Stochastic model of thermal processes in the contact network at arc discharges occurring at high speeds of movement

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Abstract
The paper proposes a stochastic model, on the basis of which estimates are given of the parameters at which extreme situations occur due to the interruption of the electrical contact between the electro-rolling stock current collector (EPS) and a contact wire for the wear and tear of the contact network as a result of acts of arcing. The model takes into account the influence of random factors, which are temporary and sometimes repetitive. The probabilities of deviating from the coordinates of the breakdowns of the contact network from the values given in advance as a result of acts of arcs with defined repeatability periods are obtained.

Keywords
Weak contact, stochastic model, intensity function, repeatability period, probability asymptotics, characteristic function, multimodal distribution, unimodal distribution

1. Introduction

A topical problem encountered in the operation of an electric rolling stock is the reduction of the wear and tear of the contact network and the extension of its useful life under the influence of electric arc discharges arising from the breakdown of mechanical contact[1].

In the present operating conditions (soft soils, low air temperature for most of the year, high humidity, icing and insufficient quality of contact suspension surface treatment), it is not possible to improve the elasticity of the contact suspension. As a result, when electric rolling stock moves along high-speed motorways, there are multiple disconnections of the current conductors from the contact wire - a violation of the mechanical and electrical contact, which result in high-potential arc discharges. Repeated acts of discharges result in severe wear on the surface of the overhead wire, leading to the breakdown of the electrical contact and, in the worst case, the breakdown of the contact network.

It should be noted that the above-mentioned mechanical and electrical contact defects also lead to the deterioration of the traction equipment of the electric rolling stock [4].

According to the available static data on the overhaul of the contact suspension at the different offices of the October Railway, the frequency of major repairs for the replacement of the contact suspension is on average from 1,5 to 3 months depending on the season of operation and the flow of trains on the main line.

The paper proposes a stochastic model, which makes it possible to assess the important quantitative characteristics of said extreme situation, which is temporary and sometimes repetitive. These characteristics include the intensity function, the repeatability period and the probability of deviation

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of the disconnect coordinate from the specified value [1], [3]. The above characteristics make it possible to assess the periods of inter-service service service and the probability of wear on the overhead wires, which includes breakages from electric arc discharges, to optimize the time and cost of drip repairs and to adjust the repair plan for both the main lines and the sections of the road [9].

2. Problem statement

The key object in the study of extreme situations [5],[6],[7] arising from the breakage of the current carrier from the contact wire is the random value position of the breakpoint on the overhead wire. The model parameter is the pair $(\mu_\xi, \mathrm{T}_\xi)$, where μ_ξ – intensity function, and T_ξ – repeatability period of the random distribution ξ. The distances between the two disconnections of the current collector and the contact wire that exceed this are random. The mean of these distances is the repeatability period. Another important parameter for the distribution of extremes is the intensity function [1]. Thus, the starting point of a mathematical model describing periodic extremes [15], [16] is the pair $(\mu_\xi, \mathrm{T}_\xi)$.

2.1. Intensity function of the position of the current probe

The intensity function is defined as [1],[3]. Considers the probability that the position of the current probe on the overhead wire will be equal to or greater than a certain value s:

$$\mathbf{P}(\xi \geq s) = 1 - F_\xi(s), \qquad (1)$$

where F_ξ – random value distribution function ξ.

Enter the conditional probability that the position of the current collector's disconnect on the contact wire will lie within the interval $[s, s+\varepsilon)$, provided that its meaning is greater than or equals s, which is expressed by the following formula:

$$\mathbf{P}(\xi \in [s, s+\varepsilon) | \xi \geq s) = \frac{\mathbf{P}(\{\xi \in [s, s+\varepsilon)\} \cap \{\xi \geq s\})}{\mathbf{P}(\xi \geq s)} :=$$

$$:= \int_s^{s+\varepsilon} \mu_\xi(t) dt. \qquad (2).$$

The function μ_ξ is called the intensity function or fault intensity function. It determines the probability of a current probe being removed at a point with a coordinate s more to the right s by the amount величину ε.

