Why the Conservative Basel III Portfolio Credit Risk Model Underestimates Losses?

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Abstract

Basel II and III allow banks to use own default statistics when estimating regulatory parameters (risk-weights) for the capital adequacy ratio purpose. Bank inputs own risk estimates into the Vasicek model. It yields a distribution of credit losses. Regulator then requires a bank to take 99.9% quantile of such a distribution as a risk-measure (a risk-weight). When saying regulator we mean any Central Bank (including Bank of Russia, but no limited to it) that allow local banks to run the described approach. Having being criticized for excessive conservatism, we reveal that it still underestimates credit risk. This comes from the newly discovered fact that the default correlation may tend to co-depend with the systemic factor (for instance, with the GDP growth rate), albeit originally such co-dependence was not considered. We use the US statistics on total loans defaults for 1985-2019 to evidence the finding. The credit loss underestimation thus at least exceeds by 11% the loss estimates using the maximum (100%) correlation with the systemic factor.

Keywords 1

Basel Committee, IRB, credit risk, Vasicek, systemic factor, default correlation, crisis

1. Introduction

The Basel Committee on Banking Supervision (BCBS) sets the standards for banks in order to assure their solvability. One of key metrics is the capital adequacy ratio (CAR). It benchmarks the bank capital to the amount of risk-weighted assets (RWA). The committee requires banks via local supervisors to hold the ratio no less than a predefined minimum. It varies depending on the definition of capital. For instance, for the total capital the ratio should be no less than 10.5% of RWA [3]. Generally speaking, RWA is the product of a risk-weight (RW) and the amount of assets (put it simply, loans).

Originally in Basel I in 1988 the BCBS prescribed the risk-weight. Later Basel II in 2006 allowed banks to compute risk-weights if they passed the regulatory test (validation). Such an approach is called an internal ratings-based (IRB) one. The IRB approach is grounded on the Vasicek [1] model. It generates loss distribution. The BCBS requires banks to take 99.9% quantile of such a distribution as an IRB credit risk estimate. However, our objective is to show that mere 99.9% quantile may not result in adequately conservative results.

The reader may ask why he/she should care about the Vasicek model, specifically, if it is 33 years old in 2020. Two key facts prove why this model is indeed vital for the world financial system. First, the BCBS has explicitly included the formula of the model in legislation, in prudential standards (this can be a mostly unprecedented legal case when the law has formulas with the probability distribution functions, not limited to basic arithmetic operations). Second, credit risk for the one third of the world banking assets is modeled using this model. Let us briefly explain how we obtain such a proxy

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estimate. The world 13 mostly largest out of 30 global systemically important banks (G-SIBs, [3]) run IRB [4, p. 28]. We proxy all G-SIBs' assets to the amount of total exposure metrics used by the BCBS.[2] The G-SIBs' total assets then equal to EUR 50 trln in 2018. 69% of the G-SIBs' credit risk assets depend upon IRB [4, p. 28]. Credit risk stands for an average of 84% of the total banking risk (risk-weighted assets, RWA) ([5, p. 49] , [6, p. 62], [7, pp. 8, 66], [8, p. 73], [9, p. 31]). Let us roughly equate banking RWA to the total banking assets. Then this yields us that IRB coverage was: EUR 50 trln of total assets * 84% of credit risk * 69% of IRB coverage = EUR 29 trln in 2018 world-wide at least. We should remember that 2k banks world-wide run IRB [10].

The amount of total banking assets can be proxied as the amount of the broad money (cash and credit money created from cash). Latter one was 123% of the world GDP for 2018, whereas the world GDP equaled to USD 86 trln. The average exchange rate in 2018 was USD 1.18 for EUR 1. Then we have that the total banking assets were USD 86 trln * 123% / USD 1.18 for EUR 1 = EUR 90 trln in 2018. So far, we may conclude that the IRB coverage stands for EUR 29 trln / EUR 90 trln = 32% of the total banking assets world-wide, or 32% * 123% = 40% of the world GDP for 2018.

Alternatively, such asset volume stands for 40% of the world GDP. When we discuss credit risk underestimation, we suggest the reader to apply those estimated to the mentioned asset base to feel its importance.

