Morphology of self-similar multi-layer neural networks

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Abstract

A class of multilayer modular neural networks with self-similar structure is considered. The paper introduces the concept of morphogenesis and network regularity. Conditions for morphogenesis of the multilayer network are determined. The theorem on morphology of the weakly connected multilayer networks is proved, and invariants of system graphs for the regular self-similar networks are obtained. A rule for graph construction of the self-similar multilayer networks is proposed. It is noted that self-similar networks describe the structure of fast Fourier transform algorithms.

Keywords 1

Neural network, modularity, neocortex, self-similarity, morphogenesis, self-similar graph

1. Modularity of biological neural networks

It is known that a biological neuron does not function in isolation, but forms neural ensembles, neural modules, or nerve centers of various sizes and amount of neural cells. For the first time, the principle of neural networks modular organization was described by R. Lorente de No [1]. In 1933, R. Lorente de No established that neurons of brain cortex are united in vertical columns of cells, which are functional units connected by commonality of theirs receptor fields. Perhaps his most significant contribution was the first description of the columnar organization of the cortex (long before Vernon Mountcastle and half a century before David Hubel and Torsten Wiesel were awarded the Nobel prize in physiology or medicine in 1981).

Direct physiological results confirming the modular brain cortical structures were obtained by V. Mountcastle in 1957-59 [2]. Studies of new cortex (neocortex) in mammals conducted by J. Edelman and V. Mountcastle [3, 4] showed that the neocortex has the high degree of uniformity in its structural organization. The neocortex is present only in mammals, and the rapid increase the neocortex in humans occurred only a couple of million years ago. Any significant evolutionary changes that occur in a short period of time are provided by reiteration of existing structures.

Starting in the late 50s of the XX century, David Hubel and Swedish scientist Torsten Wiesel recorded impulses of nerve cells located in various layers of the visual cortex. In their studies of the visual cortex, Hubel and Wiesel showed that cortical cells are usually grouped in columns (columns), and that within each column, neurons perform the same functions in interpreting the pulse signal from the eyes. The columns in turn form so-called super columns, each of which occupies an area of approximately 2 x 2 mm in the cortex of the brain. the analysis takes place in a strict sequence from one cell to another, and each nerve cell is responsible for a specific detail in the whole picture [5].

In humans, the new cortex has six horizontal layers of neurons that differ in type and nature of connections. Vertically, the neurons are grouped into so-called cortex columns. At the beginning of the XX century, the German neurologist Brodman showed that in all mammals, the new cortex has 6 horizontal layers of neurons. The main structural and organizing unit of the cortex is the cortical column, which forms a neural module, whose afferent cells share a common receptor field. The number of neurons in the column is constant and for primates is about 110 [6]. The column is

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connected by 10-30 connections with other parts of the cortex. Columns are grouped into more complex formations – macro columns that preserve a certain topological order and form strictly connected distributed systems. The human neocortex contains about 600,000 macro columns.

"According to many neuroscientists, in the end, it will be proved that the unique properties of each person – the ability to feel, think, learn and remember – are contained in strictly organized networks of synaptic relationships between the neurons of the human brain" [7].

2. Self-similarity and morphogenesis

By definition, a compact topological space X is self-similar if there exist a finite set S that indexes non-surjective homeomorphisms $\{f_s\}_{s\in S}$ for which

$$X = \cup_{s\in S} f_s(X).$$

For example, let the compactum X is a segment of unit length. Position of a point in the segment is determined by the number from the interval [0, 1). In decimal notation, the number is expressed in positional form $z = 0.z_{-1}z_{-2} \dots z_{-n} \dots$ with an infinite amount of digits in general case, where $z_i = 0,1,2,\dots,9$. Let's limit the amount by a value n in the positional representation, and assume that number $0.z_{-1}$ corresponds to a segment of length 10^{-n}, and value of the number indicates position of the segment in the interval $[0, 1)$. In particular, for $n=1$ the number $0.z_{-1}$ will correspond to a segment with length 0.1, and the digit value z_{-1} indicates the segment position at X. Let's introduce for this level a set of functions $f_{z_{-1}}(X)$ which map the compactum X to all segments with length 0.1 at $z=0.z_{-1}$ positions. It is obvious that

$$X = \cup_{z_{-1}} f_{z_{-1}}(X).$$

In this example, we can replace the union symbol by direct sum symbol, since the segments do not intersect under mappings. Thus, we can write:

$$X = \bigoplus_{z_{-1}} f_{z_{-1}}(X).$$

The process can be iteratively continued by selecting already the constructed segments as compactums of next level. Then for level n we obtain:

$$X = \bigoplus_{z_{-1}} \bigoplus_{z_{-2}} \dots \bigoplus_{z_{-n}} f_{\langle z_{-1}z_{-2}\dots z_{-n}\rangle}(X)\,.$$

