

# Formation reachability area as a data vector using a dynamic model for controlling information processes in the automated control system for moving objects

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## Abstract

This article considers **the reachability area formation as a data vector** at different points in time. To solve this problem, **a new dynamic model for information processes control** was developed, which includes the processes of receiving, transmitting and processing information in the automated control system for moving objects. Mathworks Matlab was applied for developing this dynamic model. It is noted that the developed dynamic model can be modified and presented as a dynamic model for the modernization/planning/operation process. The article provides an **algorithm** for the reachability area formation as a data vector.

The reachable area formation as a data vector is necessary to solve the task of assessing and analyzing the quality indicators of automated control system for moving objects, which will increase the efficiency and validity of control decisions related to the configuration (reconfiguration) of the structures of automated control system for moving objects in dynamically changing conditions. In this research reachability areas will be considered in the space of system-technical parameters, which is formed not on the basis of physical laws, but by the logic of data processing.

## Keywords 1

Matlab; automated control system for moving objects; ACS for MO; dynamic model for information process control; dynamic model for receiving, transmitting and processing information; reachability area; formation the reachable area; data vector; software implementation.

## 1. Introduction

At present, automated control systems (ACS) for mobile objects (MO) have gained great popularity. The characteristic features of automated control system for moving objects are: multilevel, multi-connectivity, territorial distribution, structural dynamics of their main elements and subsystems, the multi-purpose nature of the functioning of modern moving objects, structural similarity and redundancy of the main elements and subsystems in automated control system for moving objects. In these

systems, the main control functions are automated, for example, the planning function, the operational control function. In general, automated control system for moving objects includes the following main elements and subsystems: control center; central control point; control points with moving objects systems; moving objects service points; service system; moving objects systems for various purposes; automated data exchange system.

At present, there is a huge number of real-life automated control systems for moving objects, which differ from each other in the

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volume and control functions. In this work, the research object is the automated control system for moving objects, which can be considered as aircraft and spacecraft of various classes.

However, the constructing problem and using real-life models and created objects is constantly growing due to the increasing complexity of the automated control system for moving objects, therefore, complex (system) modeling is used [1-5]. Modeling as a way for creating and researching models [6] makes it possible to practically eliminate the need for lengthy and expensive field tests, to abandon the use of traditional "trial and error" methods.

The functioning automated control system for moving objects is associated with the reception, processing and transmission of large amounts of data and information, which leads to the need to use a given set of moving objects (repeaters) in the control loop of such systems, providing direct information exchange between the automated control system subsystems. As an example of such a computer network, we can consider a network of spacecraft - repeaters, providing informational interaction of spacecraft - repeaters with each other and ground subscribers.

In these conditions, the formulation and solution of planning and management tasks for the processing and transmission of information to the automated control system for moving objects acquires special relevance. To solve this task, a dynamic model (DM) is being developed for information process control in the automated control system for moving objects and the reachable areas (RA) formation [1, 7-9] with its help to assess the quality indicators of the automated control system for moving objects.

## 2. Dynamic model for information processes control in the automated control system for moving objects (short description)

In [10] a task formulation was described, and in [11] a dynamic model and an algorithm for solving the task for controlling the receiving, transmitting and processing information processes in the automated control system for moving objects were considered in detail. Therefore, we will give only the basic

formulas for dynamic model for information processes control in automated control system for moving objects.

The algorithm for solving controlling information processes task in automated control system for moving objects was developed on the basis of the following methods: branches and boundaries (proposed by A. Land and J. Doig in 1960), successive approximations of Krylov-Chernousko [12]) and based on the generalized algorithm solving the controlling the processes transmission and processing data task in a dynamic network [13] which in turn was developed on the basis of the Krylov-Chernousko method [12], the Pontryagin maximum principle [14], the Hamilton function [15].