2.2. Distribution intensity function properties

From (2) for the intensity function follows the expression: $\mu_\xi(t) = \dfrac{p_\xi(t)}{1 - F_\xi(t)}$ (here $p_\xi(t) = \dfrac{dF_\xi}{dt}$ – probability density function of the distribution ξ), which specifies the following properties:

1. f the current probe position takes a value greater than a given value s, that is, the distribution function ξ less than one: $P(\xi \geq s) > 0 \Rightarrow F_\xi(s) < 1$ for any end value s, then $\int_s^{+\infty} \mu_\xi(t) dt$, would be divergent, provided that: $\mu_\xi(t) = O\left(\frac{1}{t}\right)$ and converging if: $\mu_\xi(t) = O\left(\frac{1}{t^{1+\delta}}\right), \delta > 0$;
2. from equality $\mu_\xi(t) \cdot [1 - F_\xi(t)] = p_\xi(t)$ the differential equation linking the intensity function and the density function follows: $$\frac{1}{\mu_\xi^{\,2}(t)} \cdot \frac{d\mu_\xi(t)}{dt} - 1 = \frac{1}{p_\xi(t) \cdot \mu_\xi(t)} \cdot \frac{dp_\xi(t)}{dt}.$$
3. if x_{mo} – distribution fashion, then $\dfrac{d\mu_\xi}{dt}(x_{mo}) = \mu_\xi^2(x_{mo})$.

These properties make it easy to construct an intensity function for models with modal distributions. It should be noted that the function of the density of the current probe is usually unimodal. This is because the probability of withdrawal at certain points of the wire is lower because of mechanical tension and the position of the centre of mass of the portion of the wire considered.

2.3. Period of repeatability of separation positions

If you look at a series of observations in which the position of the current probe is greater than or equal to , this deviation of the position to the right is an event of interest to us. Let's determine its probability $\mathbf{P}(\xi \geq s) = 1 - F_\xi(s)$ for p, and the probability of the opposite event – $\mathbf{P}(\xi < s) = 1 - p$.

In the experiment, we'll look at observations at regular intervals, and the experiment will stop as soon as we have an event of interest, namely the deviation to the right of the current probe s.

To interpret the results, consider the random value X_ξ – number of tests up to first to right (i.e. $\xi \geq s$), it accepts values from set $\{0,1,2,...\}$. X_ξ is subject to geometric distribution with parameter p ($X_\xi \sim \mathbf{G}(p)$). Properties of this distribution are known: distribution series $\mathbf{P}(X_\xi = k) = p(1-p)^k$; characteristic function $\varphi(t) = \frac{p}{1-e^{it}(1-p)}$; expected value $\mathbf{E}X_\xi = -i \cdot \varphi'(0) = \frac{1-p}{p}$, second starting point $\mathbf{E}X_\xi^2 = -\varphi''(0) = \frac{(1-p)\cdot(2-p)}{p^2}$ allow to find the required model parameters [8],[10].

Define the repeatability period as a mathematical expectation $\mathrm{T}_\xi := \mathbf{E}X_\xi$ [3].

It follows from the definition that for a given model the repeatability period takes the form:

$$\mathrm{T}_\xi = \frac{F_\xi(s)}{1-F_\xi(s)} \qquad (3).$$

It follows from formula (3) that for the repeatability period a bottom-up assessment is fair: $\mathrm{T}_\xi \geq 1$.

Standard deviation from repeatability period is given by expression: $\sigma_\xi := \sqrt{\mathbf{E}X_\xi^2 - \mathbf{E}^2 X_\xi} = \frac{\sqrt{F_\xi(s)}}{1-F_\xi(s)}$, or using formula (2) to accept:

$$\sigma_\xi := \sqrt{\mathrm{T}_\xi^2 + \mathrm{T}_\xi} \qquad (4).$$

Then the probability of deviating right from the set position of the current probe to the observation with the number k or in the room k:

$$\mathbf{P}(X_\xi \leq k) = 1-(1-p)^k = 1-F_\xi^k(s) \quad (5)$$

If in the capacity of k choose T_ξ, for probability (5) there is an expression:

$$\mathbf{P}(X_\xi \leq \mathrm{T}_\xi) = 1-\left(1-\frac{1}{1+\mathrm{T}_\xi}\right)^{\mathrm{T}_\xi} \quad (6)$$

