A study of two dialectical logics*

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Abstract. In this paper I study the dialectical logics **DL*** and **HEGFL**. I compare some of the most distinguishable properties of both logics and I present some uknown features of **HEGFL** with respect to classical logic, weakly connexive logics and two negation-like connectives.

Keywords: Contra-classical · **DL*** · **HEGFL** · Paraconsistent · Negation

Introduction

In this paper I study two dialectical logics, **DL*** and **HEGFL**. The former was introduced in [3] by Newton da Costa and Robert G. Wolf as a variant of the logic **DL** —a paraconsistent logic designed to formalize Hegel's dialectical principle of the unity of opposites as presented in [9]. **HEGFL**, introduced in [5], is named, inspired and based upon Elena Ficara's interpretative work of Hegel's philosophy (see [6], [7]).

Both are contra-classical logics, that is, they validate arguments that are classically invalid. In particular, they have contradictions as theorems. Nonetheless, whereas **DL*** is very standard to a certain extent —it is one of the logics obtained by the so-called "positive logic plus approach"—, **HEGFL** is of further interest as it exhibits many more contra-classical features. For example, versions of weak-connexivity, in the sense of Pizzi [12], can be defined within it, and its negation resembles a sort of demi-negation (a negation-like connective such that a double occurrence of it is equivalent to an occurrence of an intuitionistic or a classical negation).

Unlike many of the logics developed by da Costa and his collaborators, **DL*** has not received attention. This investigation can contribute to the formal understanding of **DL*** and akin logics by relating them to other contra-classical logics using more contemporary approaches. This is just a first step towards that end. (In spite of their Hegelian origins, I will offer (almost) no comments of Hegel's philosophy.)

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The plan of the paper is the following. In the first section I present the logic **DL*** in proof-theoretic and model-theoretic terms. In the second section I present the logic **HEGFL** and some of its more distinguished properties. In the third section I discuss the relation of **HEGFL** with weakly connexive logics and two "negative" connectives.

1 The dialectical logic DL*

First, I will present **DL*** in proof-theoretic terms and then in model-theoretic terms.

Proof theory

Let $\mathcal{L}$ be a propositional language with a denumerable set $PROP = \{p_1, ..., p_n\}$ of propositional variables and with the set of connectives $\{\wedge, \vee \rightarrow, \leftrightarrow\}$. The set of formulas $FORM$ is recursively defined as usual. In this paper, when either new connectives or propositional constants expand $\mathcal{L}$ I indicate it in a subscript attached to $\mathcal{L}$. For example, I write $\mathcal{L}_{\{\circ,\sim\}}$ to mean the expansion of the set of connectives by $\{\circ, \sim\}$, with the corresponding definition of the set of formulas $FORM_{\{\circ,\sim\}}$; I write $\mathcal{L}_{\{l_n\}}$ to mean the expansion of $\mathcal{L}$ by the collection $\{l_1, ..., l_n\}$ with its corresponding definition of the set $FORM_{\{l_n\}}$.

Consider positive logic defined on $\mathcal{L}$, i.e. the logic characterized by the following schemata and rule:

A1) $A \rightarrow (B \rightarrow A)$
A2) $(A \rightarrow B) \rightarrow ((A \rightarrow (B \rightarrow C)) \rightarrow (A \rightarrow C))$
A3) $(A \wedge B) \rightarrow A$
A4) $(A \wedge B) \rightarrow B$
A5) $A \rightarrow (B \rightarrow (A \wedge B))$
A6) $A \rightarrow (A \vee B)$
A7) $B \rightarrow (A \vee B)$
A8) $(A \rightarrow C) \rightarrow ((B \rightarrow C) \rightarrow ((A \vee B) \rightarrow C))$
A9) $A \vee (A \rightarrow B)$

MP) $A \rightarrow B, A/B$

The logic **DL**, based on $\mathcal{L}_{\{\circ,\sim\}}$, expands positive logic with the introduction of the following axioms for the additional vocabulary:

A10) $\sim (A \wedge B) \leftrightarrow (\sim A \vee \sim B)$
A11) $\sim (A \vee B) \leftrightarrow (\sim A \wedge \sim B)$
A12) $\circ A \wedge \circ B \rightarrow (\circ(A \rightarrow B) \wedge \circ(A \wedge B) \wedge \circ(A \vee B) \wedge \circ(\sim A))$
A13) $\circ A \wedge \circ B \rightarrow ((A \rightarrow B) \rightarrow ((A \rightarrow \sim B) \rightarrow \sim A))$
A14) $\circ A \rightarrow (\sim\sim A \rightarrow A)$
A15) $\circ \circ A \leftrightarrow \circ A$
A16) $\circ A \rightarrow ((A \vee \sim A) \wedge ((A \rightarrow B) \vee (\sim A \rightarrow B)))$
A17) $\sim (\circ A) \rightarrow (((A \vee \sim A) \rightarrow B) \vee (A \wedge \sim A))$

One can define a strong negation using $\rightarrow$, $\circ$, $\wedge$ and $\sim$ in this terms: $\neg A =_{def} A \rightarrow (\circ B \wedge B \wedge \sim B)$; in using $\neg$ one obtains all the theorems of classical logic. Consequently, one can recover classical logic within **DL**, thus making classical logic a sublogic of **DL**.

DL*, based on $\mathcal{L}_{\{l_n, k_n, \circ, \sim\}}$, in its turn expands **DL** by adding the following axioms for the two families of propositional constants $\{k_1, k_2, ...k_n\}$ and $\{l_1, l_2, ..., l_n\}$ ($k_i \neq l_j$ for any $i, j \in \omega$, where ω is the lowest transfinite ordinal number):

A18) $\neg(k_i \vee \sim k_i)$
A19) $l_j \wedge \sim l_j$
A20) $\circ k_{i+1}$

Model theory

Let $v : PROP \longrightarrow \{1, 0\}$ be a set of valuation functions from the set of propositional variables to the set $\{1, 0\}$. v is extended to $FORM_{\{l_n, k_n, \circ, \sim\}}$ by the following interpretation functions i:

$i(p) = v(p)$, for every $p \in PROP$
$i(A \rightarrow B) = 1$ iff $i(A) = 0$ or $i(B) = 1$
$i(A \wedge B) = 1$ iff $i(A) = i(B) = 1$
$i(A \vee B) = 1$ iff $i(A) = 1$ or $i(B) = 1$
$i(\sim (A \wedge B)) = 1$ iff $i(\sim A) = 1$ or $i(\sim B) = 1$
$i(\sim (A \vee B)) = 1$ iff $i(\sim A) = i(\sim B) = 1$
If $i(\circ A) = i(\circ B) = 1$ then $i(\circ(A \rightarrow B)) = i(\circ(A \wedge B)) = i(\circ(A \vee B)) = i(\circ(\sim B)) = 1$
$i(\circ A) = 1$ iff $i(\circ \circ A) = 1$
If $i(\circ A) = 1$ then $i(\sim\sim A \rightarrow A) = 1$
If $i(A) = i(\sim A)$ then $i(\circ A) = 0$
If $i(A) \neq i(\sim A)$ then $i(\sim (\circ A)) = 0$
$i(k_i) = 0$ and $i(\sim k_i) = 0$
$i(l_j) = 1$ and $i(\sim l_j) = 1$

A formula A is valid if for every interpretation function i, $i(A) = 1$. An interpretation i is a model for a set of formulas Γ if for every A in Γ, $i(A) = 1$. If for every model of Γ, $i(A) = 1$, then A is a semantic consequence of Γ, written as '$\Gamma \models_{\text{DL*}} A$'. Let '$\models_{\text{DL*}} A$' represent that A is a semantic consequence of any set of formulas, i.e., that it is valid. Using the previous definitions, da Costa and Wolf prove the soundness and completeness of **DL*** with respect to the interpretations i.

Furthermore, the features of the interpretations have as a byproduct a decision method similar to the decision method for the hierarchy of logics $\mathbf{C}_n$ ($1 \leq n < \omega$) (see [1], [2]): to prove whether a formula is a theorem of **DL*** one considers not just the value of the components of the formula, but also of their respective negations and the appearance of the connective $\circ$ in all the formulas.[1]

[1] On account of lack of space, I will omit the details of this decision method.