In this expression, the tuple $\langle z_{-1}z_{-2} \dots z_{-n}\rangle$ defines a multidimensional index for the set of non-surjective maps which in sequence separate the compactum X into not intersected parts. For a positional number notation, correspondence between the value of a number z and its positional representation $z \leftrightarrow \langle z_{-1}z_{-2} \dots z_{-n}\rangle$ is one-to-one.

The classical concept of the self-similarity discussed above is focused on servicing mathematical models for fractals, and it is not quite suitable for representing of self-similar non-fractal objects, so there is a need for introduce a generalizing definition which includes the fractals as a special case. If we turn to biology, then morphogenesis of living systems can serve as a suitable concept. In biology the morphogenesis refers to the process of new generations organisms and changing of their forms during individual evolution. It is an inherent property of the self-organizing systems that have a property of integrity.

In applied areas of biology, the narrower definition is used: morphogenesis is generation and directed development of a population which consists of taxonomic groups of individual organisms. Here the taxonomic group (τάξις from the Greek is "order, arrangement, organization") is a strata in a hierarchical classification consisting of discrete ordered objects that are combined on the basis of common properties and features. Without setting ourselves the task of constructing strict mathematical model of the morphogenesis, we introduce a working definition which is sufficient for the morphological synthesis of self-similar networks.

Definition 1. We will say that morphogenesis is set on a population of indexed objects, if for each object of the population there exist an exact "parent-child" mapping which uniquely indexes its child objects.

For example, let $X = X_0 = \bigoplus_{z_0} A_{z_0}$ is the initial indexed population of objects, then directed evolution of the population along generations is determined by a sequence of the indexed child objects:

$$X_0 = \bigoplus_{z_0} A_{z_0}, X_1 = \bigoplus_{z_1} \bigoplus_{z_0} A_{\langle z_1 z_0 \rangle}, \dots, X_{n-1} = \bigoplus_{z_{n-1}} \dots \bigoplus_{z_1} \bigoplus_{z_0} A_{\langle z_{n-1} \dots z_1 z_0 \rangle}, \dots$$

If X is a compact topological space defined by a segment of unit length and objects of the population are represented as parts of the segment, then all generations of the population coincide with this space, so that

$$X = X_0 = X_1 = \dots = X_{n-1} = \dots$$

and besides that if we require finiteness of rule set for indexing population objects, then the concept of the morphogenesis is transformed to the definition of self-similarity on a compactum. Note now that if we restrict ourselves only to the finiteness condition for choosing the number of the indexing rules, this leads us to self-similar growing populations which are not related with fractals. If now you remove the condition of finiteness under the choice of indexing rules, the process of population growth due to morphogenesis, in general case, will not be already self-similar.

In given uncertainty of generation number, it is difficult to answer on the question about the finiteness of the rule set for indexing. In this case, it is necessary to introduce additional restrictions to garantee the self-similar process, for example, if for all morphogenesis generations, the "parent-child" mappings are coincided, then obviously morphogenesis is trivially self -similar. However, the class of populations generated by the trivial self-similar morphogenesis is rather narrow, and for practical purposes it is advisable to expand.

Definition 2. Morphogenesis is called as regular if the parent-child mappings in each generation are coincided for all objects and are uniquely determined by the order number of the generation.

Obviously, trivial self-similar morphogenesis is a special case of regular one, when the "parent-child" mappings for all generations coincide. Further discussion will be mainly related to regular morphogenesis. There is a clear auto-simulation in mathematical description of regular and trivial self-similar morphogenesis, so for regular morphogenesis therefor we will also use the term self-similar, making the necessary explanations in cases where it is necessary to highlight differences.