To prove this we briefly remind the baseline Vasicek [11] model in section 2. Then we describe the US data in section 3. It allows us to see that one of the core assumptions of the model is violated. We then proceed to section 4 where we suggest a revision to the baseline model. We show implications for the credit risk measurement and regulation. Section 5 concludes the paper.

2. Modeling Methodology

The baseline Vasicek [1] model assumes that systemic factor Y and the idiosyncratic one ε_i determine the asset return r_{Ai} of a bank (corporate) borrower, i.e.

$$r_{Ai} = Y\sqrt{R_i} + \varepsilon_i\sqrt{1 - R_i}, \tag{1}$$

where $r_{Ai} \sim N(0;1)$; $N(0;1)$ – the cumulative distribution function for standard normal (Gaussian) distribution with mean zero and variance of one; and $N^{-1}(\quad)$ is it's inverse;

$Y \sim N(0;1)$ – the *systemic (common) factor* that co-depends with asset value, i.e.,

$$\mathrm{Cor}(A_i; Y) = R_i \tag{2}$$

where $R_i \in [0;1]$ is the value of 'asset correlation'. Please, pay attention that Vasicek did not use lower subscript 'i' for the asset correlation value as he considered homogenous loan pool with infinite number of loans. It is the BCBS who added it later. For comfort, we wish to keep the subscript from start.

A notable fact is that Vasicek never provided an example of such a systemic factor Y. Neither the BCBS does this [13]. Nevertheless, the researchers try to proxy it. Some of them try using stock market index [14]. Others use the Gross Domestic Product (GDP) or its growth rate [15], [16, pp. 847, table 5], [17, pp. 343, table 5].

$\varepsilon_i \sim N(0;1)$ – the *idiosyncratic factor* that co-depends with asset value, i.e.,

$$\mathrm{Cor}(A_i; \varepsilon_i) = 1 - R_i \tag{3}$$

ε_i is independent both to the systemic factor, i.e., $\mathrm{Cor}(Y; \varepsilon_i) = 0$, and to the other idiosyncratic factors, i.e., $\mathrm{Cor}(\varepsilon_i; \varepsilon_j) = 0$ given $\forall j{:}\ i \neq j$.

A default case is called when the asset value breaches the debt held by the borrower. This results in the following probability of not paying back the loan where

$$p_i(PD_i; \alpha) = N\left(\frac{N^{-1}(PD_i) + N^{-1}(1-\alpha)\sqrt{R_i}}{\sqrt{1-R_i}}\right) \tag{4}$$

[2] https://www.bis.org/bcbs/gsib/gsib_assessment_samples.htm; https://www.bis.org/bcbs/gsib/hl_ind_since_2013.xlsx

where $\propto$ is the confidence level (quantile of the systemic risk factor realisation). BCBS sets it to 0.1%, or $1-\alpha=99.9\%$. For formula derivation and already embedded shortcomings' discussion, please, refer to [3]. Several researchers claimed that such a level yields excessively high portfolio credit risk losses estimates [4, p. 221], [5, p. 93], [2].

Here we would like to briefly explain the essence of the above formula (4). When there is no correlation of asset value and systemic factor, i.e. $\mathrm{Cor}(A_i;Y)=R_i=0$, the probability of not paying back the loan equals to the marginal default probability, $p_i(PD_i;\propto)=PD_i$. Otherwise, for the case of positive asset correlation, there is a 'bonus' to the marginal default probability depending upon the tightness of codependence R_i.

Originally Vasicek [6] does not define R_i. We may consider that he deemed it constant. Nevertheless, the regulator (it can be any Central Bank, including, but not limited to the Bank of Russia) adjusts it by introducing the following dependence and it restricts the weight (asset correlation) to belong to the range of [2%; 30%], see Annex 1 in [3].:

$$\mathrm{Cor}(PD_i;R_i)<0 \text{ or equivalently } \mathrm{Cor}(R_i;\varepsilon_i)>0 \tag{5}$$

The reader may argue that such a drastic violation of the model assumption should imply the decision to ultimately abandon the model for the prudential use. We do share such reader's opinion. However, the BCBS proceeds with such an adjustment. Even more, by 2020 we know that one third of the world banking assets' credit risk evaluation depends upon this model.