A remarkable property of **DL*** is that it is *dialetheic* —or "negation-inconsistent"—, that is, it validates both at least one formula and its negation —l_j and $\sim l_j$. Besides, the following schemas are not theorems of **DL***:
$(A \vee \sim A)$
$\sim (A \wedge \sim A)$
$(A \vee \sim A) \leftrightarrow \sim (A \wedge \sim A)$
$\sim\sim A \leftrightarrow A$
whereas the following arguments hold:
$\circ A \wedge A \wedge \sim A \models_{\mathbf{DL*}} B$
$\circ A \wedge A \wedge \sim A \models_{\mathbf{DL*}} \sim B$

2 The dialectical logic HEGFL

The logic **HEGFL**, introduced in [5], is based on $\mathcal{L}_{\{\sim\}}$ and it is defined model-theoretically as follows. **HEGFL**-valuations v_{HEGFL} are functions $v : PROP \longrightarrow \mathcal{V}_{HEGFL}$ where $\mathcal{V}_{HEGFL} = \{c, c^*, 1\}$, $c<1$ and $c^*<1$. Considering the appropriate version of the set of formulas defined on this language, **HEGFL**-valuations are extended to $FORM_{\{\sim\}}$ according to the tables of the Table 1.

	$\sim$
1	c^*
c	c^*
c^*	c

$\wedge$	1	c	c^*
1	c^*	c^*	1
c	c^*	c^*	1
c^*	1	1	c^*

$\rightarrow$	1	c	c^*
1	c	c	c^*
c	c	c	c^*
c^*	c	c	c

$\leftrightarrow$	1	c	c^*
1	c^*	c	1
c	c	c^*	1
c^*	1	1	c^*

Table 1. Truth tables for **HEGFL**

In **HEGFL**, logical consequence is defined as follows:
$\Gamma \models_{\mathbf{HEGFL}} A$ if and only if, A is not true only if any $B \in \Gamma$ is not true
HEGFL is also dialetheic; actually, it validates all contradictions. Some of the most notable features of **HEGFL** are summarized on Table 2.

Of special interest in a Hegelian logic are the principles concerning double negation. Estrada-González and Ramírez-Cámara observe that $A \models_{\mathbf{HEGFL}} \sim\sim A$ does not hold, but $\sim\sim A \models_{\mathbf{HEGFL}} A$ does. Also, they say that one could venture the hypothesis that this is so because while affirmation (A) alone still says nothing about its sublimation $(\sim\sim A)$ —for negation $(\sim A)$ is needed for

	HEGFL
Reflexivity $A \models_{\text{HEGFL}} A$	✓
Transitivity if $\Gamma, A \models_{\text{HEGFL}} B$ and $\Delta \models_{\text{HEGFL}} A$ then $\Gamma, \Delta \models_{\text{HEGFL}} B$	✓
Monotonicity if $\Gamma \models_{\text{HEGFL}} A$, then $\Delta, \Gamma \models_{\text{HEGFL}} A$	✓
Deduction Equivalence $A_1, ..., A_n, A \models_{\text{HEGFL}} B$ iff $A_1, ..., A_n \models_{\text{HEGFL}} A \to B$	×
Definability of the biconditional $\forall v, v(A \leftrightarrow B) = v((A \to B) \wedge (B \to A))$	×
ECQ 1 $A \wedge \sim A \models_{\text{HEGFL}} B$	×
ECQ 2 $A, \sim A \models_{\text{HEGFL}} B$	×
EFQ $A \models_{\text{HEGFL}} B$, for all A such that $v(A) = 0$ for all v	×
$\wedge$**-Elimination** $A \wedge B \models_{\text{HEGFL}} A$; $A \wedge B \models_{\text{HEGFL}} B$	×
Adjunction $A, B \models_{\text{HEGFL}} A \wedge B$	×

Table 2. Properties of **HEGFL**

that—, $\sim\sim A \models_{\text{HEGFL}} A$ should hold because affirmation is somehow maintained in its sublimation. If this were along the right lines, $\sim\sim A \models_{\text{HEGFL}} \sim A$ should be valid in **HEGFL**. And it is. Moreover, in **HEGFL** one has that both $\sim A \models_{\text{HEGFL}} \sim\sim A$ and $\sim A \models_{\text{HEGFL}} A$ are valid. Why $A \models_{\text{HEGFL}} \sim\sim A$ fails and $\sim A \models_{\text{HEGFL}} \sim\sim A$ holds? Because in the latter A is not missing, at least not in the same way as $\sim A$ is missing in the former: in having $\sim A$, one somehow has A too, as it is the very proposition being negated, so the logical step to the sublimation can be done.[2] Finally, Estrada-González and Ramírez-Cámara analyze Ficara's "Dialectical law of double negation": $\sim\sim A \models_{\text{HEGFL}} A \wedge \sim A$. This should hold in a Hegelian logic because "we capture the true nature of concepts only when, by negating their negation, we gain them in their completeness, which is contradictory." It can be easily verified that such a Dialectical law of double negation holds in **HEGFL**.

