnerability of objects in the urban environment.

The expert survey identified climate phenomena that significantly contribute to the vulnerability of Moscow's urban environment (table 1) [1].

Table 1

Climate phenomena that significantly contribute to the vulnerability of Moscow's urban environment

Objects of the urban environment	Climate phenomena
*O*1 – natural environment	*P*1 – sleet, heavy ice and frost deposits, freezing rain; *P*2 – tornado; *P*3 – very strong wind, squall; *P*4 – hurricane wind; *P*5.1 – abnormally hot weather
*O*2 – population	*P*5.2 – extremely high temperature; *P*5.3 – extremely low temperature; *P*7 – sudden temperature changes; *P*8 – increased level of air pollution
*O*3 – power supply system	*P*1 – sleet, heavy ice and frost deposits, freezing rain; *P*5 – extreme temperatures (*P*5.1, *P*5.2, *P*5.3)
*O*4 – buildings and constructions	*P*5 – extreme temperatures (*P*5.1, *P*5.2, *P*5.3); *P*7 – sudden temperature changes; *P*9 – air temperature transition through 0°C
*O*5 – water supply and sewerage system	*P*2 – tornado; *P*3 – very strong wind, squall; *P*4 – hurricane wind; *P*5 – extreme temperatures (*P*5.1, *P*5.2, *P*5.3); *P*6 – prolonged heavy rain or downpour
*O*6 – heat supply system	*P*5 – extreme temperatures (*P*5.1, *P*5.2, *P*5.3)
*O*7 – transport system	*P*1 – sleet, heavy ice and frost deposits, freezing rain; *P*2 – tornado; *P*3 – very strong wind, squall; *P*4 – hurricane wind; *P*5 – extreme temperatures (*P*5.1, *P*5.2, *P*5.3); *P*6 – prolonged heavy rain or downpour
*O*8 – gas supply system	*The impact of climate phenomena is not significant*
O9 – ecosystem services	*The influence of climatic phenomena is being specified*

Stage 2. Formation of a generalized model structure for analysis of the impact of climate events on the vulnerability of the urban environment.

The generalized structure of the model for analysis of the impact of climate events on the vulnerability of the urban environment is presented in figure 1, where for each object of the urban environment O1–O9, the corresponding models "M1"– "M9" for assessing the impact of specific climate events were allocated.

By aggregating vulnerability assessments Vul_{Ok} of all objects of the urban environment Ok, $k = 1, \ldots, K$ a final assessment Vul_{rez} of the impact of significant climate events on the vulnerability of the urban environment is obtained (1)

$$Vul_{rez} = F(Vul_{O1}, \ldots, Vul_{OK}). \quad (1)$$

Stage 3. Forming the model structures in accordance with the identified patterns and typified indicators of the impact of climatic phenomena on the vulnerability of urban environment objects.

As an example of forming the model structure, the "***M1***" model for assessing the impact of climate events on the vulnerability of an *O*1 object – the natural environment is to consider

Step 1. Definition of sustainable patterns of the impact of phenomena on targets.

In the course of research [8], stable patterns of the impact of climatic phenomena on the vulnerability of the corresponding objects of the urban environment are determined, the general view of which is represented by ternary combinations of the type: "climate phenomenon" ↔ "type of vulnerability" ↔ "event".

The natural environment as an object of study *O*1 has the following patterns:

- "sleet, heavy ice-frost deposition, freezing rain" ↔ "snow load" ↔ "snowbreaker";
- "tornado" ↔ "wind loading" ↔ "windfall, windbreak";
- "very strong wind, squall" ↔ "wind loading" ↔ "windfall, windbreak";
- "hurricane wind" ↔ "wind loading" ↔ "windfall, windbreak";
- "abnormally hot weather" ↔ "the lack of humidity" ↔ "drying of the soil".

Step 2. Definition for models «***M*1**»–«***M*9**» of sets of component models that correspond to the identified patterns.