Same time the regulator does not modify any assumptions referring to the correlation with the systemic factor. In fact, this may turn out to be wrong as we are to show below. Such a deficiency may lead to at least 10% credit risk underestimation. To see this let us examine real-world data in the next section.

To do so we need to define the asset correlation. As Penikas [4] shows, we should focus on the default correlation r_i , where $R_i=r_i^2$. Then the default correlation may be evaluated as follows:

$$r_i=\frac{\mathrm{Var}(DR)}{\overline{DR}\cdot(1-\overline{DR})} \tag{6}$$

where $Var(DR)$ is the historical variance of the default rate for the asset (exposure) class that incorporates the *i*-th borrower; $\overline{DR}$ is the mean default rate for the respective time period for the same asset class.

Thus the objective is to check the hypothesis whether there is a dependency of default correlation r_i and the systemic factor Y. To remind, Basel II and III assumes it to be nil.

3. Data Evidence

Default rate is the basis to develop the probability of default model according to [5]. That is why its dynamics reflects the default correlations that we are interested in. We tried to find the longest time series of default rates in terms of data points to verify our hypothesis. Publicly available rating agencies data of Moody's and Standard and Poor's dates back to 1981, but has yearly frequency. The data on the US total loans delinquency starts from 1985, but has quarterly frequency. Thus we chose to proceed with the US data. The corresponding default rate historical dynamics is given below. The highest level was demonstrated in the world financial crisis of 2007-09, second in height were the times post savings and loan crisis of 1980s in the USA.

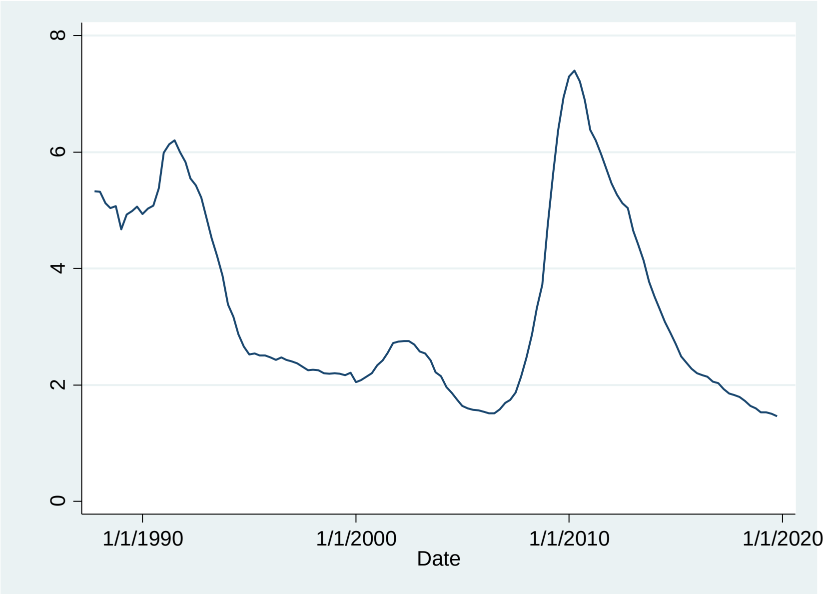

Figure 1: Historical evolution of the default rate on the US total loans.
Source: https://www.federalreserve.gov/releases/chargeoff/delallsa.htm

We proceed to analyzing codependence with the systemic factor. We proxy the systemic factor with the gross domestic product (GDP) quarterly growth rate. We take notional amount of GDP for the US as it has the corresponding depth of time series (the data for the real GDP is much shorter).[3] First, we wish to check whether there is a codependence of the systemic factor and the probability of default, or the default rate. The figure below rejects such a hypothesis. This coincides with the assumptions from [1].

