Equilibrium and Optimality in International Trade Models under Monopolistic Competition: the Unified Approach

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Abstract

We study the homogeneous model of international trade under the monopolistic competition of producers. The utility function assumes additive separable. The transport costs are of "iceberg types". We consider both market equilibrium and social optimality and study the idea of Sergey Kokovin (NRU HSE): "the search for equilibrium is equivalent to the problem of optimization, but revenue, not utility". For the case of two countries, we show that (i) in symmetric market equilibrium, the elasticity of production costs is a convex combination of the elasticities of normalized revenue in individual consumption; while (ii) in symmetric social optimality, the elasticity of production costs is a convex combination of the elasticities of sub-utility of individual consumption. These generalize the well-known facts in closed economy under monopolistic competition: "in equilibrium, the elasticity of revenue equals the elasticity of total costs" and "in optimality, the elasticity of revenue equals the elasticity of utility". Moreover, we find that, in symmetric market equilibrium, the "inverse" elasticities of production costs is a convex combination of the "inverse" elasticities of normalized revenue in individual consumption. It turns out that the last result can be generalized in the case of international trade of several countries.

Keywords

Monopolistic Competition, International Trade, Market Equilibrium, Social Optimality

1. Introduction

The concept of monopolistic competition, introduced by Chamberlin [1], widely develops now, starting with the famous paper by Dixit and Stiglitz [2] for the case of a closed economy, by Krugman [3, 4] for the international trade and Melitz [5] for the heterogeneous case. Usually, monopolistic competition models study two concepts: (i) the *market equilibrium*, see, e.g. [6, 7, 8, 9, 10, 11, 12]; (ii) the *social optimality*, see, e.g. [13, 14, 15]. Note that Osharin and Verbus [16] used weighted inverse elasticities of substitution tackling other issues, and this idea seems to be quite natural and general.

In this paper we study a unified approach to both market equilibrium and social optimality. It can allow to clarify the nature of these concepts.

The paper is organized as follows. Section 2 lays out the model for the case of two countries and contains some preliminary considerations. Here we describe the main assumptions of monopolistic competition (Section 2.1), the consumers (Section 2.2) and producers (Section 2.3). Moreover, we consider the symmetric case (Section 2.4), define the concept of symmetric Equilibrium and symmetric Optimality (Section 2.5). In Section 3 we formulate the results for the case of international trade between two countries. In Proposition 1 the conditions are formulated that satisfy (i) in symmetric

Modeling and Analysis of Complex Systems and Processes - MACSPro'2020, October 22–24, 2020, Venice, Italy & Moscow, Russia

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market equilibrium, the elasticities of production costs and the elasticities of normalized revenue in individual consumptions; (ii) in symmetric social optimality, the elasticities of production costs and the elasticities of sub-utility of individual consumptions. It turns out that these conditions have the identical form. Therefore, the search for equilibrium is equivalent to the problem of optimization, but revenue, not utility[1]. Corollary 1 shows that (i) in symmetric market equilibrium, the elasticity of production costs is a convex combination of the elasticities of normalized revenue in individual consumption; while (ii) in symmetric social optimality, the elasticity of production costs is a convex combination of the elasticities of sub-utility of individual consumption[2]. Let us note that the obvious disadvantage of the formulas in Proposition 1 and Corollary 1 is the poor interpretability of the coefficients s^H_{equ}, s^F_{equ}, s^H_{opt}, s^F_{opt}, see (27)-(30). We hope that Proposition 2 does not have this disadvantage. It turns out that it is necessary to compare not elasticities (of costs, revenues, utility), but their "inverse" values, (1/elasticities). Due to Proposition 2, in symmetric market equilibrium, the "inverse" elasticities of production costs is a convex combination of the "inverse" elasticities of normalized revenue in individual consumption. Moreover, the coefficients of this convex combination have a clear meaning: they are the ratio of total domestic consumption to the size of the firm. Moreover, Proposition 2 can be generalized to the case of several countries, see Section 4, Proposition 3. Section 5 concludes.

2. The basic model of open economy

In this section we set the basic monopolistic competition model for open economy (international trade case case). Let be two countries, H ("big") and F ("small").