*A mathematical model for controlling the receiving, transmitting and processing information processes in automated control system for moving objects, written in the form differential equations system:*

$$\dot{x}_i^{(o,1)}(t) = u_i^{(o,1)}(t); \quad (2.1)$$

$$\dot{x}_{ij\rho l}^{(n,1)}(t) = u_{ij\rho l}^{(n,1)}(t); \quad (2.2)$$

$$\dot{x}_{j\rho l}^{(n,2)}(t) = u_{j\rho l}^{(n,2)}(t); \quad (2.3)$$

$$\dot{x}_{ij\rho l}^{(n,3)}(t) = \omega_{ij\rho l}^{(n,1)}(t)e_{ij}(t); \quad (2.4)$$

$$\dot{x}_{j\rho l}^{(n,4)}(t) = \omega_{j\rho l}^{(n,2)}(t). \quad (2.5)$$

Where (2.1) is an auxiliary equation that shows where the dynamic model is located;

(2.2) is an streaming information transmission/reception model;

(2.3) is an streaming information processing model;

(2.4) is an auxiliary equation, which shows from which node to which one is receiving/transmitting information;

(2.5) is an auxiliary equation that shows in which node the information processing is carried out.

*Quality control Indicators for receiving/transmitting and processing information processes in the automated control system for moving objects:*

$$J_1 = \sum_{l=1}^L \sum_{\rho=1}^P \sum_{j=1}^n \int_{t_l}^{t_{l+1}} \gamma_{j\rho l}(\tau) \omega_{j\rho l}^{(n,2)}(\tau) d\tau$$

);

$$J_2 = \sum_{l=1}^L \sum_{\rho=1}^P \sum_{j=1}^n \int_{t_l}^{t_{l+1}} \beta_{j\rho l}(\tau) \omega_{j\rho l}^{(n,2)}(\tau) d\tau$$

);

$$J_3 = \frac{1}{2} \sum_{l=1}^L [a_l^{(o,1)} - x_l^{(o,1)}(t_{fl})]^2; \quad (2.8)$$

$$J_4 = \frac{1}{2} \left[ \sum_{l=1}^L \sum_{i=1}^n \sum_{\substack{j=1 \\ j \neq i}}^n \left( x_{ijl} - \sum_{\rho=1}^P x_{ij\rho l}^{(n,1)}(t_{fl}) \right)^2 + \right. \\ \left. + \sum_{l=1}^L \sum_{j=1}^n \left( g_{jl} - \sum_{\rho=1}^P x_{j\rho l}^{(n,2)}(t_{fl}) \right)^2 \right] \quad (2.9)$$

$$J_5 = -\frac{1}{2} \sum_{l=1}^L \sum_{\rho=1}^P \sum_{j=1}^n \sum_{i=1}^n (T_l - x_{ij\rho l}^{(n,3)}(t_{fl}))^2 \quad (2.10)$$

$$J_6 = -\frac{1}{2} \sum_{l=1}^L \sum_{\rho=1}^P \sum_{j=1}^n (T_l - x_{j\rho l}^{(n,4)}(t_{fl}))^2. \quad (2.11)$$

Form Indicator (2.6) is the information processing directive terms functional; (2.7) is the information processing completeness functional, which characterizes the total quality of the processed information; (2.8) allows you to evaluate the completeness (quality) of processing a given information amount; (2.9) allows us to estimate the fulfillment boundary conditions completeness (in fact, the Mayer functional); (2.10) and (2.11) make it possible to estimate the uniformity (unevenness) of the use of information and computing resources of the automated control system for moving objects on the planning interval.

*Hamiltonian (Hamilton function):*

$$H(\mathbf{x}(t), \boldsymbol{\eta}(t), \mathbf{u}(t), t) = \\ = (H_1 + H_2 + H_3 + H_4) \rightarrow \max_{\mathbf{u} \in Q}, \quad (2.12)$$

where  $\mathbf{u} = \| u_l^{(o,1)}, u_{ij\rho l}^{(n,1)}, u_{j\rho l}^{(n,2)}, \omega_{ij\rho l}^{(n,1)}, \omega_{j\rho l}^{(n,2)} \|$ ,  $i, j = 1, \dots, n$ ,  $\rho = 1, \dots, P$ ,  $l = 1, \dots, L$ ;  $Q$  – is the admissible controls area determined by relations (2.1)–(2.8).