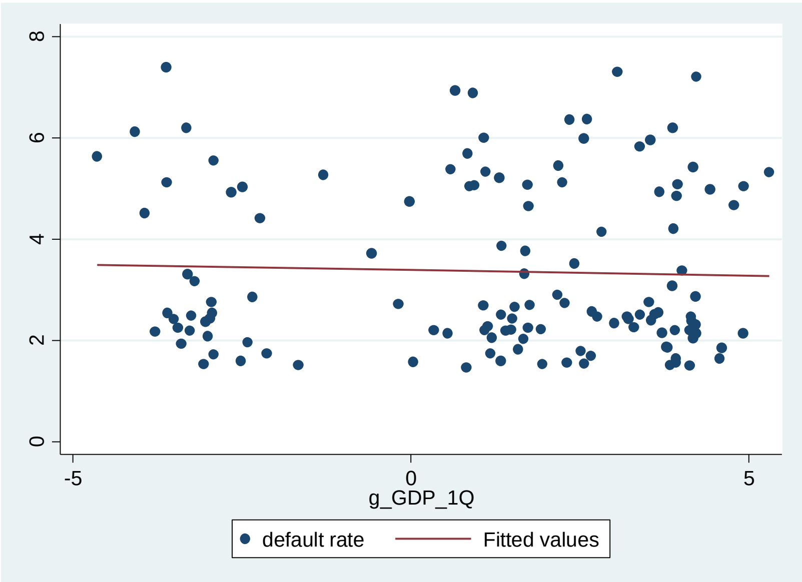

Figure 2: Systemic factor seems to be independent to the default rate.

Second, we extract default correlation using the above mentioned formula (6) and benchmark it against GDP growth as a systemic factor. We use a rolling window of two years (eight quarters, 8Q) for both variables to have negative GDP growth rates. In addition, we check that lower window size does not capture the dependency we discuss (we present regression calibration for such a case below

[3] URL: https://fred.stlouisfed.org/series/NA000334Q; code - NA000334Q.

for comparison). Larger window size results in significant data sample shrinkage and we do not consider it.

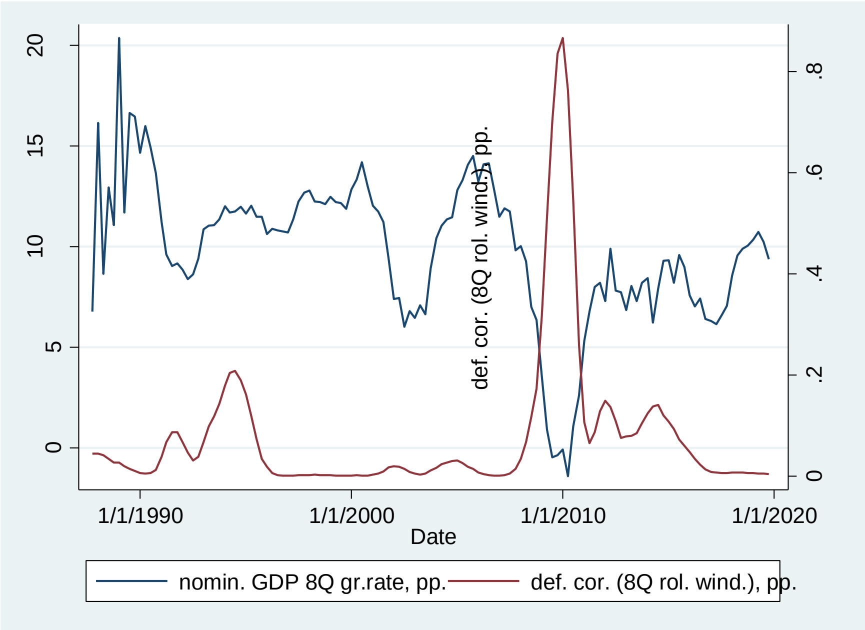

Figure 3: Default correlation R counter-depends with the systemic factor.

R here and further on stands for the default correlation r_i.

We may take a look at the dependence in the scatter plot format in addition to the time perspective. The below figure shows that the negative dependence is mostly driven by the 'outliers' observed during economic crisis times.