So, for example, for the model «***M*1**» (for object *O*1) the following set of the component models is defined **"*M*1-*Pi*"**, $i = 1, \ldots, N_{O1}$:

- **"*M*1-*P*1"**: "sleet, heavy ice-frost deposition, freezing rain" ↔ "snow load" ↔ "snowbreaker";
- **"*M*1-*P*2"**: "tornado" ↔ "wind loading" ↔ "windfall, windbreak";
- **"*M*1-*P*3"**: "very strong wind, squall" ↔ "wind loading" ↔ "windfall, windbreak";
- **"*M*1-*P*4"**: "hurricane wind" ↔ "wind loading" ↔ "windfall, windbreak";
- **"*M*1-*P*5.1"**: "abnormally hot weather" ↔ "the lack of humidity" ↔ "drying of the soil".

Step 3. Setting unified indicators of the impact of climate events on the vulnerability of objects from the corresponding event.

Input unified indicators for each of the models **"*M*1-*Pi*"**, $i = 1, \ldots, N_{O1}$ are:

- Fr_{Pi} – frequency of climate phenomenon Pi;
- Sc_{Pi} –scale of the impact of this climate phenomenon Pi.

The output indicator for each of these models «***M*1-*Pi***», $i = 1, \ldots, N_{O1}$ is an indicator of vulnerability $Vul_{Pi}^{(O1)}$ from an event corresponding to this climate phenomenon.

Step 4. Setting an indicator for an aggregate assessment of an object's vulnerability to all climate events affecting it (2)

$$Vul_{O1} = f_1(Vul_{P1}^{(O1)}, \ldots, Vul_{P_{N_{O1}}}^{(O1)}) \quad (2)$$

The structure of the model **"*M*1"** of the impact assessment of the climate phenomena on the vulnerability of the object O_1 (of the natural environment) is presented in figure 2.