2.1. Main assumptions of Monopolistic Competition

As it is usual in monopolistic competition, we assume that (cf. [1, 2, 3])

- consumers are identical, each endowed with one unit of labor;
- labor is the only production factor; consumption, output, prices etc. are measured in labor;
- firms are identical, but produce "varieties" ("almost the same") of good;
- each firm produces one variety as a price-maker, but its demand is influenced by other varieties;
- each variety is produced by one firm that produces a single variety;
- each demand function results from additive utility function;
- number (mass) of firms is big enough to ignore firm's influence on the whole industry/economy;
- free entry drives all profits to zero;
- labor supply/demand in each country is balance;
- trade in each country is balance.

[1] As far as the author knows, this idea was first formulated several years ago by Sergey Kokovin, Center for Market Studies and Spatial Economics, National Research University Higher School of Economics.

[2] Thus Corollary 1 generalizes the well-known facts in closed economy monopolistic competition: "in equilibrium, the elasticity of revenue equals the elasticity of total costs" and and "in optimality, the elasticity of revenue equals the elasticity of utility".

2.2. Consumers

Let

- L^H be the number of consumers in country H,
- L^F be the number of consumers in country F.

As usual, we assume that $L^H \geq L^F$. Analogously, let

- N^H be the number (mass) of firms in country H,
- N^F be the number (mass) of firms in country F.

Note that L^H and L^F are parameters (the known constants) while N^H and N^F are the variables determined endogenously. Moreover, let us recall that, in monopolistic competition models, number of firms is big enough. Therefore, instead of standard "number of firms is N^H (or N^F)" we consider the intervals $[0, N^H]$ and $[0, N^F]$ with uniformly distributed firms.

Now we introduce four kinds of the *individual* consumption and prices. Let for every $k, l \in \{H, F\}$[3],

- x_i^{kl} be the amount of the variety produced in country k by firm $i \in [0, N^k]$ and consumed by a consumer in country l,
- p_i^{kl} be the price of the unit of the variety produced in country k by firm $i \in [0, N^k]$ and consumed by a consumer in country l.

Let $u(\cdot)$ be a sub-utility function. As usual, we assume that $u(\cdot)$ is twice differentiable and satisfies the conditions

$$u(0) = 0,\ u'(\xi) > 0,\ u''(\xi) < 0,$$

i.e., it is strictly increasing and strictly concave. Further, let $w^H = w$ be the wage rate in country H while the wage rate in country F be w^F normalizing to one, $w^F = 1$.

In country H, the problem of representative consumer is

$$\int_0^{N^H} u\left(x_i^{HH}\right) di + \int_0^{N^F} u\left(x_i^{FH}\right) di \longrightarrow \max$$

s.t.

$$\int_0^{N^H} p_i^{HH} x_i^{HH} di + \int_0^{N^F} p_i^{FH} x_i^{FH} di \leq w,$$

while, in country F, the problem of representative consumer is

$$\int_0^{N^F} u\left(x_i^{FF}\right) di + \int_0^{N^H} u\left(x_i^{HF}\right) di \longrightarrow \max$$

s.t.

$$\int_0^{N^F} p_i^{FF} x_i^{FF} di + \int_0^{N^H} p_i^{HF} x_i^{HF} di \leq 1.$$

Using First Order Conditions, we get the inverse demand functions

$$p_i^{kl}\left(x_i^{kl}, \lambda^l\right) = \frac{u'\left(x_i^{kl}\right)}{\lambda^l}, i \in [0, N^k], k, l \in \{H, F\}, \tag{1}$$

where λ^H, λ^F are Lagrange multipliers.

[3]Hereinafter, due to the tradition of monopolistic competition, we use the notation x_i for the function $x(i)$, etc.

2.3. Producers

To introduce the production amount of the firms (the "size" of the firm), let us introduce the parameter $\tau \geq 1$ as transport costs of "iceberg type"[4]. Each firm in each country produces for consumers in each country. Thus,

$$Q_i^H = L^H x_i^{HH} + \tau L^F x_i^{HF}, i \in [0, N^H], \tag{2}$$

is the size of firm $i \in [0, N^H]$ in country H, while

$$Q_i^F = L^F x_i^{FF} + \tau L^H x_i^{FH}, i \in [0, N^F] \tag{3}$$

is the size of firm $i \in [0, N^F]$ in country F.