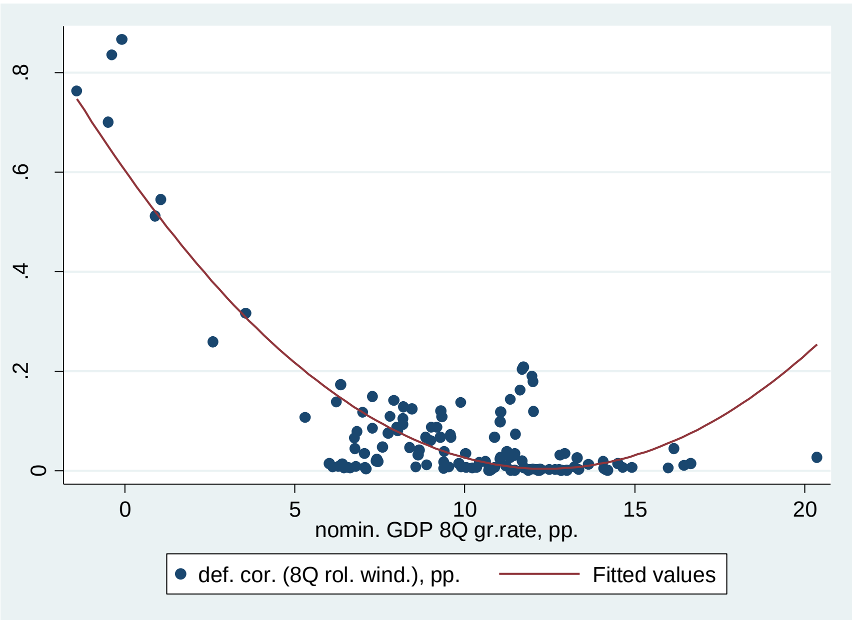

Figure 4: Default correlation rises in economic crisis times.

Thus we cannot reject the hypothesis that there is no dependence of default correlation and the systemic factor, i.e.:

$$\mathrm{Cor}(\mathrm{R_i}; \mathrm{Y}) < 0 \tag{7}$$

3.1.1. Regression Output

To model the observed dependence we run the descriptive regression model and present its outcome in Table 1 below. The model adjusted R-squared is 46%. Using a shorter window (e.g. one quarter) does not reveal the dependence of default correlation and systemic factor, see Table 2.

Table 1
Default Correlation Dependency with the Systemic Factor (8Q window).

R	Coef.	Std. Err.	t	P>|t|	[95% Conf. Interval]	
g_GDP_8Q	-0.09698	0.005474	-17.71	0	-0.10781	-0.08614
g_GDP_8Q_sq	0.003919	0.000298	13.16	0	0.00333	0.004508
_cons	0.603465	0.02644	22.82	0	0.55114	0.655789

Table 2
Default Correlation Dependency with the Systemic Factor (1Q window).

R	Coef.	Std. Err.	t	P>|t|	[95% Conf. Interval]	
g_GDP_8Q	-0.00403	0.005195	-0.78	0.44	-0.01431	0.006254
g_GDP_8Q_sq	8.12E-05	0.002168	0.04	0.97	-0.00421	0.004372
_cons	0.082749	0.023042	3.59	0	0.037151	0.128348

The resulting residuals' distribution from the model for the 8Q-window is close to the normal one, see Figure 5.

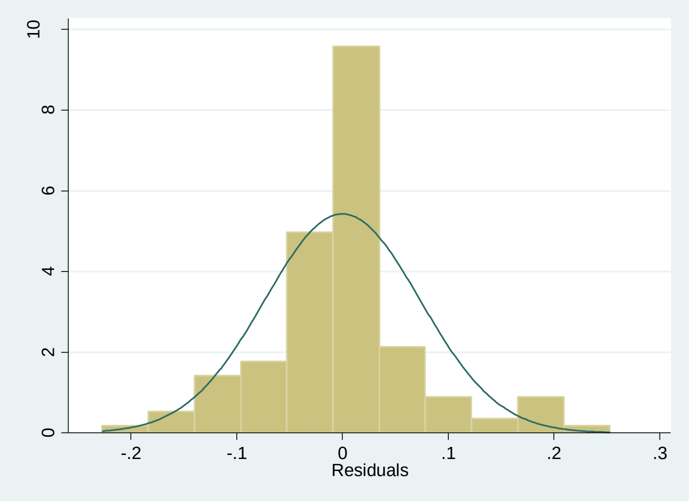

Figure 5: Regression Model Residuals Distributions.