Probability asymptotics (6) at large T_ξ: $\mathrm{T}_\xi \to +\infty$ has the form:

$$\lim_{\mathrm{T}_\xi \to +\infty} \mathbf{P}(X_\xi \leq \mathrm{T}_\xi) = 1 - \frac{1}{e} \approx 0{,}63212 . \quad (7)$$

The formulas (5) and (6) make it possible to estimate the probability of deviation from the specified position of the current probe detachment to the right, with certain repeatability periods. In addition, the formulae provide valid parameter estimates by the maximum likelihood method [11],[12],[14] Multimodal distributions should be chosen as model distributions:

- for partitions located on one block, it is sufficient to choose a unimodal distribution with density:

$$p_\xi(t) = f(t-t_0)\cdot \mathbf{1}_{[0,L]} \qquad (8),$$

  where $t_0 \in [0, L]$, L – length of block - area;
- it is sufficient to use linear combinations of functions of the form (8) for breaks occurring on an extended section comprising several blocks.

2.4. Simulation example: case of one block - fixed length section

The separation density function in this case belongs to a two-parameter family $\{(h, t_0)\}$ distributions and has the form:

$p_\xi(t;h,t_0) = C\left[(t-t_0)^2 + h\right]\cdot \mathbf{1}_{[0,L]}$, here $t_0 = L/2$, h – the variation of the contact wire from the equilibrium position (can be determined by statistical evaluation). Random density normalization constant ξ, specified by the normalization condition:

$$\int_{[0,L]} p_\xi(t)dt = 1, \quad p_\xi(t;h,L) = C\left[(t-L/2)^2 + h\right]\cdot \mathbf{1}_{[0,L]}$$

and takes on the importance $C = \frac{12}{L(L^2+12h)}$.

The distribution function has the form:

$$F_\xi(s;h,L) = C\cdot\left[\frac{1}{3}\cdot\left(\left(s-L/2\right)^3 + L^3/8\right) + h\cdot s\right],$$

$0 \leq s \leq L$.

Repeatability period in this model:

$$\mathrm{T}_\xi(s;h,L) = \frac{4(s-L/2)^3 + \frac{3}{2}L^3 + 12sh}{\frac{3}{2}L^3 - 4(s-L/2)^3 + 12h(L-s)} \quad (9).$$

Probability of right deviation from a specified value:

$$\mathbf{P}(X_\xi \leq \mathrm{T}_\xi(s;h,L)) =$$

$$= 1 - \left(\frac{\frac{3}{2}L^3 + 4(s-L/2)^3 + 12hs}{3L(L^2+4h)}\right)^{\mathrm{T}_\xi} \quad (10),$$

here $T_{\xi} = \dfrac{4(s-L/2)^3 + \dfrac{3}{2}L^3 + 12sh}{\dfrac{3}{2}L^3 - 4(s-L/2)^3 + 12h(L-s)}$.

Formulas (9), (10) express the functional dependence of the repeatability period and the probability of the right deviation of the current probe from a given position to the observation with the number k or in the room k from that of the s on a wire, here $0 \le s \le L$. These functional relationships are complex and cumbersome for numerical estimates, which is particularly important for applications, the type. Given the symmetry in the probabilistic model described by the unimodal distribution (8), limit values were found (9), (10).

2.4.1. Limit value of function $\mathrm{T}_{\xi}(s;h,L)$ when $s \to +0$

As a result of the cut-off $s \to +0$ repeatability period for the left end of the block:
$\mathrm{T}_1(h,L) = \lim\limits_{s \to +0} \mathrm{T}_{\xi}(s;h,L)$.

Here $\mathrm{T}_1(h,L) = \dfrac{L^2}{2(L^2+12h)} = \dfrac{1}{2+24h/L^2}$ (11).