Let the production costs be determined for each firm in each country by the *increasing* twice differentiable function V. Then the profits π_i^H, $i \in [0, N^H]$, of firm i in country H and π_i^F, $i \in [0, N^F]$, of firm i in country F are

$$\pi_i^H = L^H p_i^{HH} x_i^{HH} + L^F p_i^{HF} x_i^{HF} - wV\left(Q_i^H\right), i \in [0, N^H], \tag{4}$$

$$\pi_i^F = L^F p_i^{FF} x_i^{FF} + L^H p_i^{FH} x_i^{FH} - V\left(Q_i^F\right), i \in [0, N^F]. \tag{5}$$

Of course, the firms choose inverse demand functions (1) as the prices. Let us introduce so-called "revenue per consumer"

$$R_\lambda(\xi) := \frac{u'(\xi) \cdot \xi}{\lambda}. \tag{6}$$

Let us substitute (1) in (4) and (5). Using (6), we get

$$\pi_i^H = L^H R_{\lambda^H}\left(x_i^{HH}\right) + L^F R_{\lambda^F}\left(x_i^{HF}\right) - wV\left(Q_i^H\right), \tag{7}$$

$$\pi_i^F = L^F R_{\lambda^F}\left(x_i^{FF}\right) + L^H R_{\lambda^H}\left(x_i^{FH}\right) - V\left(Q_i^F\right). \tag{8}$$

The Labor Balances in countries H and F are, correspondingly,

$$\int_0^{N^H} V\left(Q_i^H\right) di = L^H, \tag{9}$$

$$\int_0^{N^F} V\left(Q_i^F\right) di = L^F. \tag{10}$$

2.4. Symmetric case

Let us recall that the consumers are assume identical, the producers are assumed identical. Thus, as usual, we consider the symmetric case. More precisely, we omit index i in consumption, inverse demand functions (1), sizes of the firms (2), (3), profits (7), (8) and Labor Balances (9), (10). This way (1)-(3) and (7)-(10) are

$$p^{kl}\left(x^{kl}, \lambda^l\right) = \frac{u'\left(x^{kl}\right)}{\lambda^l}, k, l \in \{H, F\}, \tag{11}$$

$$Q^H = L^H x^{HH} + \tau L^F x^{HF}, \tag{12}$$

$$Q^F = L^F x^{FF} + \tau L^H x^{FH}, \tag{13}$$

[4] To sell in another country y units of the goods, the firm must produce $\tau \cdot y$ units. "During transportation, the product melts like an iceberg ..."

$$\pi^H = L^H R_{\lambda^H}\left(x^{HH}\right) + L^F R_{\lambda^F}\left(x^{HF}\right) - wV\left(Q^H\right), \tag{14}$$

$$\pi^F = L^F R_{\lambda^F}\left(x^{FF}\right) + L^H R_{\lambda^H}\left(x^{FH}\right) - V\left(Q^F\right), \tag{15}$$

$$N^H V\left(Q^H\right) = L^H, \tag{16}$$

$$N^F V\left(Q^F\right) = L^F. \tag{17}$$

Moreover, the Trade Balance (“export equals import”) is

$$L^F N^H p^{HF}\left(x^{HF}, \lambda^F\right) x^{HF} = L^H N^F p^{FH}\left(x^{FH}, \lambda^H\right) x^{FH},$$

i.e., using (6), (11), (16), (17),

$$R_{\lambda^F}\left(x^{HF}\right) V\left(Q^F\right) = R_{\lambda^H}\left(x^{FH}\right) V\left(Q^H\right). \tag{18}$$

As to the Social Welfare (total utility), it is

$$U = L^H \cdot \left(N^H u\left(x^{HH}\right) + N^F u\left(x^{FH}\right)\right) + L^F \cdot \left(N^F u\left(x^{FF}\right) + N^H u\left(x^{HF}\right)\right) =$$

$$\left(L^H u\left(x^{HH}\right) + L^F u\left(x^{HF}\right)\right) N^H + \left(L^F u\left(x^{FF}\right) + L^H u\left(x^{FH}\right)\right) N^F,$$

i.e., substituting (16) and (17),

$$U = L^H \cdot \frac{L^H u\left(x^{HH}\right) + L^F u\left(x^{HF}\right)}{V\left(Q^H\right)} + L^F \cdot \frac{L^F u\left(x^{FF}\right) + L^H u\left(x^{FH}\right)}{V\left(Q^F\right)}. \tag{19}$$