The maximizing task for Hamilton function of the form (2.12), depending on the situation that develops during the distribution of resources in automated control system for moving objects, is decomposed into 4 (in our case) particular optimization tasks of the following form:

$H_1$  is confirms that time intervals are being set.

$H_4$  is an optimization subtask "about purposes", which can be reduced to an integer linear programming task [16-17].

$H_2$  is subtask receiving/transmitting information (solved using linear programming [18]).

$H_3$  is subtask processing information (solved using linear programming [18]).

Let us consider in more detail the "about purposes" task is the task about best distribution of work between the same number of performers task. This task belongs to the scientific direction is operations research. Operations Research is a scientific methods set for solving the tasks about organizational systems effective control. The main methods for finding optimal solutions include mathematical programming, in particular, for example, linear programming, linear integer (Boolean) programming.

When solving the "about purposes" task, an optimal purpose is sought from the maximum overall performance condition, which is equal to the sum performance performers.

In the process for control production, the tasks about appointing performers for various types of work often arise, for example: the workers selection and the candidates appointment for vacant positions, the equipment distribution between regions, the trains distribution along routes.

With regard to our dynamic model, the "about purposes" task is formulated as follows. The spacecraft can receive/transmit or process information simultaneously.

It is known that the  $j^{\text{th}}$  node for the  $i^{\text{th}}$  spacecraft will store the information amount equal to  $c_{ij}$  units.

It is required to distribute spacecraft in such a way as to maximize the information received amount (to minimize information loss). At the same time, on each spacecraft there is a bandwidth limitation for the communication channel, and the spacecraft cannot simultaneously receive/transmit or process information.

The variable  $x_{ij}$  ( $i, j = \overline{1, n}$ ) is such that:

$x_{ij} = 1$ , if a decision is made to transfer information;

$x_{ij} = 0$ , if no decision is made to transfer information.

Then the model for this task takes the following form:

$$\begin{aligned} \sum_{i=1}^n \sum_{j=1}^n c_{ij} x_{ij} &\rightarrow \max, \\ \sum_{j=1}^n x_{ij} &= 1, \quad i = \overline{1, n} \\ \sum_{i=1}^n x_{ij} &= 1, \quad i = \overline{1, n} \\ x_{ij} &\in \{0, 1\}, \quad i, j = \overline{1, n}. \end{aligned}$$

The software implementation (in particular, the mathematical programming task) of the described dynamic model for obtaining the source data for calculating the reachability area as a data vector is shown in the next section.

### 3. Software implementation of dynamic model for information process control in the automated control system for moving objects in Matlab

Mathworks Matlab [19-20] was chosen as the software, as it is excellent for designing and analyzing systems and working with computational mathematics and matrix, and the built-in graphics provide visualization and better understanding of the data. In addition, Matlab contains predefined functions in the Optimization Toolbox for solving the linear programming tasks and the "about purposes" tasks (integer linear programming tasks), which are used in the developed dynamic model for information process control in automated control system for moving objects. It is worth noting that during the development of the software module, Matlab's capabilities for working with matrices used to speed up the work of the dynamic model.

The main parameters for developed dynamic model are:

- information amount;
- penalty coefficients for the deadlines;
- performance for receiving/transmitting information;
- information processing performance;
- directive terms.

In a linear equation, the integral is the summation. Therefore, in formulas (2.6) and (2.7), the integration is carried out automatically. In formulas (2.1) and (2.5), the derivative is the sum at all times. There is no time in the software implementation of the

dynamic model, and the switching of planning intervals is carried out automatically. In addition, the software module implements verification of technical and technological limitations of the mathematical model.

Let's dwell on the Optimization Toolbox in Matlab for solving linear programming problems and the "about purposes" task (integer linear programming task).

Linear programming tasks in the Optimization Toolbox in Matlab are solved using the `linprog()` function.