2.4.2. Limit value of $\mathrm{T}_{\xi}(s;h,L)$ when $s \to L/2$

For the middle of the block $s = L/2$ the repeatability period dependent on model parameters will be:

$$\mathrm{T}_2(h,L) = \lim_{s \to L/2} \mathrm{T}_{\xi}(s;h,L) = 1 \quad (12).$$

2.4.3. Limit value of $\mathrm{T}_{\xi}(s;h,L)$ when $s \to L-$

For the right end of block - section a $s = L$ the repeatability period of the model will be:

$$\mathrm{T}_3(h,L) = \lim_{s \to L-} \mathrm{T}_{\xi}(s;h,L) = \frac{2(L^2+12h)}{L^2} = 2 + \frac{24h}{L^2} \quad (13).$$

Due to the symmetry of the partitions, the limits of the repeatability period on the right and left ends are linked by the ratio: $\mathrm{T}_3(h,L) = \mathrm{T}_1^{-1}(h,L)$.

From (11), (12), (13) it follows that when the bends are small h: $h \to 0+$ The repeatability period will accept the following values at the appropriate points on the overhead wire: $\mathrm{T}_1(h,L) = 1/2$, $\mathrm{T}_2(h,L) = 1$, $\mathrm{T}_3(h,L) = 2$.

The above dependency graphics are shown on figure 1.

Accepted here as $L = 1200$ meters, then $L/2 = 600$ meters.

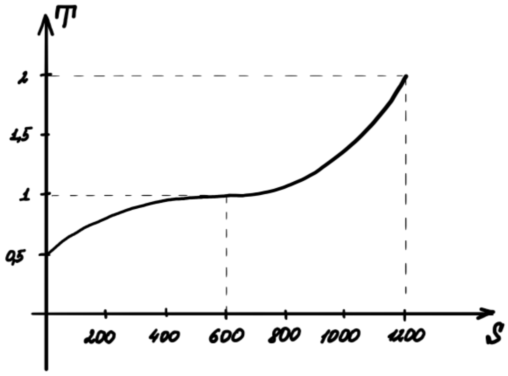


Figure 1: **Repeatability period dependent on contact position on overhead wire.**

Repeatability period limits are tabulated for ease of reference 1.

Table 1 Repeatability period limit values

Limit value $\mathrm{T}_{\xi}(s;h,L)$	Extreme contact wire position	Limit value when $h \to 0+$
$\mathrm{T}_1(h,L) = \dfrac{1}{2+24h/L^2}$	$s \to +0$	$\mathrm{T}_1(0,L) = 1/2$
$\mathrm{T}_2(h,L) = 1$	$s \to L/2$	$\mathrm{T}_2(h,L) = 1$
$\mathrm{T}_3(h,L) = 2 + \dfrac{24h}{L^2}$	$s \to L-0$	$\mathrm{T}_3(h,L) = 2$

The asymptotic formulas for the repeatability period indicate its limitation to a segment $s \in [0;L]$, corresponding to the length of the block - section ($L = 1200$ meters). This feature of the repeatability period makes it possible to predict the frequency of major maintenance activities to replace worn-out parts of the network. It should be noted that in the operation of the network, the optimization of the cost of repairs is important, not only at the cost of the work carried out, but also at the cost of the time spent. The latter means that it is more advantageous to repair several sections in parallel (in one period) than to replace the overhead wire consecutively (after some time to return the repair crew to the same section). The above-mentioned mode of repair makes it possible to substantially reduce the cost of idling of electrified rolling stock. Thus, the replacement of the overhead wire on at least one block - the section leads to the dysfunction of a fairly long stretch of the railway network, resulting in economic losses for the enterprises using the company's services «Russian Railways» as the main carrier.