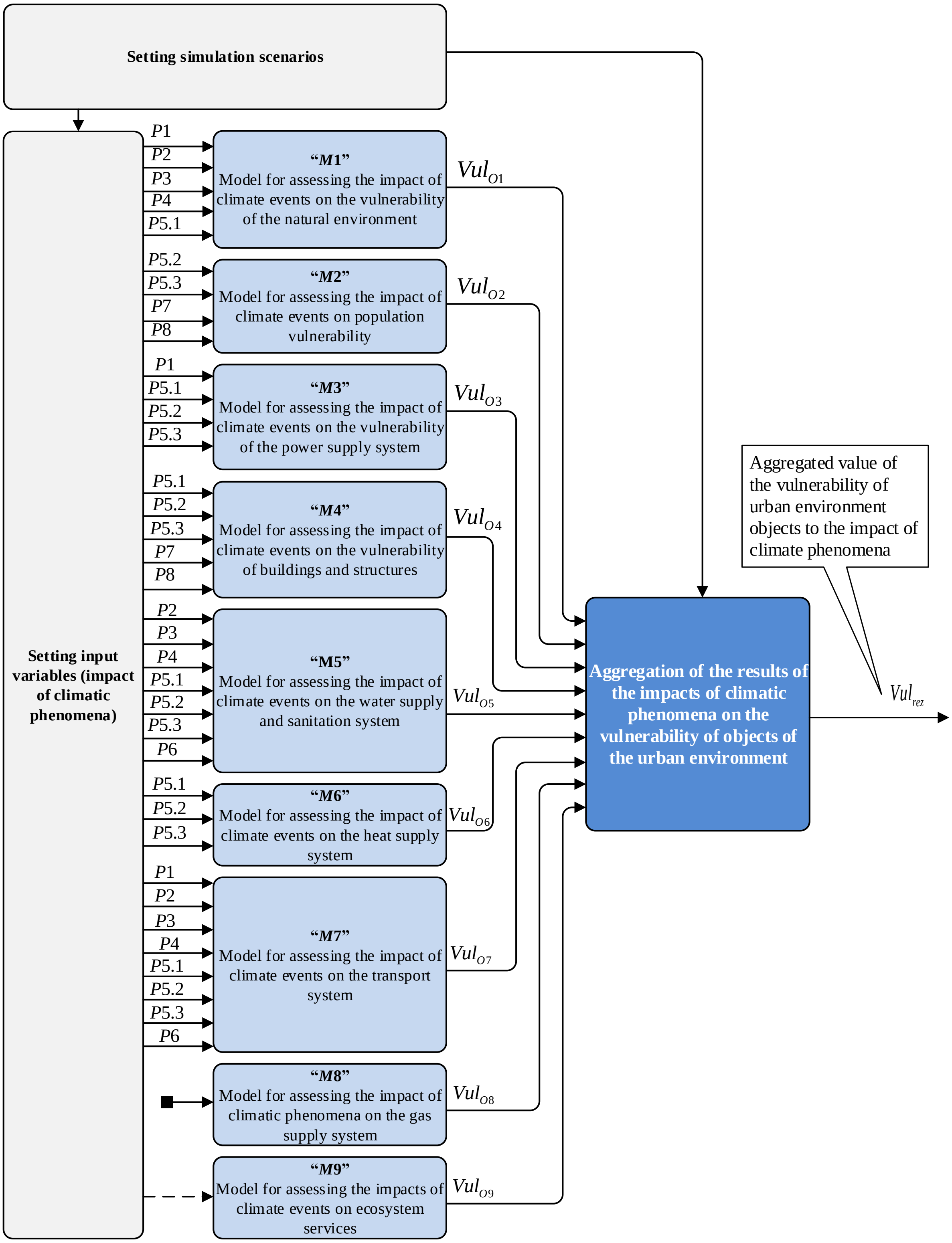

Figure 1: Generalized structure of the model for analysis of the impact of climate events on the vulnerability of the urban environment

Stage 4. Forming of the structure of a fuzzy Bayesian model.

For the object *O*1 (of the natural environment) is formed the structure of the fuzzy Bayesian model (Figure 3).

Frequency Fr_{Pi} and scale Sc_{Pi} indicators of the climate phenomena impact *Pi* on the vulnerability of urban objects are presented at the 1st level of the model. The fuzzy values of these indicators are strictly typified in the form of term sets {*L* – low, *M* – middle, *H* – high}.

The next level of the model contains indicators Vul_{Pi} of objects' Ok, $k = 1, \ldots, K$ o climate events. Indicators also have fuzzy values described by term sets {L – low, M – middle, H – high}. A feature of the representation of fuzzy conditional probabilities is the way of their obtaining, namely, in the form of a task by an expert or using an automatic generation procedure according to expert rules.

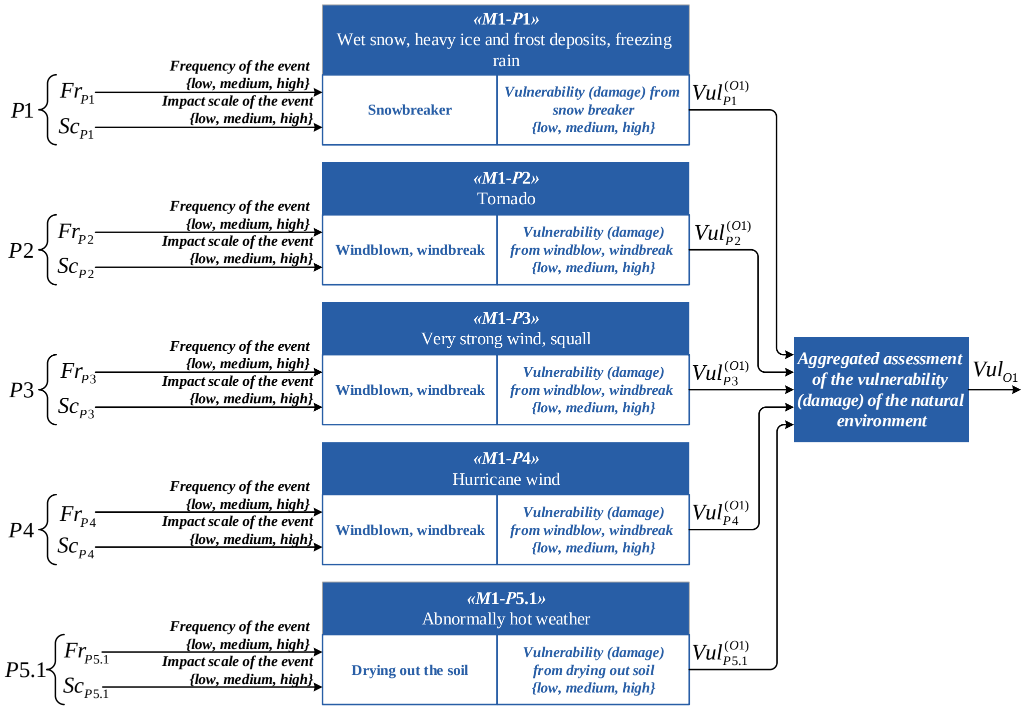

Figure 2: Structure of the model for assessing the impact of climate events on the vulnerability of the natural environment