3 (Other) Contra-classical features of HEGFL

A *contra*-**L** logic of a given logic **L** is a logic that, over the same underlying language, validates arguments that are not valid according to **L**. In particular, a contra-classical logic is a logic that validates arguments that, over the same language as that of classical logic, are not classically valid. Examples of such logics are well known (see, for instance [4], for a (non-exhaustive) list of contra-classical logics): syllogistic, over-complete logics, super-contracting logics, Abelian logic, dialetheic logics, and so on. **DL*** and **HEGFL** turn out to be contra-classical, as both are dialetheic. But the story does not end there.

Another special kind of contra-classical logics are connexive logics.[3] A *connexive logic* is characterized by the validity of the following schemas:

A1) $\sim (A \to \sim A)$

[2] To see why $A \models_{\text{HEGFL}} \sim\sim A$ and $A \models_{\text{HEGFL}} \sim A$ are invalid, just assign $v(A) = 1$. From this assignation it follows that $v(\sim\sim A) = c$ and $v(\sim A) = c$.

[3] For an introductory survey, see [11] and [15] .

A2) $\sim(\sim A \to A)$
B1) $(A \to B) \to\sim (A \to\sim B)$
B2) $(A \to\sim B) \to\sim (A \to B)$

together with the invalidity of the following schema:

SI) $(A \to B) \to (B \to A)$.

A1) is known as *Aristotle's Thesis* and A2) as *Variant of Aristotle's Thesis*. B1) is known as *Boethius' Thesis* and B2) *Variant of Boethius' Thesis*.

Neither **DL*** nor **HEGFL** validate Aristotle's or Boethius' theses, they only invalidate the last schema. However, in an alike spirit to the logic **DL***, one can define classical negation in **HEGFL**, and not only that, but also a classical conditional, that in certain combinations with the primitive negation and conditional in the formulas of **HEGFL** can validate or invalidate some connexive schemas.

The key to obtain the classical negation and conditional is by drawing upon the partition between designated and antidesignated set of values as follows. In **HEGFL** one can define the constants $\mathbf{1}$, $\mathbf{c}$, $\mathbf{c^*}$ as $\mathbf{1} =_{def} A \wedge \sim A$, $\mathbf{c} =_{def} \sim (A \wedge \sim A)$ and $\mathbf{c^*} =_{def} \sim\sim (A \wedge \sim A)$—let $\mathbf{Cx}$ denote any constant. Considering that in (anti)designatedness jargon, classical negation exchanges designated values for antidesignated values and vice versa, classical negation can be defined as $\neg A =_{def} ((A \leftrightarrow \mathbf{1}) \wedge \mathbf{c^*})$, because one obtains the following table:

A	$((A \leftrightarrow \mathbf{1}) \wedge \mathbf{c^*})$[4]
1	$c*$
c	1
$c*$	1

By the same token, considering that classical conditional is antidesignated if and only if the antecedent is designated and the consequent antidesignated, one can define classical conditional as $(A \supset B) =_{def} ((\neg\neg A \to (\mathbf{Cx} \wedge \neg\neg B)) \to\sim ((\mathbf{Cx} \wedge \neg\neg A) \to \neg\neg A)) \wedge \mathbf{c^*})$, because one has the following table

A	B	$((\neg\neg A \to (\mathbf{Cx} \wedge \neg\neg B)) \to\sim ((\mathbf{Cx} \wedge \neg\neg A) \to \neg\neg A)) \wedge \mathbf{c^*})$
1	1	1
1	c	$c*$
1	$c*$	$c*$
c	1	1
c	c	1
c	$c*$	1
$c*$	1	1
$c*$	c	1
$c*$	$c*$	1

[4] Note, however, that classical negation so defined conflates $c*$ and c. Therefore, once one adds it to **HEGFL**, one cannot obtain the values of the original formula A by double negation elimination.

Given the well-known fact that in the model theory structure of classical logic the set $\{\neg, \supset\}$ is a functionally complete set of connectives, and that in **HEGFL** one can model that structure by appealing to the division of designated and antidesignated values, one can define all classical logic within **HEGFL**, what makes classical logic a sublogic of **HEGFL**, as in the case of **DL***.