3. Morphogenesis of multi-layer networks

Let a multi-layer network graph Γ has n layers. Denote by $\{A_i^m\}$ set of the vertices in layer m, where $m = 0, 1, \dots, n\text{-}1$. The vertex set for input (zero) layer is called as network afferent and is denoted *Eff(Γ)*, and the set of vertices of final layer is called as network efferent and denoted *Eff(Γ)*. Let A_i^m be some vertex of the network in layer m. Let's call as afferent of the vertex (hereinafter denoted as $Aff(A_i^m)$) the vertex subset of the network afferent are connected by arcs to the vertex A_i^m, thus $Aff(A_i^m) \subset Aff(\Gamma)$. Similarly, we introduce concept efferent of the vertex $Eff(A_i^m)$ as a subset of network efferent vertices are connected by arcs to the vertex A_i^m, thus $Eff(A_i^m) \subset Eff(\Gamma)$. The afferents and efferents of vertex will also be called as terminal vertex projections.

Denote as $\Gamma^{-1}(A_i^m)$ receptor neighborhood of vertex A_i^m, and as $\Gamma(A_i^m)$ its axon neighborhood. We set graph construction rule using the following expressions:

$$\begin{aligned} Aff(A_i^m) &= \bigoplus_{A_k^{m-1} \in \Gamma^{-1}(A_i^m)} Aff\left(A_k^{m-1}\right), \\ Eff(A_i^m) &= \bigoplus_{A_k^{m+1} \in \Gamma^{-1}(A_i^m)} Eff\left(A_k^{m+1}\right) \end{aligned} \tag{1}$$

These rules were called as network weak connectivity conditions [8] because their implementation generates the weakly connected networks. The direct sum symbol in (1) emphasizes that for any vertex in the network, the terminal projections of vertices its graph neighborhood do not intersect. In this case, the populations are the afferent and efferent projections of the network vertices on the terminal layers of the network graph. In fact, both the expressions (1) are dual to each other, and if one of them is true, the other is bound to be true as well. Let's prove this statement.

Theorem 1. About duality.

Proof. Let the first condition from (1) be met for all vertices of the network, but the second condition is not met, then there are at least a pair of vertices C_1, C_2 (Figure 1) belonging to the axon neighborhood of the vertex *B*, such that $Efr(C_1) \cap Efr(C_2) \neq \emptyset$. But this means that in some layer *M*, located between the vertex *B* and the efferent layer, there is a vertex *M'*, that is connected by arcs with the vertices C_1, C_2 Let N_1, N_2 be the vertices of the receptor neighborhood for *M'*, then it is obvious that $Afr(N_1) \cap Afr(N_2) \neq \emptyset$. This contradicts the accepted assumption that the first condition of the expression (1) is fulfilled, it proves the duality of the conditions.

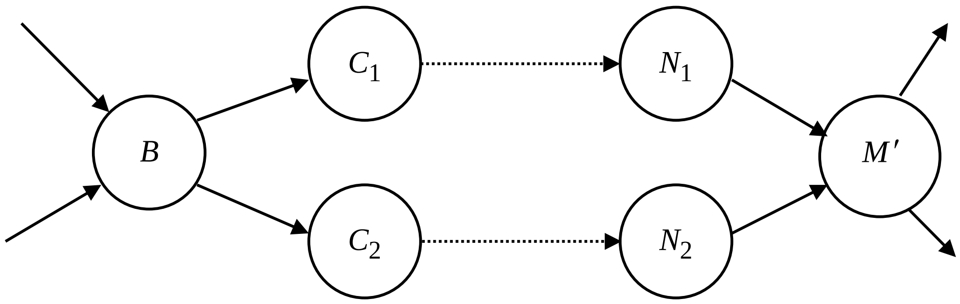

Figure 1: Parallel paths in modular network

The following theorem justifies the use of the term "weakly connected network".

Theorem 2. There are no parallel paths in weakly connected networks.

Proof. Let's assume the opposite. Let two parallel paths converge at the vertex B, passing through its neighboring vertices A_1 and A_2. By the condition of parallelism, the paths have an intersection point at some vertex *A'* that precedes the vertices A_1 and A_2. Hence, the afferents of the vertices and intersect so that $Afr(A_1) \cap Afr(A_2) = \mathrm{Afr}(A')$, but this contradicts the condition of weak connectedness. Since the statement is true for any vertex, including terminal ones, it follows that parallel paths are not possible for the entire network.

