Model of General Equilibrium in Multisector Economy with Monopolistic Competition and Hypergeometric Utilities

Vasily M. Goncharenko[a], Alexander B. Shapoval[a] and Larisa V. Lipagina[b]

[a]*National Research University Higher School of Economics, 20 Myasnitskaya Ulitsa, Moscow 101000, Russia*
[b]*Financial University, Leningradsky pr., 49, 125167, Moscow, Russia*

Abstract

We consider a general equilibrium model in a multisector economy with n high-tech sectors where single-product firms compete monopolistically producing a differentiated good. Homogeneous sector is characterized by perfect competition. Workers attempt to find a job in high-tech sectors because of higher wages. However, it is possible for them to remain unemployed. Wages of employees in high-tech sectors are set by firms as a result of negotiations. Unemployment persists in equilibrium by labor market imperfections. All the consumers have identical preferences defined by a separable utility function of general form. We find the conditions such that the general equilibrium in the model exists and is unique. The conditions are formulated in terms of the elasticity of substitution between varieties of the differentiated good. We show that basic properties of the model can be described using families of hypergeometric functions.

Keywords

General equilibrium, monopolistic competition, variable elasticity of substitution, general utility function, hypergeometric functions

1. Introduction

In this paper we study a model of a multi-sector economy with monopolistic competition in n high-tech (industrial) sectors and perfect competition in the agricultural (traditional) sector. In our model the consumer preferences are described by utility functions of a general type. If certain conditions are met for elasticity of substitution between goods in industrial sectors, the existence of a symmetrical general equilibrium can be proved in the model under the assumption that workers are mobile within their sectors, but cannot move from sector to sector.

Let us note that modeling of monopolistic competition is related to balancing between the need to complicate the basic models to obtain adequate theoretical predictions and the possibility to get analytical results. In particular, there is a natural question about the choice of the consumer utility function. Going beyond the utility with constant substitution elasticity introduced in [1], which does not allow describing the effects of market size, special utility functions were used in [2], [3]. As a result, a deep theoretical analysis of the proposed structural models was obtained. In [4], [5] the analytical approaches were developed to build a general equilibrium in economies where consumers are empowered by separable preferences of a general type. However, the extension of their approaches to the case of multi-sector economics, when the questions of trade or the heterogeneity of economic agents are addressed, meets serious analytical difficulties. To identify the effects related to variable

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vgoncharenko@hse.ru (V.M. Goncharenko); abshapoval@gmail.com (A.B. Shapoval); llipagina@fa.ru (L.V. Lipagina)

elasticity of substitution (VES) between products, it is sufficient to introduce a non-specific separable utility function at the lower level and consider the Cobb-Douglas utility at the upper level. This is what we do in this work.

We are going to show that the basic properties of monopolistic competition models are described using utility functions, which are a family of hypergeometric functions. We will demonstrate our idea using the general equilibrium model which shows the dependence of the employment structure on consumer demand and easily extends to other models.

Hypergeometric functions and series, as well as their generalizations, play an important role in many fields of modern applied mathematics. Although they first appeared in the middle of the seventeenth century as elementary generalizations of converging series for the sum of a decreasing geometric progression, and then were used by L. Euler in the study of solutions of differential equations, now hypergeometric functions can be found in a broad range of mathematical knowledge varying from algebraic topology to theoretical economics. A modern overview of applied research where these functions and their properties play a key role can be found, for example, in [6].

A power series of the form

$$1+\frac{ab}{c}\frac{z}{1}+\frac{a(a+1)b(b+1)}{c(c+1)}\frac{z^2}{2!}+\frac{a(a+1)(a+2)b(b+1)(b+2)}{c(c+1)(c+2)}\frac{z^3}{3!}+\dots. \tag{1}$$

is called the *hypergeometric series* of the complex variable $z \in \mathbb{C}$, depending on the parameters $a, b \in \mathbb{C}$ and $c \in \mathbb{C} \setminus \{0, -1, -2, \dots\}$ (see [7, 8]).