With the new connectives thus introduced, and combining them with the basic connectives of **HEGFL** in the characteristic schemas of connexive logic, the following schemas are valid:

$$
\begin{array}{cc}
\neg(A \to\sim A) & \neg(\neg A \to A) \\
\neg(A \to \neg A) & \neg(\sim A \to A) \\
 & \\
(A \to B) \supset\sim (A \to\sim B) & (A \to B) \supset \neg(A \to \neg B) \\
(A \to B) \supset\sim (A \to \neg B) & (A \to B) \supset \neg(A \supset\sim B) \\
(A \to B) \supset\sim (A \supset\sim B) & (A \to B) \supset \neg(A \supset \neg B) \\
(A \to B) \supset\sim (A \supset \neg B) & (A \to B) \supset \neg(A \to\sim B) \\
(A \to B) \supset \neg(A \to\sim B) & (A \to B) \supset \neg(A \to \neg B) \\
 & \\
(A \to\sim B) \supset\sim (A \to B) & (A \to \neg B) \supset \neg(A \supset B) \\
(A \to \neg B) \supset\sim (A \to B) & (A \to\sim B) \supset \neg(A \to B) \\
(A \to \neg B) \supset\sim (A \supset B) & (A \to \neg B) \supset \neg(A \to B) \\
(A \to\sim B) \supset \neg(A \to B) & (A \supset\sim B) \supset \neg(A \to B) \\
(A \to\sim B) \supset \neg(A \supset B) & (A \supset \neg B) \supset \neg(A \to B)
\end{array}
$$

To prove the validity of all these schemas it is enough to note that the schemas with the form $A \to B$ are always antidesignated, so the schemas with the form $\neg(A \to B)$ are always designated. By the same reason, when a schema with the form $A \to B$ appears as antecedent of a classical conditional, the classical conditional is designated. The proofs of the rest of the schemas can be made by simply making its tables and checking if they turn out to be true under every assignement of values to its propositional variables.

But one also obtains the unfortunate validation of the following schemas, which are versions of symmetry:

$$
\begin{array}{cc}
(A \to B) \supset (B \supset A) & (A \to B) \supset (B \to A)
\end{array}
$$

and the invalidation of the following schemas (to avoid tedious repetition of several formulas, when possible I use the following notation $(A \underset{\supset}{\to} B) \underset{\supset}{\to} \underset{\neg}{\sim} (A \underset{\supset}{\to} \underset{\sim}{\neg} B)$ to say that from the connectives that appear between formulas one can choose any of them):

$$
\begin{array}{cc}
(A \supset B) \supset\sim (A \supset \neg B) & (A \supset B) \supset\sim (A \to\sim B) \\
(A \supset B) \supset\sim (A \supset\sim B) & \\
(A \supset B) \supset \neg(A \supset\sim B) & (A \supset B) \supset\sim (A \to \neg B) \\
(A \supset B) \supset \neg(A \supset \neg B) & (A \underset{\supset}{\to} B) \to \underset{\neg}{\sim} (A \underset{\supset}{\to} \underset{\sim}{\neg} B)
\end{array}
$$

$$(A \supset\sim B) \supset\sim (A \supset B)$$
$$(A \supset\sim B) \supset\sim (A \rightarrow B)$$
$$(A \supset\sim B) \supset \neg(A \supset B)$$
$$(A \supset \neg B) \supset\sim (A \supset B)$$

$$(A \supset \neg B) \supset\sim (A \rightarrow B)$$
$$(A \supset \neg B) \supset\sim (A \supset B)$$
$$(A \overset{\rightarrow}{\supset} \overset{\neg}{\sim} B) \rightarrow \overset{\sim}{\neg} (A \overset{\rightarrow}{\supset} B)$$

$$(A \supset B) \rightarrow (B \supset A)$$
$$(A \supset B) \rightarrow (B \rightarrow A)$$
$$(A \supset B) \supset (B \supset A)$$

$$(A \supset B) \supset (B \rightarrow A)$$
$$(A \rightarrow B) \rightarrow (B \supset A)$$
$$(A \rightarrow B) \rightarrow (B \rightarrow A)$$