Added-value plots (avplots in Stata software) confirm strong factor significance, see Figure 6.

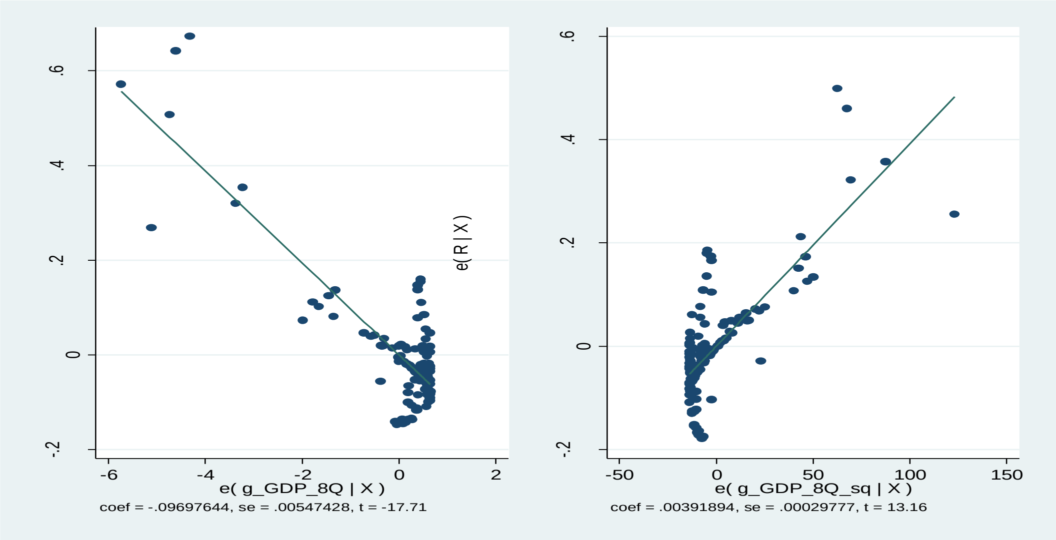

Figure 6: Added-Value Plots for the Regression Model.

Our finding is particularly striking for the following reason. The default correlation by definition implies that there is a concentration of two event types: defaults and non-defaults. Defaults occur mostly in crisis and non-defaults take place in normal times. When the default correlation rises in crises, this means that the defaults are observed even more often than expected against the presence of the positive default correlation and even much more compared to the zero default correlation. Such a situation of default correlation rise in crisis was coined by Longin and Solnik [6]. They seem to be the first, to the best of author's knowledge, to demonstrate that equity quotes tend to fall in a more synchronous manner than they tend to rise. This gives the basis to apply Clayton copula for modeling joint probability distributions [7]. Longin and Solnik studied the consequences of the Asian 1997 crisis. Comparable finding is available in Andrievskaya and Penikas [11]. They demonstrate that risk correlation was high within the Russian banking system in 2000-2004 when the overall sovereign credit risk level was high (more specifically, when Russia as a country had a speculative credit rating – below BBB- in Standard&Poors scale- from the world leading credit rating agencies). However, when Russia was promoted to the investment grade in September 2005, the risk correlation decreased. A more recent finding is delivered by the Bank of International Settlement representatives [12]. In July 2020 Aramonte and Avalos demonstrated that the default risk correlation rose to 60% because of pandemics and lockouts lasting first half of the year. For comparison, they state that in normal times their indicator was around 10% and even during the world financial crisis of 2007-09 it did not exceed 40%. Let us then discuss how such a revealed dependence may impact regulatory risk estimates.

4. Vasicek Loan Portfolio Loss Model Revision

As of now, we see that we need to incorporate two stylized facts into the [1] model that were unforeseen from start:

1. Positive correlation of the idiosyncratic factor ε_i and the default (asset) correlation. This is a requirement from the regulator, see formula (5).
2. Negative correlation of the systemic factor Y and default (asset) correlation, see formula (7).