2.5. Symmetric Equilibrium and Symmetric Optimality

For equilibrium, producers choose inverse demand functions (11) and maximize profits (14), (15). So First Order Conditions (FOC)

$$\frac{\partial \pi^H}{\partial x^{HH}} = 0, \quad \frac{\partial \pi^H}{\partial x^{HF}} = 0, \quad \frac{\partial \pi^F}{\partial x^{FF}} = 0, \quad \frac{\partial \pi^F}{\partial x^{FH}} = 0 \tag{20}$$

and Second Order Conditions (SOC)

$$\frac{\partial^2 \pi^H}{\partial x^{HH} \partial x^{HH}} < 0, \quad \frac{\partial^2 \pi^H}{\partial x^{HH} \partial x^{HH}} \cdot \frac{\partial^2 \pi^H}{\partial x^{HF} \partial x^{HF}} - \left(\frac{\partial^2 \pi^H}{\partial x^{HH} \partial x^{HF}}\right)^2 > 0, \tag{21}$$

$$\frac{\partial^2 \pi^F}{\partial x^{FF} \partial x^{FF}} < 0, \quad \frac{\partial^2 \pi^F}{\partial x^{FF} \partial x^{FF}} \cdot \frac{\partial^2 \pi^F}{\partial x^{FH} \partial x^{FH}} - \left(\frac{\partial^2 \pi^F}{\partial x^{FF} \partial x^{FH}}\right)^2 > 0 \tag{22}$$

hold.

Like in the standard monopolistic competition framework, the firms enter into the market until their profit remains positive. Therefore, free entry implies the zero-profit condition

$$\pi^H = 0, \qquad \pi^F = 0. \tag{23}$$

By definition, the *symmetric market equilibrium* is a bundle

$$\left(x^{HH}_{equ}, x^{HF}_{equ}, x^{FF}_{equ}, x^{FH}_{equ}, p^{HH}_{equ}, p^{HF}_{equ}, p^{FF}_{equ}, p^{FH}_{equ}, \lambda^H_{equ}, \lambda^F_{equ}, N^H_{equ}, N^F_{equ}, w_{equ}\right)$$

satisfying the following:

- the rational consumption conditions (11);
- the rational production conditions (20)-(22);
- the free entry condition (23);
- the labor balance conditions (16), (17);
- the trade balance condition (18).

For optimality, the Social Welfare (19) is maximized. So First Order Conditions (FOC)

$$\frac{\partial U}{\partial x^{HH}} = 0, \quad \frac{\partial U}{\partial x^{HF}} = 0, \quad \frac{\partial U}{\partial x^{FF}} = 0, \quad \frac{\partial U}{\partial x^{FH}} = 0 \tag{24}$$

and Second Order Conditions (SOC)

$$\frac{\partial^2 U}{\partial x^{HH} \partial x^{HH}} < 0, \quad \frac{\partial^2 U}{\partial x^{HH} \partial x^{HH}} \cdot \frac{\partial^2 U}{\partial x^{HF} \partial x^{HF}} - \left(\frac{\partial^2 U}{\partial x^{HH} \partial x^{HF}} \right)^2 > 0, \tag{25}$$

$$\frac{\partial^2 U}{\partial x^{FF} \partial x^{FF}} < 0, \quad \frac{\partial^2 U}{\partial x^{FF} \partial x^{FF}} \cdot \frac{\partial^2 U}{\partial x^{FH} \partial x^{FH}} - \left(\frac{\partial^2 U}{\partial x^{FF} \partial x^{FH}} \right)^2 > 0 \tag{26}$$

hold.

The *symmetric social optimality* is a bundle

$$\left(x_{opt}^{HH}, x_{opt}^{HF}, x_{opt}^{FF}, x_{opt}^{FH}, N_{opt}^{H}, N_{opt}^{F} \right)$$

satisfying the following:

- the rational welfare conditions (24)-(26);
- the labor balance conditions (16), (17).

3. Results

Let us introduce

$$s^H \left(x^{HH}, x^{HF}, A^H, A^F \right) := \frac{L^H A^H \left(x^{HH} \right)}{L^H A^H \left(x^{HH} \right) + L^F A^F \left(x^{HF} \right)},$$

$$s^F \left(x^{FF}, x^{FH}, A^H, A^F \right) := \frac{L^F A^F \left(x^{FF} \right)}{L^F A^F \left(x^{FF} \right) + L^H A^H \left(x^{FH} \right)},$$

where A^H and A^F are real functions.