In general, the entered conditions (1) are not limited only by regularity. We will keep the term "weak connectivity conditions" for them, but for generality we will sometimes refer to them as the conditions of multi-layer network morphogenesis.

For multilayer weakly connected networks, the number of the layers depends on the number of vertices in the layers. Let's explain nature of this dependency using a example of regular network. We will assume that the network consists of one component graph. Any projection relation of layer vertices on the terminal afferent field of the network, splits the set of the layer vertices into classes. Each class includes the layer vertices that have a common afferent set. let's call these classes as afferent domains and denote, $Dom_p\left(A^m_{z^m}\right) = X^m_{i^m}$ here $A^m_{z^m}$ one of the domain vertex representatives, $i^m$is ordinal number of the domain in layer *m*. The vertices of an afferent domain have a projection on the input layer of the network. To indicate the projection of the layer domains on network afferent we will use the left directed arrow above the symbol of the afferent domain:

$$\overleftarrow{X}^m_{i^m} \subset Aff(\Gamma).$$

The iterative process from layer *n-1* to layer 0 generated by morphogenesis conditions (1) sequentially splits domain projections of each layer into smaller parts, indexing the partition in the layer by means of bit number i_m. The network will be regular if the numbers of generated parts depends only on layer number. For the last layer of the one-component network, there is only one afferent domain, so $\overleftarrow{X}^{n-1} = Aff(\Gamma)$. Union of afferent projections of domains from each layer always forms the full afferent of the network , so we have:

$$Aff(\Gamma) = \overleftarrow{X}^{n-1} = \bigoplus_{i_{n-1}} \overleftarrow{X}^{n-2}_{i_{n-1}} = \bigoplus_{i_{n-1}} \bigoplus_{i_{n-2}} \overleftarrow{X}^{n-3}_{\langle i_{n-1} i_{n-2} \rangle} = \cdots = \bigoplus_{i_{n-1}} \bigoplus_{i_{n-2}} \dots \bigoplus_{i_2} \bigoplus_{i_1} \overleftarrow{X}^{0}_{\langle i_{n-1} i_{n-2} \dots i_1 \rangle}$$

where the transition rule from the positional-digit form $i^{n-k} = \langle i_{n-1} i_{n-2} \dots i_{n-k+1} \rangle$ to the number establishes a one-to-one correspondence of the domain ordinal number to the tuple of indexes. In zero layer, the domains coincide with their afferent projections, and each domain consists of only one vertex:

$$Dom_p\left(A^0_{i^0}\right) = \overleftarrow{X}^0_{i_{\langle i_{n-1} i_{n-2} \dots i_1\rangle}} = A^0_{i^0}.$$

Any vertex number of zero layer is determined by the tuple $i^0 = \langle i_{n-1} i_{n-2} \dots i_1\rangle$. Suppose that for all *m*, the index i_m takes only two values 0 and 1, then the terminal afferent layer has to $N=2^{n-1}$ vertices.

Similarly. The projection relation of the vertices from layer *m* to terminal efferent field of the network, splits the vertices into efferent domain classes, which we denote as $Dom_g\left(A^m_{z^m}\right) = Y^m_{j^m}$, where j^m is the ordinal number of a efferent domain in layer *m*. The terminal projection of the efferent domain in output layer $\vec{Y}^m_{j^m} \subset Eff(\Gamma)$ is denoted by the right directed arrow above the domain symbol. There exist only one efferent domain for the zero layer $\vec{Y}^0 = Eff(\Gamma)$. Due to the expression (1) the iterative process from zero layer to *n-1* layer sequentially splits the terminal domain projections of from each layer into smaller parts, indexing the partition in the layer by bit number j_m. Union of the domain projections from each layer to output layer always forms the full efferent of the network, so we have:

$$Eff(\Gamma) = \vec{Y}^0 = \bigoplus_{j_0} \vec{Y}^1_{j_0} = \bigoplus_{j_0} \bigoplus_{j_1} \vec{Y}^2_{\langle j_0 j_1\rangle} = \dots = \bigoplus_{j_0} \bigoplus_{j_1} \dots \bigoplus_{j_{n-1}} \bigoplus_{j_{n-2}} \vec{Y}^{n-1}_{\langle j_0 j_1 \dots j_{n-1} j_{n-2}\rangle}.$$