To amalgamate the two requirements we suggest the following modification of the asset distribution formula:

$$A_i = Y \cdot r_i + \varepsilon_i \cdot (1 - r_i), \tag{8}$$

where $r_i = N(\varepsilon_i - Y)$ is the normal distribution cumulative density function. For the lowest values of Y, the weight is the largest reflecting positive co-dependence. The lowest values of ε_i (the higher marginal default probabilities) are assigned close to zero weight all other things being equal.

The figure below presents the distributions for the asset value from the baseline [1] model (case A) and modified version from formula (8) (case B). The key finding is that the distribution goes to the left, i.e. lower values of the asset value are observed more often implying more frequent cases of not paying back the loan. Part II of the below figure compares the modified model to the worst case as it seemed before, i.e. to asset correlation of R=100%. In fact, it implies normal distribution of asset values reflecting the Gaussian distribution of the systemic factor.

However, the modified model from formula (8) incorporates the observed negative dependence of default correlation and systemic factor. This means that the asset value goes down more often either when systemic factor has negative values with higher weight, or when the marginal default probability is high and it is also assigned higher weight compared to the baseline.

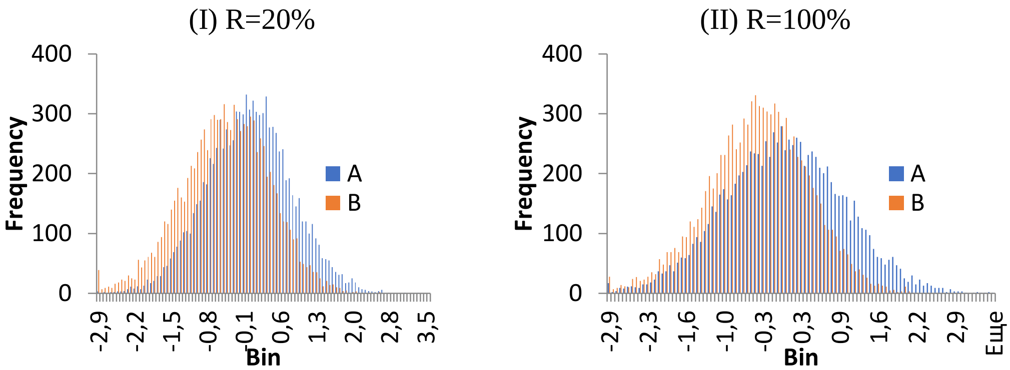

Figure 7: Loss distribution for baseline and adjusted models.

More precisely, the table below compares the quantiles of the loss distributions. It shows that for the mean values of asset correlation imposed by the regulator (e.g. R=20%), the loss underestimation is around 30% where as it is at least 10% when the most severe case of R=100% is considered (please, check,

Table 3).

Table 3
Quantile estimates for the baseline Vasicek model and its modification.

	Vasicek model [1]			Benchmark of the proposed approach to the Vasicek one [1]	
quantile	**R=20%**	**R=100%**	**Revised model**	**R=20%**	**R=100%**
0,1%	-2,65	-3,05	-3,20	1,21	1,05
0,5%	-2,20	-2,62	-2,76	1,25	1,05
1,0%	-1,97	-2,33	-2,55	1,30	1,10
5,0%	-1,37	-1,65	-1,90	1,39	1,15
10,0%	-1,06	-1,28	-1,55	1,46	1,21
			MEAN	**1,32**	**1,11**

Thus, the finding of the paper suggests that 100% asset correlation is not the worst case for any longer.

5. Conclusion

Basel IRB credit risk regulation uses a model that combines the impacts of systemic and idiosyncratic factors. The model is often blamed for over-conservative credit risk treatment as it requires systemic risk realization at the 99.9% confidence level [2]. However, no one before us has shown that there is statistically significant negative dependence of systemic risk factor realization and default correlation. By accounting for this finding we arrive at the credit loss distribution that is shifted to the left, i.e. to the more severe losses domain. This implies that actual losses might occur even larger than those prescribed by the 99.9% confidence level. It means such a model de facto is not as conservative as claimed before. Path to model adjustment is suggested together with the impact assessment. We find that the credit risk underestimation is at least 10% if we consider the worst case of 100% asset correlation and is up to 30% when benchmarked to the current asset correlation levels prescribed by the BCBS [8].

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