2.5. Limit values for deviation from a specified value

Based on the expression (10) for deviation probabilities and repeatability time limits (11), (12), (13), the probability limits are determined from:

$$\mathbf{P}\left(X_\xi \le \mathrm{T}_j(h,L)\right) = 1 - \left(1 - \frac{1}{1+T_j(h,L)}\right)^{T_j(h,L)}$$

In the least case with low bending values h: $h \to 0+$ refer:

$$\mathbf{P}\left(X_\xi \le \mathrm{T}_1(h,L)\right) \to (3-\sqrt{3})/3;$$
$$\mathbf{P}\left(X_\xi \le \mathrm{T}_2(h,L)\right) \to 1/2;$$
$$\mathbf{P}\left(X_\xi \le \mathrm{T}_3(h,L)\right) \to 5/9.$$

These limits for probabilities indicate that the probability of a deviation increases with the coordinate of the detachment.

On figure chart of deviation probability from coordinates s.

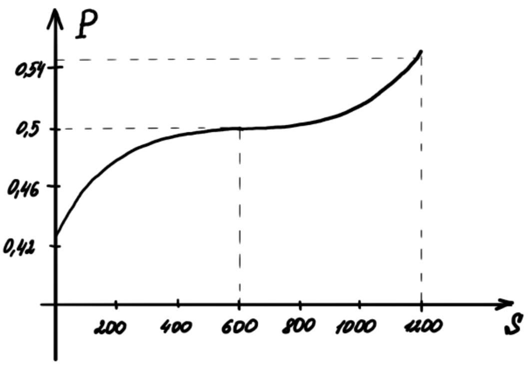


Figure 2: **Probability of deviation chart along the overhead wire**.

This relationship reflects the fact that at the end of repeatability, as is the case for most contact networks, the probability of deviation varies on a subset of the segment $[0,1]$, but it does not accept values close to 0 or 1. Probability as a function s – slowly changing in segment $[0,L]$, i.e. by the length of the wiring function. At the point $L/2$ probability function is bent, which means increasing the rate of growth of the function when approaching the right end.

Thus, for this unimodal two-parameter model (8), probability limit values $\mathbf{P}\left(X_\xi \le \mathrm{T}_j(h,L)\right)$ when $h \to 0+$ is not dependent on parameter L distributions of type (8) and have a uniform form for the whole family of distributions. Localize the values of the probability function on a segment $[0{,}42;0{,}56]$ results in high accuracy forecast of wear periods and major maintenance of contact suspension.

2.6. Severance intensity function

Intensity function of the probability of the current probe being removed at a point with a coordinate s to the right s by the amount ε, introduced in paragraph 2.1 for this model is:

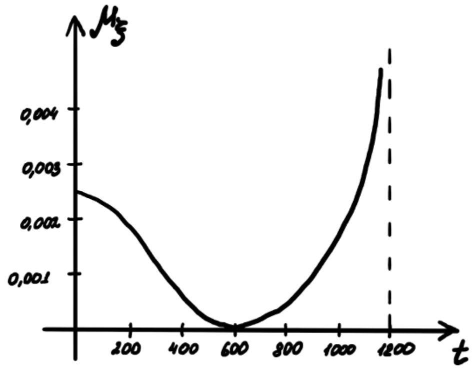
$$\mu_\xi(s;h,L) = \frac{C\left[(s-L/2)^2 + h\right]\cdot \mathbf{1}_{[0,L]}}{1 - C\left[\frac{1}{3}\left((s-L/2)^3 + L^3/8\right) + hs\right]} \quad (12),$$

here $\mathbf{1}_{[0,L]}$ – segment indicator function $[0,L]$. Here $C = \dfrac{12}{L\left(L^2+12h\right)}$ – is the standard density constant of a random distribution ξ, specified by the normalization condition: $\int_{[0,L]} p_\xi(t)dt = 1$,

$$p_\xi(t;h,L) = C\left[(t-L/2)^2 + h\right]\cdot \mathbf{1}_{[0,L]}.$$

Following, on figure 3 is the graph of the intensity function μ_ξ for the following values of the distributions: $h = 0{,}05$ meters, $L = 1200$ meters.

Figure 3: **Intensity function graph**



Analyzing the intensity function and its graph, you can see that it has at least in the middle of the segment $[0,L]$, which is fully consistent with the type of probability distribution and the presence of the latter mode also at a point $t_0 = L/2$.