Let us denote

$$s_{equ}^{H} = s^H \left(x_{equ}^{HH}, x_{equ}^{HF}, R_{\lambda_{equ}^{H}}, R_{\lambda_{equ}^{F}} \right), \tag{27}$$

$$s_{equ}^{F} = s^F \left(x_{equ}^{FF}, x_{equ}^{FH}, R_{\lambda_{equ}^{H}}, R_{\lambda_{equ}^{F}} \right), \tag{28}$$

$$s_{opt}^{H} = s^H \left(x_{opt}^{HH}, x_{opt}^{HF}, u, u \right), \tag{29}$$

$$s_{opt}^{F} = s^F \left(x_{opt}^{FF}, x_{opt}^{FH}, u, u \right). \tag{30}$$

As usual, let $\mathcal{E}_g(\xi) = \dfrac{g'(\xi)\,\xi}{g(\xi)}$ be the elasticity of function g. Note that

$$\mathcal{E}_{R_{\lambda^H}}(\xi) = \mathcal{E}_{R_{\lambda^F}}(\xi) = \mathcal{E}_R(\xi),$$

where $R(\xi) = u'(\xi)\,\xi$ is "normalized" revenue. Let

$$q^H_{equ} = L^H x^{HH}_{equ}, \quad q^F_{equ} = L^F x^{FF}_{equ}, \quad q^H_{opt} = L^H x^{HH}_{opt}, \quad q^F_{opt} = L^F x^{FF}_{opt}$$

be total domestic consumptions while

$$Q^H_{equ} = L^H x^{HH}_{equ} + \tau L^F \cdot x^{HF}_{equ}, \quad Q^F_{equ} = L^F x^{FF}_{equ} + \tau L^H x^{FH}_{equ},$$

$$Q^H_{opt} = L^H x^{HH}_{opt} + \tau L^F \cdot x^{HF}_{opt}, \quad Q^F_{opt} = L^F x^{FF}_{opt} + \tau L^H x^{FH}_{opt}$$

be firm sizes[5].

Note that in propositions below, we assume that market equilibrium and social optimality exist (and, moreover, are unique)[6].

Proposition 1. 1. *In symmetric market equilibrium, the elasticities of normalized revenue in individual consumptions and the elasticities of production costs satisfy the conditions*[7]

$$s^H_{equ} \cdot \mathcal{E}_R\left(x^{HH}_{equ}\right) = \frac{q^H_{equ}}{Q^H_{equ}} \cdot \mathcal{E}_V\left(Q^H_{equ}\right),$$

$$\left(1 - s^H_{equ}\right) \cdot \mathcal{E}_R\left(x^{HF}_{equ}\right) = \left(1 - \frac{q^H_{equ}}{Q^H_{equ}}\right) \cdot \mathcal{E}_V\left(Q^H_{equ}\right),$$

$$s^F_{equ} \cdot \mathcal{E}_R\left(x^{FF}_{equ}\right) = \frac{q^F_{equ}}{Q^F_{equ}} \cdot \mathcal{E}_V\left(Q^F_{equ}\right),$$

$$\left(1 - s^F_{equ}\right) \cdot \mathcal{E}_R\left(x^{FH}_{equ}\right) = \left(1 - \frac{q^F_{equ}}{Q^F_{equ}}\right) \cdot \mathcal{E}_V\left(Q^F_{equ}\right).$$

2. *In symmetric social optimality, the elasticities of sub-utility of individual consumptions and the elasticities of production costs satisfy the conditions*[8]

$$s^H_{opt} \cdot \mathcal{E}_u\left(x^{HH}_{opt}\right) = \frac{q^H_{opt}}{Q^H_{opt}} \cdot \mathcal{E}_V\left(Q^H_{opt}\right),$$

$$\left(1 - s^H_{opt}\right) \cdot \mathcal{E}_u\left(x^{HF}_{opt}\right) = \left(1 - \frac{q^H_{opt}}{Q^H_{opt}}\right) \cdot \mathcal{E}_V\left(Q^H_{opt}\right),$$

$$\left(1 - s^F_{opt}\right) \cdot \mathcal{E}_u\left(x^{FF}_{opt}\right) = \frac{q^F_{opt}}{Q^F_{opt}} \cdot \mathcal{E}_V\left(Q^F_{opt}\right),$$

$$s^F_{opt} \cdot \mathcal{E}_u\left(x^{FH}_{opt}\right) = \left(1 - \frac{q^F_{opt}}{Q^F_{opt}}\right) \cdot \mathcal{E}_V\left(Q^F_{opt}\right).$$

[5] The definition of firm sizes see in (12), (13).