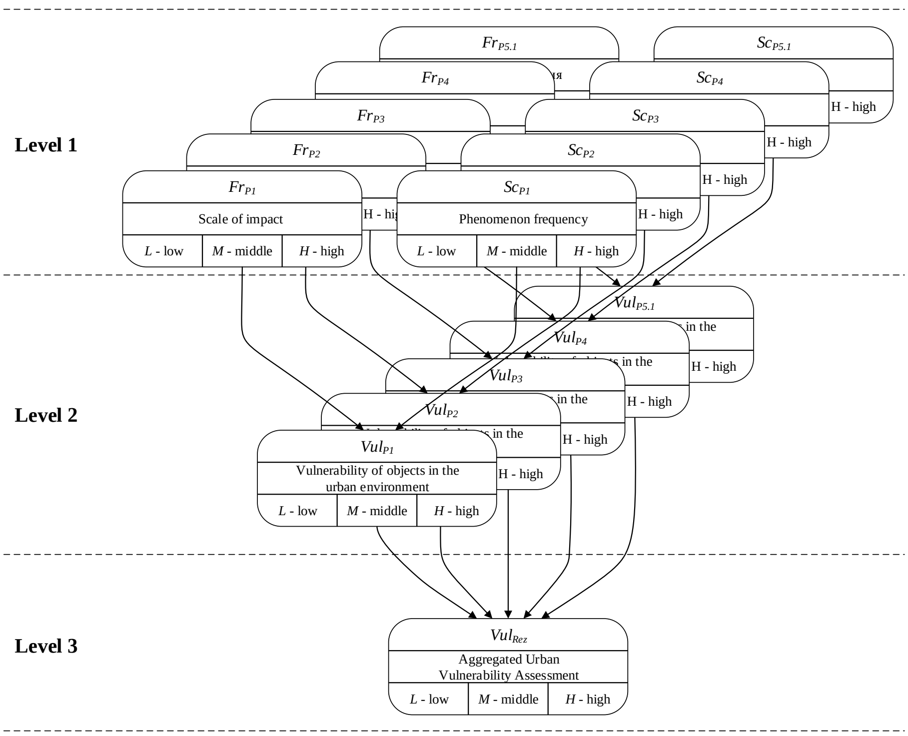

Figure 3: Structure of fuzzy Bayesian model for analysis of the impact of climate events on environmental vulnerabilities

The final indicator Vul_{rez} of the vulnerability of the urban environment to the impact of climatic phenomena is located at the 3rd level of the model.

Stage 5. Setting fuzzy a priori probabilities for all indicators of the first level of the model, namely, indicators of the frequency Fr_{Pi} and scale Sc_{Pi} indicators of climate phenomena *Pi* on the vulnerabilities of the corresponding objects of the urban environment.

The generated fuzzy a priori probabilities for the *P*2 tornado are presented in tables 2, 3.

Table 2
Fuzzy a priori probability Fr_{P2}

Fr_{P2}	$P(Fr_{P2})$
L	[0.2; 0.2; 0.6]
M	[0.2; 0.6; 1]
H	[0.6; 1; 1]

Table 3
Fuzzy a priori probability Sc_{P2}

Sc_{P2}	$P(Sc_{P2})$
L	[0; 0; 0.5]
M	[0; 0.5; 1]
H	[0.5; 1; 1]

Graphical interpretation of indicators of the frequency and scale of the impact of a climate phenomenon *P2* is presented in figures 3 and 4, respectively.