If the complex plane is cut along the ray $[1; +\infty)$, then the series (1) defines an analytical function in the complex plane, which is denoted by ${}_2F_1(a, b; c; z)$ and is called *hypergeometric function.* The indexes in the entry ${}_2F_1(a, b; c; z)$ denote the number of parameters in the numerator and denominator of the power series coefficients (1), respectively. Let us remark that *generalized hypergeometric functions* ${}_pF_q(a_1, \dots, a_p; b_1, \dots, b_q; z)$ are also used in applied economics problems (see, for example, [8])

$${}_pF_q(a_1,\dots,a_p;b_1,\dots,b_q;z)=\sum_{n=0}^{\infty}\frac{(a_1)_n\dots(a_p)_n}{(b_1)_n\dots(b_q)_n}\cdot\frac{z^n}{n!}=\sum_{n=0}^{\infty}\frac{\prod_{k=1}^{p}(a_k)_n}{\prod_{k=1}^{q}(b_k)_n}\cdot\frac{z^n}{n!},$$

where $(c)_n$ – the Pohhammer symbol that is calculated according to the rule

$$(c)_n=\prod_{k=0}^{n-1}(c+k)=c(c+1)\dots(c+n-1).$$

It is easy to see that the row for ${}_0F_0(z)$ is an exponent, and ${}_1F_0(a; z)$ is a row for $(1-z)^{-a}$. Therefore, the hypergeometric functions are often considereded as generalizations of the exponential ones. Further, for simplicity of notation when working with hypergeometric functions, indexes will be omitted: $F(a, b; c; z) = {}_2F_1(a, b; c; z)$.

Applications of hypergeometric functions in economics are also diverse. Thus, solutions for generalizations of the Sollow model of economic growth can be written in terms of hypergeometric functions (see [9]). Hypergeometric functions play a key role in describing solutions to the Uzava-Lucas model of endogenous growth in a two-sector economy, [10]. In finance, the variety of solutions of the Black-Scholes model containing all previously known analytically solvable cases were obtained (see [11]) as well as new families of solutions.

Note that we are not the first to use hypergeometric functions in models of monopolistic competition. For example, in a recent paper [12], hypergeometric functions set examples of demand functions in which the distribution of firm productivity coincides with the distribution of sales. But it is we who propose to use a family of such functions in order to exhaust all the studied values of the elasticity of substitution between products.

The use of a family of hypergeometric functions allows, on the one hand, to use the accumulated methods of their research in analytics, and, on the other hand, allows us to draw conclusions that are stable with respect to the variation of the utility function.

2. Model

Supply. Let us consider an economy consisting of n hi-tech sectors where N_i ($i = 1, \dots, n$) single-product firms monopolistically compete in each sector, and a homogeneous sector with the perfect competition. Due to the perfect competition firms price their products at the marginal cost in the homogeneous sector. Prescribing the index 0 to this sector, we assume that prices p_0 are equal to wages w_0 as productivity can be assigned to 1.

In the i-th hi-tech sector a firm produces a good ξ_i with sector specific fixed costs c_i^{φ} and variable costs associated with wages $w(\xi_i)$ of employees working with an inverse productivity c_i^{v}. The price $p(\xi_i)$ can be found from the optimization problem to maximize the profit $\pi(\xi_i)$

$$\pi(\xi_i) = p(\xi_i)Q(\xi_i) - c_i^{v} Q(\xi_i)w(\xi_i) - c_i^{\varphi} w(\xi_i) \longrightarrow \max, \tag{2}$$

where $Q(\xi_i)$ is the aggregate demand for the good ξ_i by all the consumers in the economy. Labor is supposed to be the only production factor and a firm hires

$$l(\xi_i) = c_i^{v} Q(\xi_i) + c_i^{\varphi} \tag{3}$$

workers to produce $Q(\xi_i)$ goods.

The mass N_i of firms in the i-th sector is defined by the free entry condition $\pi(\xi_i) = 0$. In each hi-tech sector we have L_i employed and L_i^u unemployed workers. If L_0 is a number of workers in the homogeneous sector and L_{n+1} is the total number of unemployed, that is,

$$L_{n+1} = \sum_{i=1}^{n} L_i^u,$$

then the total number $\mathcal{L}$ of workers in the economy

$$\mathcal{L} = L_0 + L_{n+1} + \sum_{i=1}^{n} L_i.$$

Demand. The aggregate demand $Q(\xi_i)$ for a good ξ_i is made of individual preferences of consumers. Their incomes depend on the sector where they work. So, there are $n + 2$ types of the incomes y_i, $i = 0, 1, \dots, (n + 1)$ in the economy. We suppose that a consumer with an income y_j decides upon her demand in two steps.