[6] The questions of the existence and uniqueness of equilibrium and optimality are separate problems (often not quite simple), which is not the subject of this study.

[7] The definitions of s^H_{equ}, s^F_{equ} see in (27), (28).

[8] The definitions of s^H_{opt}, s^F_{opt} see in (29), (30).

The following Corollary generalizes the well-known facts in closed economy monopolistic competition: "in equilibrium, the elasticity of revenue equals the elasticity of total costs" and "in optimality, the elasticity of revenue equals the elasticity of utility".

Corollary 1. 1. *In symmetric market equilibrium, the elasticity of production costs is a convex combination of the elasticities of normalized revenue in individual consumption, i.e.,*

$$s_{equ}^{H} \cdot \mathcal{E}_R\left(x_{equ}^{HH}\right) + \left(1 - s_{equ}^{H}\right) \cdot \mathcal{E}_R\left(x_{equ}^{HF}\right) = \mathcal{E}_V\left(Q_{equ}^{H}\right),$$

$$s_{equ}^{F} \cdot \mathcal{E}_R\left(x_{equ}^{FF}\right) + \left(1 - s_{equ}^{F}\right) \cdot \mathcal{E}_R\left(x_{equ}^{FH}\right) = \mathcal{E}_V\left(Q_{equ}^{F}\right).$$

2. *In symmetric social optimality, the elasticity of production costs is a convex combination of the elasticities of sub-utility of individual consumption, i.e.,*

$$s_{opt}^{H} \cdot \mathcal{E}_u\left(x_{opt}^{HH}\right) + \left(1 - s_{opt}^{H}\right) \cdot \mathcal{E}_u\left(x_{opt}^{HF}\right) = \mathcal{E}_V\left(Q_{opt}^{H}\right),$$

$$s_{opt}^{F} \cdot \mathcal{E}_u\left(x_{opt}^{FF}\right) + \left(1 - s_{opt}^{F}\right) \cdot \mathcal{E}_u\left(x_{opt}^{FH}\right) = \mathcal{E}_V\left(Q_{opt}^{F}\right).$$

Let us note that the obvious disadvantage of the formulas in Proposition 1 and Corollary 1 is the poor interpretability of the coefficients (27)-(30). We hope that the proposition below does not have this disadvantage.

Proposition 2. *In symmetric market equilibrium, the elasticities of normalized revenue in individual consumptions and the elasticities of production costs satisfy the conditions*

$$\frac{q_{equ}^{H}}{Q_{equ}^{H}} \cdot \frac{1}{\mathcal{E}_R\left(x_{equ}^{HH}\right)} + \left(1 - \frac{q_{equ}^{H}}{Q_{equ}^{H}}\right) \cdot \frac{1}{\mathcal{E}_R\left(x_{equ}^{HF}\right)} = \frac{1}{\mathcal{E}_V\left(Q_{equ}^{H}\right)},$$

$$\frac{q_{equ}^{F}}{Q_{equ}^{F}} \cdot \frac{1}{\mathcal{E}_R\left(x_{equ}^{FF}\right)} + \left(1 - \frac{q_{equ}^{F}}{Q_{equ}^{F}}\right) \cdot \frac{1}{\mathcal{E}_R\left(x_{equ}^{FH}\right)} = \frac{1}{\mathcal{E}_V\left(Q_{equ}^{F}\right)}.$$

Moreover, Proposition 2 can be generalized to the case of several countries.

4. The case of several countries

Let, for K countries, $I = \{1, \ldots, K\}$. Let, for $k \in I$ and $l \in I$,

- N^k be the mass of firms in country k,
- x_i^{kl} be the amount of variety produced in country k by firm $i \in [0, N^k]$ and consumed in country l by a consumer,
- p^{kl} be the corresponding prices,
- L^k be the number of consumer in country k,
- $q_i^{kl} = \tau^{kl} L^l x_i^{kl}$ be the total output for firm $i \in [0, N^k]$ in country k for selling in country l, $\tau^{kl} \geq 1,\ \tau^{kk} = 1$,
- w^k be the wage in country k.