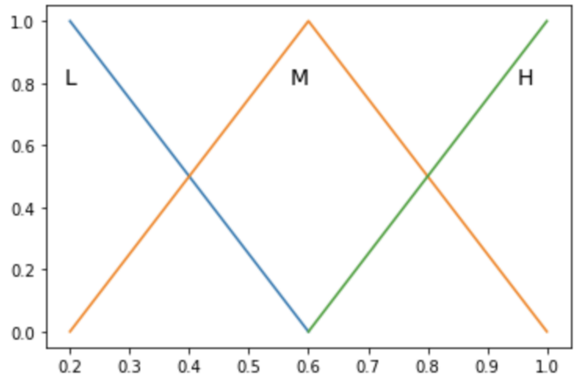

Figure 3: Indicator Fr_{P2}

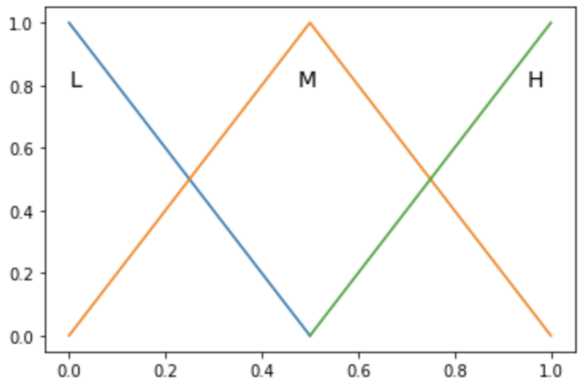

Figure 4: Indicator Sc_{P2}

Stage 6. Forming fuzzy conditional probabilities for indicators of vulnerability Vul_{Pk} of vulnerability of the corresponding objects of the urban environment *Ok*, $k = 1, \ldots, K$.

Tables of fuzzy conditional probabilities for nodes-descendants of the second level of the model $P4$, Vul_{P2}, …, $Vul_{P5.1}$ are built by automatic filling taking into account preliminary expert settings. Flexible configuration of obtaining tables of fuzzy conditional probabilities is achieved by the following algorithm.

Stage 1. Drawing up a table of the degree of significance of factors.

The table of the degree of significance of climate phenomena is based on expert assessment in order to identify parameters that make a significant contribution to the outcome of the model. The values range from [0, 1]. The table is also used to exclude certain parameters from consideration if it is necessary to evaluate the performance of the model without taking into account their impact. To do this, it is enough to assign a zero value to the selected parameter. In this way, a change in the model operation is achieved while preserving the initial structure.

An example of disabling some climate phenomena in the original model is presented in table. 4.

Table 4

Degree of significance of factors

	$P1$	$P2$	$P3$	$P4$	$P5.1$
k	0	1	0	0	1

Step 2. Filling in the table for evaluating the influence of parameters depending on the selected term.

Tables for evaluating the influence of parameters depending on the selected term are also filled in by experts in order to level the subsequent subjective change in factors during the filling of tables of conditional probabilities of huge sizes, where the monotonous process of work prevails, which increases the degree of fatigue of experts.

As an example, the table for evaluating the influence of a parameter for a factor $P2$ is presented (table 5).

Table 5

Evaluating the influence of parameters for factor $P2$

	Fr^*_{P2}	Sc^*_{P2}	Vul^*_{P2}
L	0,3	0,2	0,2
M	0,5	0,4	0,4
H	0,7	0,8	0,8

Step 3. Multiplication the values of the corresponding variables.

Multiplication the values of the corresponding variables allows to automatically take into account the degree of significance of a particular factor and the influence of its parameters, calculated by the formula (3):

$$Vul^k_{Pi} = k_{Pi} \cdot Vul_{Pi}\{LMH\}.$$

The exclusion of one of the variables of the first layer, as well as its complete removal from the model is not possible due to the peculiarities of the considered problem of analysis of climate phenomena.

Step 4. Entering of the received works in the table of active factors, taking into account the exclusion of zero factors.

Creating a table of active factors makes it easier to exclude factors that are not taken into account in the further work of the model, without having to manually change the conditional probability tables. So, taking into account the model with two climate phenomena, the values of the active factors, taking into account the influence of parameters, are presented in table 6.

Table 6

Current factors taking into account the influence of parameters

	Vul_{P2}^{k}	$Vul_{P5.1}^{k}$
L	0,2	0,2
M	0,4	0,4
H	0,8	0,8

Step 5. Choosing a rule for the interaction of parameters.

Choosing a rule for the interaction of parameters makes it possible, depending on the search over term sets, to present an objective assessment of the change in parameters. To obtain the general conditional probability for a variable Vul_{Pi} is used the rule (4):

$$Vul'_{Pi} = prod(\min(Fr^{*}_{Pi}, Sc^{*}_{Pi}), Vul^{k}_{Pi}).$$

Step 6. Setting fuzzy numbers and entering them in the table of fuzzy conditional probabilities. The final step is to generate fuzzy numbers that will be displayed in the table of fuzzy conditional probabilities necessary for the correct operation of the fuzzy Bayesian model. The calculated fuzzy conditional probabilities of the climate phenomenon "tornado" for assessing the vulnerability of urban objects are presented in table 7.