Consider a linear programming task:

$$\begin{cases} f^T \cdot x \rightarrow \max \\ A \cdot x \leq b \\ Aeq \cdot x = beq \\ lb \leq x \leq ub \end{cases} \quad (3.1)$$

Basic inputs to `linprog`:

- coefficient vector for objective function  $f$ ;
- inequality constraints matrix  $A$ ;
- inequality constraints right-hand sides of the vector  $b$ ;
- inequality constraints matrix  $Aeq$ ;
- inequality constraints right-hand sides of the vector  $beq$ ;
- vector  $lb$ , limiting the permissible plan  $x$  from below;
- vector  $ub$ , limiting the permissible plan  $x$  from above.

At the system output (3.1) the function `linprog` gives the optimal plan  $x$  and the objective function optimal value  $fval$ . It is also possible to set an initial guess  $x_0$ .

If one of the input parameters is absent, then in Matlab it should be replaced by square brackets `[]`, except for the case when it is the last parameter in the list. In addition, it is possible to set additional settings, in particular, the solution algorithm. Matlab solves linear programming problems in two ways: the Large-Scale Algorithm and the Simplex method.

Integer linear programming tasks in the Optimization Toolbox in Matlab are solved using the `intlinprog()` function.

Consider an integer linear programming task:

$$\begin{cases} f^T \cdot x \rightarrow \max \\ A \cdot x \leq b \\ Aeq \cdot x = beq \\ lb \leq x \leq ub \\ x \in \mathbb{Z} \end{cases} \quad (3.2)$$

where  $x$  is a vector with some integer coordinates. For an integer linear programming task, all coordinates of a vector  $x$  must be integers, and for a Boolean programming task, they must take values 0 or 1.

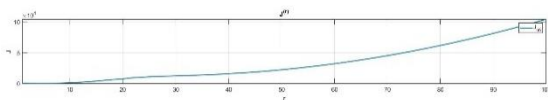
Basic inputs to `intlinprog`:

- basic input data as `linprog`;
- indices set `intcon`, at which the plan  $x$  variables are integer.

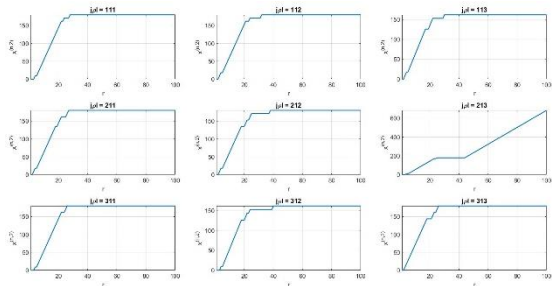
At the system output (3.2) the function `intlinprog` like `linprog` gives the optimal plan  $x$  and the objective function optimal value `fval`.

Due to the absence of a static model, some constants for the dynamic model for information process control in automated control system for moving objects were taken from [21-22].

The graph for the generalized quality indicator for control information process in automated control system for moving objects is shown at Figure 1, and the streaming information processing model is shown at Figure 2.



**Figure 1:** Generalized quality indicator for control information process in automated control system for moving objects



**Figure 2:** Streaming information processing model

Thus, with the help of the described software implementation of the developed dynamic model for information process control in the automated control system for moving objects, the source data were obtained for calculating the reachability area as a data vector.

#### 4. Reachability area formation as a data vector using dynamic model for information processes control in automated control system for moving objects

A vector data model is a digital point, line, and polygonal representation features as a set of coordinate pairs (vectors) that describe the features geometry.

It is proposed to construct and approximate reachability area based on the data obtained from the developed dynamic model for information processes control in the automated control system for moving objects [23-25]. Reachability area will be considered in the space of system-technical parameters, which is formed not on the basis of physical laws, but on the basis of various technologies for receiving, transmitting and processing information.

Based on the construction and approximation for reachability areas many problems in the optimal control theory are solved [8, 23, 26, 27].

The redistribution task, the modernization task, and the schedule task (control theory in scheduling theory [28-30]) are reduced to constructing or evaluating reachability areas, which subsequently serves as the basis for the development various numerical algorithms for finding a solution to a boundary or optimization problem for the considered dynamical system. However, the practical construction of reachable areas [31-33] especially in complex dynamic systems of large dimension is a very difficult task even when using modern computers. Therefore, in practice methods are often used to approximate these reachability areas with the required accuracy instead of directly constructing them.