First, she differentiates between production sectors represented by consumption indices H_{ij}, $i = 0, \dots, n$, maximizing the upper-tier utility

$$U = H_{0j}^{\beta_0} H_{1j}^{\beta_1} \dots H_{nj}^{\beta_n}. \longrightarrow \max, \tag{4}$$

Here the exponents $\beta_i \in (0,1)$, $i = 0, 1, \dots, n$ are such that $\sum\limits_{i=0}^{n} \beta_i = 1$. Because of Cobb-Douglas form of utility the consumers spend their income proportionally to the exponents β_i that is, a consumer with income y_j spends $\beta_i y_j$ for the good of the i-th sector.

Second, each consumer from the j-th sector where $j = 0, \dots, n+1$ chooses the demand $q_j(\xi_i)$ for each $i = 1, \dots, n$ maximizing the consumption index

$$H_{ij} = \int\limits_{N_i} u_i\left(q_j(\xi_i)\right) d\xi_i \longrightarrow \max, \tag{5}$$

with a low-tier four times differentiable increasing concave utility function $u_i(\varkappa)$, Optimization problem (5) subject to budget constraint

$$\int\limits_{N_i} p(\xi_i) q_j(\xi_i) d\xi_i \le \beta_i y_j \tag{6}$$

reflects preferences for the i-th sector differentiated good.

Elasticity of substitution between hi-tech varieties. The function

$$\sigma_i(\varkappa) = -\frac{u_i'(\varkappa)}{u_i''(\varkappa)\varkappa}$$

to be interpreted *the elasticity of the substitution* between hi-tech goods turns out to be key part to write down the solution of the consumer's problem (5), (6). Namely, its first order conditions with respect to the price $p(\xi_i)$ for a particular good ξ_i are

$$\mathrm{E}_{p(\xi_i)} q_j(\xi_i) = -\sigma_i(q_j(\xi_i)), \tag{7}$$

so that the elastisity $\mathrm{E}_{p(\xi_i)} q_j(\xi_i)$ of the demand $q_j(\xi_i)$ with respect to its price $p(\xi_i)$ is opposite to the elasticity of substitution.

Exploring a multi-sector economy, we introduce the mean value of individuals' elasticity of substitution

$$\mathfrak{S}(\xi_i) = \sum_{j=0}^{n+1} \frac{q_j(\xi_i) L_j}{Q(\xi_i)} \sigma_i(q_j(\xi_i)) \tag{8}$$

between varieties (MES) where $Q(\xi_i)$ is the aggregated demand $Q(\xi_i) = \sum\limits_{j=0}^{n+1} q_j(\xi_i) L_j$. One can easily check that property (7) is extended to $\mathfrak{S}(\xi_i)$:

$$\mathrm{E}_{p(\xi_i)} Q(\xi_i) = -\mathfrak{S}(\xi_i), \tag{9}$$

Labor Market. Workers are motivated to find a job in the hi-tech sectors as wages there are larger. Nevertherless, to be employed in each sector including homogeneous one, they have to get some sector specific skills. So, workers should choose a sector they would like to get a job, get required skills, and then enter the sector job market. As soon as the sector is chosen they can't change their choice. Let us remark that an arbitrary number of workers can be employed in the homogeneous sector. In any hi-tech sector firms hire the required number of employees while the other become to be unemployed. We assume that the labor market exhibits some frictions, so that rejected candidates cannot find a job in another sectors as they are not qualified for it. Following Stole and Zwiebel [13], one can write out the wages $w(\xi_i) = w_i$ obtained as a result of bargaining:

$$w(\xi_i) = \left(\frac{p(\xi_i)}{c_i^v} + w_0 \right) \frac{l(\xi_i)^2 - (c_i^\varphi)^2}{2l(\xi_i)^2}. \tag{10}$$

We assume that workers' choice of the labor market is balanced on average. As the probabilities to be employed and unemployed respectively in sector i, $i = 1, \dots, n$, are $L_i/(L_i + L_i^u)$ and $L_i^u/(L_i + L_i^u)$ they face identical *expected* incomes that coincide with the incomes got by workers employed in the homogeneous sector:

$$\frac{y_i L_i}{L_i + L_i^u} + \frac{y_{n+1} L_i^u}{L_i + L_i^u} = y_0. \tag{11}$$