Table 7

Fuzzy conditional probability Vul_{P2}

n	Frl_{P2}	Sc_{P2}	$P(Vul_{P2}'=L)$	$P(Vul_{P2}'=M)$	$P(Vul_{P2}'=H)$
1	*L*	*L*	[0.04; 0.04; 0.08]	[0.04; 0.08; 0.16]	[0.08; 0.16; 0.16]
2	*L*	*M*	[0.06; 0.06; 0.12]	[0.06; 0.12; 0.24]	[0.12; 0.24; 0.24]
3	*L*	*H*	[0.06; 0.06; 0.12]	[0.06; 0.12; 0.24]	[0.12; 0.24; 0.24]
4	*M*	*L*	[0.04; 0.04; 0.08]	[0.04; 0.08; 0.16]	[0.08; 0.16; 0.16]
5	*M*	*M*	[0.08; 0.08; 0.16]	[0.08; 0.16; 0.32]	[0.16; 0.32; 0.32]
6	*M*	*H*	[0.1; 0.1; 0.2]	[0.1; 0.2; 0.4]	[0.2; 0.4; 0.4]
7	*H*	*L*	[0.04; 0.04; 0.08]	[0.04; 0.08; 0.16]	[0.08; 0.16; 0.16]
8	*H*	*M*	[0.08; 0.08; 0.16]	[0.08; 0.16; 0.32]	[0.16; 0.32; 0.32]
9	*H*	*H*	[0.14; 0.14; 0.28]	[0.14; 0.28; 0.56]	[0.28; 0.56; 0.56]

This approach allows to configure flexibly the model depending on the influencing factors and the degree of their influence on the outcome, as well as to turn off nodes if necessary. The structure of the model does not change. This significantly reduces the time required to prepare the model, simplifying the work of experts.

Stage 7. Calculation of fuzzy unconditional probabilities.

Based on the obtained tables of fuzzy conditional probabilities, fuzzy unconditional probabilities of each indicator of the model are calculated. Below are expressions (4)-(6) in general form for obtaining fuzzy unconditional probabilities of the indicator Vul_{Pi}:

$$P(Vul_{Pi} = L) \cong \widetilde{\bigoplus_{Fr_{Pi}, Sc_{Pi}}} P(Fr_{Pi}, Sc_{Pi}, Vul_{Pi} = L), \tag{4}$$

$$P(Vul_{Pi} = M) \cong \widetilde{\bigoplus_{Fr_{Pi}, Sc_{Pi}}} P(Fr_{Pi}, Sc_{Pi}, Vul_{Pi} = M), \tag{5}$$

$$P(Vul_{Pi} = H) \cong \widetilde{\bigoplus_{Fr_{Pi}, Sc_{Pi}}} P(Fr_{Pi}, Sc_{Pi}, Vul_{Pi} = H), \tag{6}$$

As a result, fuzzy unconditional probabilities of operating indicators are obtained. The calculation results of the assessment of the vulnerability of the natural environment from the factor $P2$ are entered in table 8 and for $P5.1$ in table 9.

Table 8
Fuzzy unconditional probability Vul_{P2}

Vul_{P2}	$P(Vul_{P2})$
L	[0.212; 0.282; 0.422]
M	[0.424; 0.564; 0.844]
H	[0.848; 1.128; 1.288]

Table 9
Fuzzy unconditional probability $Vul_{P5.1}$

$Vul_{P5.1}$	$P(Vul_{P5.1})$
L	[0.235; 0.298; 0.438]
M	[0.457; 0.597; 0.877]
H	[0.915; 1.195; 1.339]

The indicators are presented in Figures 5 and 6, respectively.