The problem of representative consumer in country $k \in I$ is

$$\sum_{l \in I} \int_0^{N^l} u\left(x_i^{lk}\right) di \longrightarrow max$$

s.t.

$$\sum_{l \in I} \int_0^{N^l} p_i^{lk} x_i^{lk} di \leq w^k.$$

For symmetric case, FOC is

$$u'\left(x^{lk}\right) - \lambda^k p^{lk} = 0,$$

where λ^k is the corresponding Lagrange multiplier.

For a producer in country $k \in I$, total output is

$$Q^k = \sum_{l \in I} q^{kl}.$$

Let us substitute the inverse demand function

$$p^{kl} = \frac{u'\left(x^{kl}\right)}{\lambda^l}$$

in profits. Then the profit of a producer in country k is

$$\pi^k = \sum_{l \in I} \frac{L^l}{\lambda^l} \cdot R\left(x^{kl}\right) - w^k \cdot V\left(Q^k\right).$$

(Let us recall that $R(\xi) = u'(\xi) \cdot \xi$ is "normalized" revenue.)

To define symmetric equilibrium, as usual, we write first and second order conditions, free entry conditions, labor and trade balances.

It turns out that first second order conditions and free entry conditions allow to generalize Proposition 2.

Let x_{equ}^{kl} be equilibrium consumption and

$$q_{equ}^{kl} = \tau^{kl} L^l x_{equ}^{kl}.$$

Proposition 3. *For country $k \in I$, in symmetric market equilibrium, the elasticities of normalized revenue in individual consumptions and the elasticities of production costs satisfy the condition*

$$\sum_{l \in I} \frac{q_{equ}^{kl}}{Q_{equ}^k} \cdot \frac{1}{\mathcal{E}_R\left(x_{equ}^{kl}\right)} = \frac{1}{\mathcal{E}_V\left(Q_{equ}^k\right)}.$$

5. Conclusion

In this paper, we study, in the monopolistic competition framework, the homogeneous model of international trade with additively separable utility function for each consumer.

One of the most interesting topic in these studies is the so-called "comparative statics", i.e., the influence of the models' parameters (market size, transport costs, etc.) on the equilibrium and optimal variables: consumption, firm sizes, market sizes, social welfare, etc., see, e.g. [17, 18, 19, 20, 21, 22, 23].

Instead, we study a unified approach to both market equilibrium and social optimality.

The following results are obtained.

- For the case of international trade between *two* countries,
  - in symmetric market equilibrium, the elasticities of production costs and the elasticities of normalized revenue in individual consumptions; moreover, the elasticity of production costs is a convex combination of the elasticities of normalized revenue in individual consumption;
  - in symmetric social optimality, the elasticities of production costs and the elasticities of sub-utility of individual consumptions; moreover, the elasticity of production costs is a convex combination of the elasticities of sub-utility of individual consumption;
  - in symmetric market equilibrium, the "inverse" elasticities of production costs is a convex combination of the "inverse" elasticities of normalized revenue in individual consumption; moreover, the coefficients of this convex combination have a clear meaning: they are the ratio of total domestic consumption to the size of the firm.
- The last result generalizes to the case of international trade between several countries.

Therefore, we generalize the well-known facts in closed economy monopolistic competition: "in equilibrium, the elasticity of revenue equals the elasticity of total costs" and and "in optimality, the elasticity of revenue equals the elasticity of utility". It can allow to clarify the nature of these concepts.

Finally, it would be also glad to know whether the best choice for the two economies can gives the best choice for each economy separately. This can be the topic of future research.

Acknowledgments

The author is very grateful to the anonymous reviewers for very useful suggestions for improving the text. The study was carried out within the framework of the state contract of the Sobolev Institute of Mathematics (project no. 0314-2019-0018). The work was supported in part by the Russian Foundation for Basic Research, projects 18-010-00728 and 19-010-00910.

References

[1] E. H. Chamberlin, The Theory of Monopolistic Competition: A re-Orientation of the Theory of Value, 1st. ed., Harvard University Press, Cambridge, MA, 1933.