In output layer, the efferent domains coincide with their projections, and each domain consists of only one vertex:

$$Dom_g\left(A^{n-1}_{j^{n-1}}\right) = \vec{Y}^{n-1}_{\langle j_0 j_1 \dots j_{n-1} j_{n-2}\rangle} = A^{n-1}_{j^{n-1}}.$$

The vertex number is determined by the tuple $j^{n-1} = \langle j_0 j_1 \dots j_{n-1}\rangle$. The partitioning procedure ensures one-to-one correspondents the number of the efferent domain to the tuple of indexes $j^m = \langle j_0 j_1 \dots j_{m-1}\rangle$ for each layer. Iterative process by *m* in the forward and reverse direction, associated with sequential indexing of domains in each layer. Below as example are shown symbolic formulas for indexing afferent and efferent domains for a four- layer network:

$$X^0\langle i_1 i_2 i_3\rangle \longleftarrow X^1\langle i_2 i_3\rangle \longleftarrow X^2\langle i_3\rangle \longleftarrow X^3,$$
$$Y^0 \longrightarrow Y^1\langle j_0\rangle \longrightarrow Y^2\langle j_0 j_1\rangle \longrightarrow Y^3\langle j_0 j_1 j_2\rangle.$$

Theorem 3. About morphology of weekly connected network. In each layer of a weekly connected network with single connectivity component, the afferent and the efferent domains intersect in all possible pair combinations at one vertex exactly. That is, for each pair combination of afferent and efferent domains, there exist a single vertex $A^m_{z^m}$ such that

$$Dom_p\left(A^m_{z^m}\right) \cap Dom_g\left(A^m_{z^m}\right) = A^m_{z^m}.$$

Proof. We need to prove that all paired combinations of different type domains of each layer have a non-empty intersection and that this intersection consist of only one vertex of the layer.

Let's prove the first. Let's assume that a layer has a pair of domains of different types that do not intersect in the layer. In this case, for the afferent domain of the layer, there are vertices that are not connected to part of the network's efferent vertices, and so the network contains two right-hand vertex cones that are not connected to each other in all subsequent layers. On the other hand, the vertices of the layer's efferent domain are not connected to part of the network's afferent vertices, and the network has two left cones that are not connected in the previous layers. So if the domains do not intersect, then there are at least two unrelated components in the network, but this contradicts the condition about the one-component network theorem.

Let's prove the second. Assume that the domains intersect at two vertices A^m_1 and. A^m_2. Then this pair of vertices has a common afferent neighborhood. That is in layer *m-1*, there is at least one vertex A^{m-1}_k connected with the vertices and A^m_1, A^m_2 and by morphogenesis condition (1) its efferent must be equal to the direct sum of the efferents of the vertices of its axon neighborhood. This direct sum includes the vertices A^m_1andA^m_2, but since they are assumed to belong to the same efferent domain of the layer, $Eff(A^m_1) \cap Eff(A^m_2) \neq \emptyset$ is performed for them (see Figure 2). Thus, we have come to a violation of the morphogenesis condition (1) for efferent vertices. Similarly, this position is proved using the efferent neighborhood for vertices A^m_1 and A^m_2.

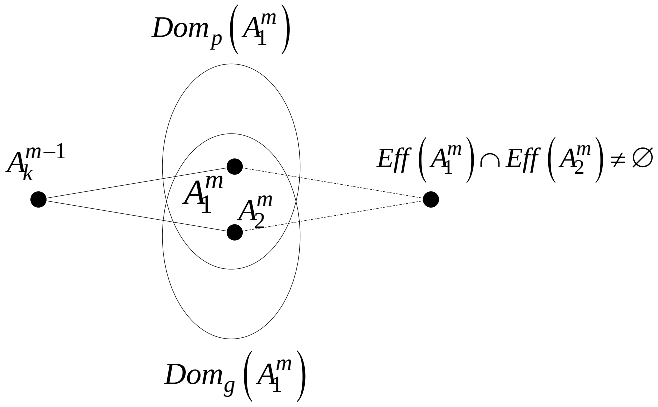

Figure 2: Theorem about the morphology of a weakly connected network

From this theorem it follows that for any self-similar network, there is a one-to-one correspondence between the vertex order number within a layer and indexes of the layer's pair of intersecting different type domains. The correspondence can be set by a tuple, for example:

$$z^m = \langle j_0 j_1 \dots j_{m-1} j_{m+1} j_{m+2} \dots j_{n-1} \rangle. \tag{2}$$

Any index permutation is allowed in this tuple.