A flat tax with some rate $\alpha \in (0, 1)$ is applied to the wages of all employed workers (including workers employed in the homogeneous sector) and distributed equally between unemployed agents as an unemployment benefit. This sets the (netto) income of employed in the hi-tech sectors, employed in the homogeneous sector, and unemployed workers to, respectively,

$$y_i = (1 - \alpha) w_i, \; i = 1 \dots, n, \quad y_0 = (1 - \alpha) w_0,$$
$$y_{n+1} = \frac{\alpha (L_0 w_0 + L_1 w_1 + \dots + L_n w_n)}{L_{n+1}}. \tag{12}$$

3. Definition of Equilibrium and Assumptions

The set of prices $\{\hat{p}(\xi_i)\}$, individual demands $\{\hat{q}_j(\xi_i)\}$, firms' outputs $\{\hat{s}(\xi_i)\}$, the number $\hat{N}_i$ of firms, wages $\hat{w}_i$, incomes $\hat{y}_j$, the numbers $\hat{L}_i$ of workers in each sector, and the number $\hat{L}_i^u$ of rejected job market candidates in hi-tech sectors ($i = 1, \dots, n$, $j = 0, 1, \dots, n+1$, $\xi_i \in [0, \hat{N}_i]$) constitute a *general equilibrium*, if the following conditions are satisfied:

- For any fixed $j = 0, 1, \dots, n+1$, individual demands $\{\hat{q}_j(\xi_i)\}_{\xi_i \in [0, N_i], i=1,\dots,n}$ solve a consumer's problem (4)–(6) with $p(\xi_i) = \hat{p}(\xi_i)$, $N_i = \hat{N}_i$, $y_j = \hat{y}_j$, $i = 1, \dots, n$, $\xi_i \in [0, \hat{N}_i]$;
- For any fixed $i = 1, \dots, n$, and $\xi_i \in [0, \hat{N}_i]$ a firm's optimization problem with respect to a single price $p(\xi_i)$ is defined by Equation (2), where $w(\xi_i) = \text{const} = \hat{w}_i$ and $Q(\xi_i) = \sum_{j=0}^{n+1} q_j(\xi_i) \hat{L}_j$ depends implicitly on $p(\xi_i)$. Namely, for any fixed $j = 0, 1, \dots, n+1$, we define individual demands $\{q_j^*(\eta_k)\}_{\eta_k \in [0, \hat{N}_k], k=1,\dots,n}$ as the solutions of a consumer's problem (4)–(6) with
  - the price for the specific variety ξ_i equalled to $p(\xi_i)$,
  - the equilibrium prices for the other varieties: $p(\eta_k) = \hat{p}(\eta_k)$ if $k \neq i$ or $k = i$ and $\eta_i \neq \xi_i$,
  - and $N_k = \hat{N}_k$, $k = 1, \dots, n$, $y_j = \hat{y}_j$.

  Then these $q_j^*(\xi_i)$ are substituted for $q(\xi)$ into the Equation for $Q(\xi_i)$. Finally, we require that $\hat{p}(\xi_i)$ solves a firm's optimization problem formulated above.
- The market clearance condition is hold: $\hat{s}(\xi_i) = \sum_{j=0}^{n+1} \hat{q}_j(\xi_i) \hat{L}_j$, $i = 1, \dots, n$.
- Equations (3)–(10) hold with $L_j = \hat{L}_j$, $L_i^u = \hat{L}_i^u$, $y_j = \hat{y}_j$, $w_i = \hat{w}_i$, $p(\xi_i) = \hat{p}(\xi_i)$, and $Q(\xi_i) = \hat{s}(\xi_i)$, where $i = 1, \dots, n$, $j = 0, 1, \dots, n+1$, $\xi_i \in [0, \hat{N}_i]$. Additionally, $\int_{\hat{N}_i} l(\xi_i)\, d\xi_i = \hat{L}_i$.
- Firms are free to enter: $\hat{p}(\xi_i)\hat{s}(\xi_i) - c_i^v \hat{s}(\xi_i)\hat{w}(\xi_i) - c_i^\varphi \hat{w}(\xi_i) = 0$, $i = 1, \dots, n$, $\xi_i \in [0, \hat{N}_i]$.

We discuss the existence and uniqueness of the equilibrium in the model under the following Assumptions:

Assumption 1. Differentiable functions $\sigma_i(\varkappa)$ are supposed to be monotonous and satisfy the condition

$$\sigma_i(\varkappa) > 1, \quad \text{for all } i = 1, \dots, n \text{ and } \varkappa \geq 0. \tag{13}$$

Assumption 2. If L_j is the equilibrium number of employed workers in sector j then we assume that $L_j \geq 1$ and

$$|\sigma_i'(\varkappa)| < \frac{L_j}{2nC_i} \quad \text{for all } i = 1, \dots, n,\ j = 0, \dots, n+1, \text{ and } \varkappa \geq 0, \tag{14}$$

where $C_i = c_i^{\varphi}/c_i^{v}$ is the ratio of the fixed to variable costs.