I can make a case for these results. In [12] Claudio Pizzi has characterized a *weak system of connexive logic* as a logic that validates Boethius' thesis in the form $(A \rightarrow B) \supset\sim (A \rightarrow\sim B)$ (also known as *weak Boethius' thesis* in the context of consequential implication, see [14]), where '$\rightarrow$' represents here a "connexive conditional", meaning by this a conditional that is true if and only if there is some kind of connection —that can be analytic, nomic, and so on— between the antecedent and the consequent. In Pizzi's perspective, the desirable conditions of any such kind of connection can be established without spelling out beforehand that very same informal notion of connection. According to Pizzi, *the minimal property of connection of any connexive conditional* is the following: that if A and B are (say, analytically) connected, then A and $\sim B$ are not (analytically) connected (a property that validates Boethius' thesis).[5]

As I have remarked, the **HEGFL** conditional $\rightarrow$ is not a connexive conditional, as it, together with **HEGFL**'s primitive negation $\sim$, does not validate any connexive schema. However, in using the classical conditional and negation, not only some variants of Aristotle's thesis, but also the weak Boethius' thesis is validated, an outcome that probably would have pleased Pizzi.

Additionally, the only connexive schemas that turn out to be valid are the ones that have the connexive conditional as second main connective (as in Aristotle's theses) or in the antecedent (as in Boethius' theses). The invalid schemas are those with the classical conditional in the antecedent. Moreover, the validation of the schemas that are versions of symmetry are not so bad at it could seem *prima facie*. The schema $(A \rightarrow B) \supset (B \supset A)$ suggests that if there is a connection between the antecedent and the consequent, then the antecedent is necessary for the consequent. Similarly, $(A \rightarrow B) \supset (B \rightarrow A)$ can suggest that if there is a connection between the antecedent and the consequent, then the same can be said conversely: there is a connection between the consequent and the antecedent in any connexive conditional, a connection that holds because in the first place the connection between the antecedent and the consequent already holds.

Until now, I have not touched upon the meaning of basic connectives of **HEGFL**. As it could be seen, the connectives have very strange tables, as they show that the conjunction of two designated formulas results in an antidesignated

[5] In [13], Pizzi defines the notion of connection as follows: A and B are connected when either A is subalternant of B or B is a subalternant of A, that is, when A implies B or vice-versa.

formula. Perhaps one could think that those connectives are not the intended connectives, that is, that the symbols used do not represent what they were supposed to represent. For instance, negation is never true under any assignment to its immediate subformula, and nevertheless this does not entail that negation is always 'false', because there are two completely different antidesignated values that can take its place. To keep this distinction, one could identify $c*$ with non-truth and c with falsity. Under this interpretation, the satisfiability conditions of negation

A	$\sim A$
1	c^*
c	c^*
c^*	c

are the following:

$\sim A$ is not true iff A is either true or false

$\sim A$ is false iff A is not true

What such interpretation amounts to is to the falsity conditions of a demi-negation connective (see [8],[10]), that is, a unary connective $\circlearrowleft$ such that $i(\neg A) = i(\circlearrowleft\circlearrowleft A)$. The first condition resembles a connective that can be read as "it is not exhaustive that..." or "the formula A lacks a classical truth value". In a sense, the negation of **HEGFL** is kindred with two different "negative" connectives, i.e. connectives such that i) are unary connectives and ii) tweak at least one of the evaluation conditions of negation, either its truth conditions — $\sim A$ is true if and only if A is false— or its falsity conditions — $\sim A$ is false if and only if A is true—.

Conclusions and future work

In this paper I made a quick study of two dialectical logics, **DL*** and **HEGFL**. I presented **DL*** in proof- and model-theoretic terms, and I mentioned its more distinguished properties. Then I presented the logic **HEGFL** as another dialectical logic, very different from **DL***. Also, I discussed one form to define classical logic within **HEGFL** and I showed some unexpected connections of **HEGFL** to connexive logics and two different negative connectives.

It is worth mentioning that the decision method of **HEGFL** is truth-functional, and this characteristic lets to take few steps to establish whether an argument is valid. On the other hand, the decision method of **DL*** is not truth-functional, and this characteristic makes necessary to consider more steps to establish the validity of any argument. As a bonus, in **HEGFL** one can recover all the properties that **DL*** has with less linguistic resources.

As a future work, it remains to spell out in detail the meaning of all the connectives in **HEGFL** using a bivalent semantics. Also, **HEGFL** still lacks any proof-theoretic presentation, which could also shed light on some of its features. Finally, I deliberately ignored the discussion about to what extent **DL*** and **HEGFL** recover Hegel's philosophy. I do not aim at doing this in future work, but perhaps others could try to do it.

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