4. Self-similar network graph

From expression (2), for layer *m-1* we have:

$$z^{m-1} = \langle j_0 j_1 \dots j_{m-2} j_m j_{m+1} \dots j_{n-1} \rangle.$$

Vertices of adjacent layers m and m-1 are connected by an arc if their afferents intersect. The afferent of vertex z^{m-1} is defined by bit numbers of the tuple $\langle i_m i_{m+1} \dots i_{n-1} \rangle$, and vertex z^m is defined by bit numbers of the tuple $\langle i_{m+1} i_{m+2} \dots i_{n-1} \rangle$, the intersection of afferents is possible only if the same named bits in the given tuples coincide. The same conclusion can be implied using the conditions for intersection of vertex's efferents for adjacent layers m and$m+1$: vertices will be connected by an arc if in the tuples $\langle j_0 j_1 \dots j_{m-1} \rangle$ and $\langle j_0 j_1 \dots j_{m-1} j_m \rangle$ the same named bits coincide. The resulting rules make it easy to build graphical image of a self-similar modular network. Figure 3 shows an example of building the four-layer self-similar network for variant where all m indexes take the values i_m={0, 1} and j_m={0, 1}.

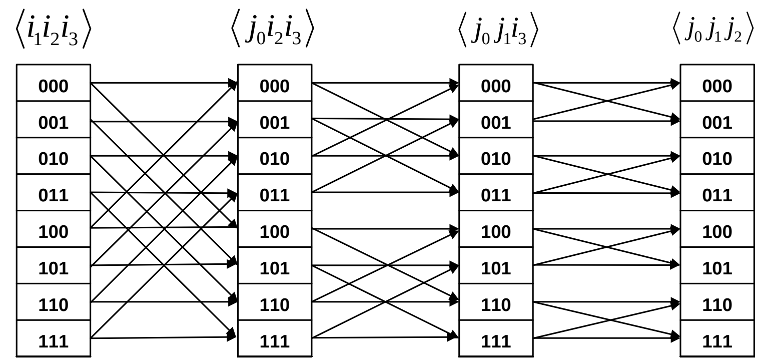

Figure 3: Four-layer self-similar modular network

The constructed model is fully described by the linguistic sentence:

$$[\langle i_1 i_2 i_3 \rangle \langle j_0 i_2 i_3 \rangle \langle j_0 j_1 i_3 \rangle \langle j_0 j_1 j_2 \rangle].$$

Here, each word of the sentence represents the tuple of the network vertex z^m. Any letter permutation is allowed in the sentence words. Denote by I_m and J_m the ordered subsets of indexes of each type in the tuple z^m, then the rule for constructing a graphical image of any n-layer self-similar network can be formulated as follow

$$m = 0,1, \dots n-1, \quad I_m \supset I_{m+1}, \quad J_m \subset J_{m+1}$$
$$I_{n-1} = \emptyset, \quad J_0 = \emptyset.$$

The resulting rule is not related with the network dimension, the structural characteristics of its modules, or topology of their links, this rule is an invariant of the morphological level of multilayer self-similar modular network [9].

5. Conclusion

The paper shows that mathematical model of self-similarity can be extended to multilayer networks. It is proved that structure of any self-similar regular network can be expressed analytically and described by a linguistic sentence. It is easy to see that the graph of the self-similar network is analogous to the fast Fourier transform graph, which is a weakly connected network also. In works [9, 10, 11, 12] algorithms for construction of weakly connected networks for fast spectral transformations are presented. The generalized genesis of nonregular weakly connected networks was considered in [8].

Self-similarity and regularity of weakly connected networks provide a unique opportunity for analytical representation of the topology for the implementing networks, which makes it possible to develop neural network training algorithms that absolutely converge in a finite number of steps. In addition, there is a variant of analytical extension of the topology of a self-similar network that leads to architectures of deep neural networks with fast absolute convergence learning algorithms [13].

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