Assumption 3. The diversity of the *equilibrium individual demands* is supposed to be limited:

$$\frac{\sigma_i(q_j(\xi_i))}{\sigma_i(q_{j'}(\xi_i))} < 2 \quad \text{for all } i = 1, \dots, n,\ j, j' = 0, \dots, n+1, \text{ and } \varkappa \geq 0. \tag{15}$$

If σ_i increases, we assume additionally that there exists some $\delta_i > 0$ such that inequalities

$$\sigma_i'(q_j(\xi_i))q_j(\xi_i) < \delta_i < \sigma_i(q_j(\xi_i)) - 1, \quad i = 1, \dots, n, \tag{16}$$

holds for all equilibrium individual demands $q_j(\xi_i)$, $i = 1, \dots, n$, $j = 0, \dots, n+1$.

Zhelobodko et al. [4] used an analogue of Assumptions 1 and 2 to prove the existence and uniqueness of the equilibrium in a single-sector economy. Assumption 2 links together the equilibrium number of employed workers in each sector, variability of the elasticity of substitution, and the ratio of fixed to variable costs. It is easy tio see that small values of C_i correspond to low fixed or/and high variable costs. Both effects can be attributed to less efficient economies.

Assumption 3 is most restrictive. It appears because the economy is multi-sector. Nevertheless, one can check that if the economy (number of individuals $\mathcal{L}$) is large enough then Assumption 3 follows from Assumption 2.

4. Equilibrium and its Properties

Proposition 1. *Let Assumptions 1–3 be satisfied. Then a general equilibrium exists, and it is unique. This equilibrium is symmetrical with respect to varieties of the i-th differentiated product: $p(\xi_i)$ and $q_j(\xi_i)$ depends on i but not on specific varieties ξ_i. We denote*

$$p_i = p(\xi_i), \quad q_{ij} = q_j(\xi_i), \quad Q_i = Q(\xi_i), \quad \mathfrak{S}_i = \mathfrak{S}(\xi_i) \tag{17}$$

the symmetrical equilibrium variables. Then they are given by the following expressions:

$$Q_i = C_i\left(\mathfrak{S}_i - 1\right), \quad i = 1, \dots, n, \tag{18}$$

$$p_i = \frac{\mathfrak{S}_i c_i^{v} w_i}{\mathfrak{S}_i - 1}, \quad i = 1, \dots, n, \tag{19}$$

$$q_{ij} = \frac{(1-\alpha)\beta_j Q_i}{L_j} = \frac{(\mathfrak{S}_j + 1)Q_i}{\mathcal{L}\mathfrak{S}_j}, \; j = 1, \dots, n, \tag{20}$$

$$q_{i0} = \frac{Q_i}{\mathcal{L}}, \tag{21}$$

$$q_{i,n+1} = \frac{\alpha Q_i}{L_{n+1}}. \tag{22}$$

The following Equations determine the equilibrium wages, number of employed and unemployed workers, and number of firms:

$$w_i = \frac{\mathfrak{S}_i + 1}{\mathfrak{S}_i} w_0, \quad i = 1, \dots, n \tag{23}$$

$$L_i = (1-\alpha)\beta_i \mathcal{L} \frac{\mathfrak{S}_i}{\mathfrak{S}_i + 1}, \quad i = 1, \dots, n, \quad L_0 = (1-\alpha)\beta_0 \mathcal{L}, \tag{24}$$

$$L_{n+1} = \mathcal{L}\left(\alpha + (1-\alpha)\sum_{j=1}^{n} \frac{\beta_j}{\mathfrak{S}_j + 1}\right), \tag{25}$$

$$L_i^u = \frac{\beta_i/(\mathfrak{S}_i + 1)}{\sum_{j=1}^{n} \beta_j/(\mathfrak{S}_j + 1)} L_{n+1}, \quad i = 1, \dots, n, \tag{26}$$

$$N_i = \frac{(1-\alpha)\beta_i \mathcal{L}}{c_i^{\varphi}(\mathfrak{S}_i + 1)}, \quad i = 1, \dots, n. \tag{27}$$