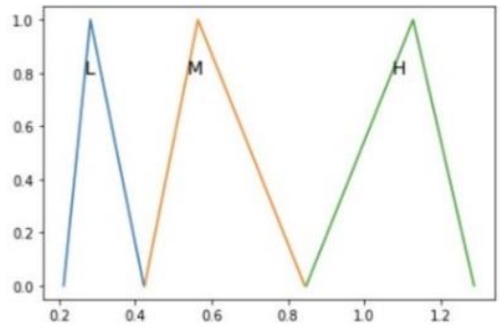

Figure 5: Fuzzy unconditional probability Vul_{P2}

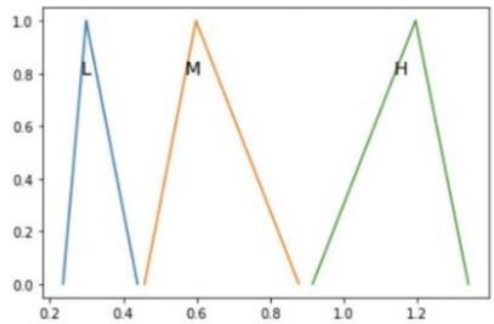

Figure 6: Fuzzy unconditional probability $Vul_{P5.1}$

Stage 8. Forming fuzzy conditional probabilities for the final indicator Vul_{rez} vulnerability of the urban environment.

The construction of a table of fuzzy conditional probabilities for the indicator of the third level of the model Vul_{rez} is performed according to the algorithm described in stage 6, taking into account some features.

Thus, the algorithm for obtaining fuzzy conditional probabilities for assessing the vulnerability of the urban environment consists of the following steps.

Step 1. Setting an assessment of the indicator impact Vul_{rez} depending on the selected term.

Step 2. The selection rules for the interaction parameters.

Taking into account the previously obtained table of active factors, the rule of interaction of parameters for the indicator Vul_{rez} has the form:

$$Vul'_{rez} = prod(\min(Vul^k_{P1}, Vul^k_{P2}, \dots, Vul^k_{P5.1}), Vul^*_{rez}) \qquad (7)$$

Table 10
Assessment of the indicator impact Vul_{rez}

	Vul^*_{rez}
L	0,2
M	0,4
H	0,8

Step 3. Setting fuzzy numbers and entering them in the table of fuzzy conditional probabilities.

The obtained results of calculations are fuzzified and entered in the table of fuzzy conditional probabilities.

In this example with the active factors P2 and P5.1, the generated table of fuzzy conditional probabilities looks as follows.

Table 11
Fuzzy conditional probability Vul_{rez}

n	Vul_{P2}	$Vul_{P5.1}$	$P(Vul'_{rez} = L)$	$P(Vul'_{rez} = M)$	$P(Vul'_{rez} = H)$
1	L	L	[0.04; 0.04; 0.08]	[0.04; 0.08; 0.16]	[0.08; 0.16; 0.16]
2	L	M	[0.04; 0.04; 0.08]	[0.04; 0.08; 0.16]	[0.08; 0.16; 0.16]
3	L	H	[0.04; 0.04; 0.08]	[0.04; 0.08; 0.16]	[0.08; 0.16; 0.16]
4	M	L	[0.04; 0.04; 0.08]	[0.04; 0.08; 0.16]	[0.08; 0.16; 0.16]
5	M	M	[0.08; 0.08; 0.16]	[0.08; 0.16; 0.32]	[0.16; 0.32; 0.32]
6	M	H	[0.08; 0.08; 0.16]	[0.08; 0.16; 0.32]	[0.16; 0.32; 0.32]
7	H	L	[0.04; 0.04; 0.08]	[0.04; 0.08; 0.16]	[0.08; 0.16; 0.16]
8	H	M	[0.08; 0.08; 0.16]	[0.08; 0.16; 0.32]	[0.16; 0.32; 0.32]
9	H	H	[0.16; 0.16; 0.32]	[0.16; 0.32; 0.64]	[0.32; 0.64; 0.64]

3. Conclusion

The method for the analysis of a fuzzy Bayesian model and its use for the analysis of multifactor phenomena in conditions of uncertainty is proposed, based on the preliminary identification of stable patterns of the impact of phenomena on target indicators, as well as on setting unified types of indicators of the impact of phenomena on target indicators, depending on the events corresponding to these phenomena and intended to assess the frequency and magnitude of the impact of corresponding events.

The created fuzzy Bayesian model is presented and the process and results of modeling and analysis of the impact of climatic phenomena on the vulnerabilities of the urban environment of Moscow using the constructed model are considered.

4. Acknowledgements

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