The expression for the intensity function (12) has a rather cumbersome and difficult form for analysis.

Give an asymptotic intensity function in the vicinity of a fashion point $t_0 = L/2$:

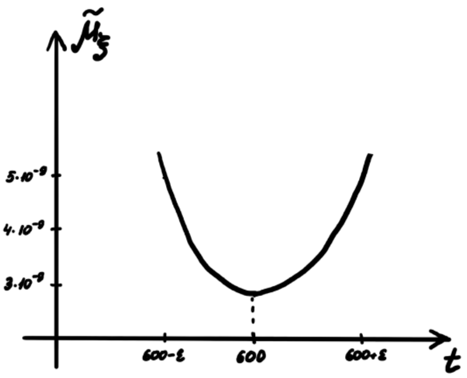
$$\mu_\xi(t;h,L) = \frac{24h}{L(L^2+12h)} + \\ + \frac{576h^2}{L^2(L^2+12h)^2}\cdot(t-L/2) + \\ + \frac{24}{L(L^2+12h)}\left(1+\frac{1728h^3}{L^3(L^2+12h)^3}\right)\cdot(t-L/2)^2 + \\ + o\left((t-L/2)^2\right) \qquad (13).$$

The main part of the formula (13) asymptotics should be designated $\tilde{\mu}_\xi$:

$$\tilde{\mu}_\xi(t;h,L) = \frac{24h}{L(L^2+12h)} + \\ + \frac{576h^2}{L^2(L^2+12h)^2}\cdot(t-L/2) + \\ + \frac{24}{L(L^2+12h)}\left(1+\frac{1728h^3}{L^3(L^2+12h)^3}\right)\cdot(t-L/2)^2 \qquad (14).$$

Graph of the main part of the asymptotics of the intensity function in the vicinity of the point $t_0 = L/2$ in the figure below 4.

Figure 4: **Graph of the main part of the asymptotics of the intensity function in the vicinity of the point** $t_0 = L/2$



Dependency graph analysis $\tilde{\mu}_\xi(t)$ shows that the main part of the intensity function in the neighborhood of the mode of the distribution has a quadratic view. Minimum value $\tilde{\mu}_\xi(t)$ is at a point $t_0 = L/2$. This is fully consistent with the fact that the intensity function acts as the density of the conditional distribution $\mathbf{P}(\xi \in [s, s+\varepsilon] | \xi \geq s)$ probabilities of lead.

Note also that in this model the intensity function has no property: $\mu_\xi(t) = O\left(\frac{1}{t}\right), t \to +\infty$. This is due to the fact that it is set in a non-trivial way on a compact segment $[0, L]$, outside which it lasts 0: the breakdowns of the current collector occur in a fixed-length block, so the asymptotics at the large coordinates of the separation points are not meaningful.

2.7. Conclusion

The task of estimating the probability characteristics of an extreme situation occurring during the operation of the contact network as a result of the detachments of the current collector from the contact wire during the movements of high-speed electric rolling stock was defined and solved, as a consequence of electric arc discharges between the overhead wire and the current collector.

Precise formulas and their asymptotic expressions have been found in important extreme cases for the probabilities of deviations from a given value, repeatability periods and intensity function for a special type of unimodal distributions, in accordance with the load distribution over the length of the wire. This makes it possible to assess the most important characteristics and to predict the occurrence of said extreme situation - breakage of the wire as a result of heavy heating with an electric arc. It is equally important to draw up an optimal plan of work for periodic major maintenance, thus minimizing the cost.

Note that this type of distribution of possible straps from the overhead wire, although approximate, is for evenly stretched contact suspensions without severe altitude variations and the absence of soft soils and underground floating lakes, It describes the processes of cutting off the current probes with sufficient precision.

The work, according to the idea of the authors, has a natural continuation, where the type of distribution will be specified according to the geometrical characteristics of the contact wire, such as the coordinates of the attachment points, the amount of bending, the curvature, the natural profile. In addition, it is intended to assess the parameters of the working distributions on the basis of statistical data from different offices and company roads «Russian Railways» [17].

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