In the frame of this paper it is impossible to give the detailed economic interpretation of the Proposition 1. Let us do several remarks. Equilibrium variables depend on the MES $\mathfrak{S}_i$. If $\mathfrak{S}_i$ is large and the demand is elastic, consumers are almost indifferent between varieties. On the contrary, small values of $\mathfrak{S}_i$ enlarge the diversity of the differentiated good. Therefore, the quantity $1/\mathfrak{S}_i$ is interpreted as the love for variety in various papers (see, f.e., Zhelobodko et al. [4])

Equation (18) also relates the output of varieties to the efficiency of the economy: more efficient economies produce more amount of specific varieties. The number of unemployed workers expectedly increases with the taxation rate α: a growth of the unemployment benefit subdues the risk of unemployment, Equation (25). The size of each sector can be measured in terms of the number of firms operated in the sector or the number of workers employed by those firms. As in other similar models, these two quantities are proportional, Equations (24) and (27).

5. Two Families of Utilities

Let us consider two families of utilities. The first of them is defined by

$$u(\varkappa) = \begin{cases} \dfrac{A}{2(A-1)}(\varkappa(\varkappa+2))^{\frac{A-1}{A}} F\left(1, 2 - \dfrac{2}{A}; 2 - \dfrac{1}{A}; -\dfrac{\varkappa}{2}\right), & \text{if } A > 1, \\ \ln\left(\varkappa + 1 + \sqrt{\varkappa^2 + 2\varkappa}\right), & \text{if } A = 1, \end{cases}$$

where $A \geq 1$ is a parameter. Then the elasticity of substitution is

$$\sigma(\varkappa) = A(1 + 1/(\varkappa + 1), \qquad A \geq 1.$$

The second family of utility functions is given by equation

$$u(\varkappa) = \begin{cases} \frac{A}{A-1}\varkappa^{1-\frac{1}{A}}(2\varkappa+1)^{1+\frac{1}{2A}} F\left(1, 2 - \frac{1}{2A}; 2 - \frac{1}{A}; -2\varkappa\right), & \text{if } A > 1, \\ 2\sqrt{2\varkappa + 1} + \ln\frac{\sqrt{2\varkappa+1}-1}{\sqrt{2\varkappa+1}+1} & \text{if } A = 1, \end{cases}$$

In this case the elasticity of substitution is $\sigma(\varkappa) = A(2 - 1/(\varkappa + 1)$. The function $u(\varkappa)$ defined above are represented respectively by the sum of two power functions up to terms of higher order:

$$u(\varkappa) = \frac{2^{-1/A}A}{A-1}\left(\varkappa^{1-\frac{1}{A}} - \frac{A-1}{2A(2A-1)}\varkappa^{2-\frac{1}{A}}\right) + O(\varkappa^{3-\frac{1}{A}}), \quad A > 1, \quad \varkappa \ll 1,$$

$$u(\varkappa) = \frac{A}{A-1}\left(\varkappa^{1-\frac{1}{A}} + \frac{A-1}{A(2A-1)}\varkappa^{2-\frac{1}{A}}\right) - O(\varkappa^{3-\frac{1}{A}}), \quad A > 1, \quad \varkappa \ll 1,$$

where minus is used ahead of big-O just to stress that the next term of the series is negative.

If an arbitrary function from any of these families represent the low-tier utility of consumers then for any choice of the economy primitives, that is, variable $\{c_i^v\}_{i=1}^n$ and fixed $\{c_i^\varphi\}_{i=1}^n$ costs and the exponents β_i of the Cobb–Douglas upper tier utility, there exists such a value of $\hat{\mathcal{L}}$ that for any $\mathcal{L} > \hat{\mathcal{L}}$ Assumption 2 holds. So, all the Assumptions 1–3 are satisfied.

6. Conclusion

In the paper we introduced the model of a multi-sector economy with high-tech sectors producing differentiated goods and firms competing monopolistically. A homogeneous sector is characterized by perfect competition. The conditions are found of symmetric equilibrium to exist and unique which allow to find individual consumer demands, product prices, wages, the number of employed and unemployed, and the number of firms in each sector. Two one-parameter families of hypergeometric functions are defined as examples of utility functions of a general type that satisfy the conditions of existence and uniqueness. The examples found correspond to the decreasing and increasing elasticity of substitution between products. When changing parameters, all possible values of the equilibrium elasticity of substitution are exhausted. So, the complete analysis of the equilibrium in the model is possible.

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