It is well known that information about the reachability area and its main characteristics essentially replace all the information necessary for solving the assessing tasks the capabilities

for any dynamic system, the stability for its functioning, the options synthesis for creating and developing these systems.

It is well known that in the general case the Pareto set has a rather complicated structure, and therefore its construction often encounters insurmountable computational difficulties.

For reachability area formation as a data vector using dynamic model for information process control in automated control system for moving objects, it is proposed to use the following algorithm:

**Step 1** Obtaining the source data from dynamic model for information process control in automated control system for moving objects.

**Step 2.** Data vector Construction obtained from dynamic model for information process control in automated control system for moving objects.

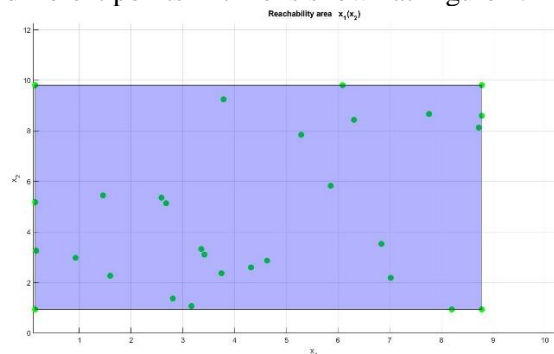
**Step 3.** Vertices determination for the future polyhedron.

**Step 4.** Formation of the set of all vertices for polyhedron.

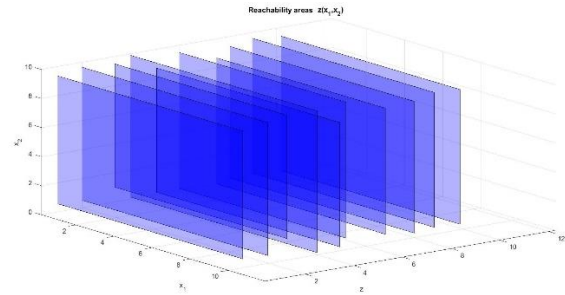
**Step 5.** Constructing reachability area based on the vertices of a polyhedron in Matlab at a specific point in time.

**Step 6.** Formation and interpretation the output results, presenting them in a form convenient for subsequent use.

Reachability area based on dynamic model for information processes control in automated control system for moving objects is shown at Figure 3, and graphical representation of changes in reachability area parameters at different points in time is shown at Figure 4.



**Figure 3:** Reachability area based on dynamic model for information process control in automated control system for moving objects



**Figure 4:** Graphical representation of changes in reachability area parameters at different points in time

Thus, the algorithm for the reachability area formation as a data vector, based on dynamic model for information process control in automated control system for moving objects is implemented in software.

## 5. Conclusion and future research

On the basis of the developed and implemented at the software level a **new dynamic model for information process control** [34], which includes the processes of receiving/transmitting and processing data and information in the automated control system for moving objects, **the reachability area formation as data vector at different points in time is performed**. On the basis of this model, the dynamic model for the process of modernization, planning, functioning for the task of managing the development of production facilities was built [35].

The reachability area formation as a data vector is necessary to solve the task of assessing and analyzing the quality indicators of the automated control system for moving objects, which will increase the efficiency and validity of management decisions related to the configuration (reconfiguration) of the structures of the automated control system for moving objects in dynamically changing conditions.

In [36], the Pareto set construction in the state space using projective geometry is performed.

An example of describing an algebraic system in geometric terms for a graphical solution is given in [37].

In the future, it is planned to develop an algorithm for the orthogonal projection of multidimensional simplexes that set the required ranges of values of target indicators on

the reachability area, built by the dynamic model to obtain and select the most preferred technologies and control programs for the elements and subsystems of the automated control system for moving objects belonging to the corresponding compromises area V. Pareto [1, 7, 38,39]. This is possible, since it was shown in [40] that for the considered class of optimal control problems (linear dynamical system) the admissible controls area and the objective function are convex, the method of successive approximations ensures monotonic convergence.

Also, solving the constructing task and approximating reachability area will also allow to solve the rescheduling task from [41], which is based on an analysis of structural and dynamic control in a complex dynamic object [42].

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