[2] A. K. Dixit, J. E. Stiglitz, Monopolistic competition and optimum product diversity, American Economic Review 67 (1977) 297–308.

[3] P. R. Krugman, Increasing returns, monopolistic competition, and international trade, Journal of International Economics 9 (1979) 469–479. doi:10.1016/0022-1996(79)90017-5.

[4] P. R. Krugman, Scale economies, product differentiation, and the pattern of trade, American Economic Review 70 (1980) 950–959.

[5] M. J. Melitz, The impact of trade on intra-industry reallocations and aggregate industry productivity, Econometrica 71 (2003) 1695–1725. doi:10.1111/1468-0262.00467.

[6] K. Behrens, Y. Murata, General equilibrium models of monopolistic competition: a new approach, Journal of Economic Theory 136 (2007) 776–787. doi:10.1016/j.jet.2006.10.001.

[7] J. Brander, P. Krugman, A 'reciprocal dumping' model of international trade, Journal of international economics 15 (1983) 313–321. doi:10.1016/S0022-1996(83)80008-7.

[8] I. Bykadorov, S. Kokovin, Can a larger market foster r&d under monopolistic competition with variable mark-ups?, Research in Economics 71 (2017) 663–674. doi:10.1016/j.rie.2017.10.006.

[9] S. Dhingra, Trading away wide brands for cheap brands, American Economic Review 103 (2013) 2554–2584. doi:10.1257/aer.103.6.2554.

[10] R. C. Feenstra, A homothetic utility function for monopolistic competition models, without constant price elasticity, Economics Letters 78 (2003) 79–86. doi:10.1016/S0165-1765(02)00181-7.

[11] M. J. Melitz, G. I. P. Ottaviano, Market size, trade, and productivity, Review of Economic Studies 75 (2008) 295–316.

[12] E. Zhelobodko, S. Kokovin, M. Parenti, J.-F. Thisse, Monopolistic competition in general equilibrium: Beyond the constant elasticity of substitution, Econometrica 80 (2012) 2765–2784. doi:10.3982/ECTA9986.

[13] S. Dhingra, J. Morrow, Monopolistic competition and optimum product diversity under firm heterogeneity, Journal of Political Economy 127 (2019) 196–232. doi:10.1086/700732.

[14] I. Bykadorov, Monopolistic competition with investments in productivity, Optimization Letters 13 (2019) 1803–1817. doi:10.1007/s11590-018-1336-9.

[15] I. Bykadorov, Social optimality in international trade under monopolistic competition, Communications in Computer and Information Science 1090 (2019) 1803–1817. doi:10.1007/978-3-030-33394-2_13.

[16] A. Osharin, V. Verbus, Heterogeneity of consumer preferences and trade patterns in a monopolistically competitive setting, Journal of Economics 125 (2018) 211–237. doi:10.1007/s00712-018-0599-z.

[17] C. Arkolakis, A. Costinot, A. Rodríguez-Clare, New trade models, same old gains?, American Economic Review 102 (2012) 94–130. doi:10.1257/aer.102.1.94.

[18] C. Arkolakis, A. Costinot, D. Donaldson, A. Rodríguez-Clare, The elusive pro-competitive effects of trade, The Review of Economic Studies 86 (2019) 46–80. doi:10.1093/restud/rdx075.

[19] I. Bykadorov, A. Gorn, S. Kokovin, E. Zhelobodko, Why are losses from trade unlikely?, Economics Letters 129 (2019) 35–38. doi:10.1016/j.econlet.2015.02.003.

[20] M. J. Melitz, S. J. Redding, Missing gains from trade?, American Economic Review 104 (2014) 317–321. doi:10.1257/aer.104.5.317.

[21] M. J. Melitz, S. J. Redding, New trade models, new welfare implications, American Economic Review 105 (2015) 1105–1146. doi:10.1257/aer.20130351.

[22] M. Mrázová, J. P. Neary, Together at last: Trade costs, demand structure, and welfare, American Economic Review 104 (2014) 298–303. doi:10.1257/aer.104.5.298.

[23] M. Mrázová, J. P. Neary, Not so demanding: Demand structure and firm behavior, American Economic Review 107 (2017) 3835–3874. doi:10.1257/aer.20160175.