The Use of E. Cartan Mechanics in Quantum Electrodynamics of a Meson Field*

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Abstract. In this paper, the mechanics of E. Cartan is used to formulate the quantum electrodynamics of the meson field. The dynamics of quantized fields are written in the form of Cartan mechanics. One of the Cartan equations - the Schrödinger equation is solved by the perturbation theory method. As a result, the processes of photon boson emission and photon boson absorption are studied.

The modern use of the tools of Cartan mechanics for the formulation of all branches of theoretical physics: mechanics, electrodynamics, quantum mechanics also involves the spread of Cartan mechanics in problems of quantum electrodynamics.

Along with the well-known mechanics of Lagrange and Hamilton, the use of Cartan mechanics tools has become very promising

This paper answers this question. To quantize the meson field, the Lagrangian and Hamiltonian formalism is used. And for quantization of the electromagnetic field, Maxwell's equations and the energy formula of the electromagnetic field are used. The type of electromagnetic current is derived from the Lagrangian invariance concerning the phase Ψ-operator of the meson field.

And the form of electromagnetic interaction of the electromagnetic field is. A_μ with a meson current from electrodynamics

Keywords: Cartan mechanics; quantum electrodynamics, meson field.

1 Introduction

Along with the well-known mechanics of Lagrange and Hamilton, the use of Cartan mechanics tools has become very promising [5].

Its application first to the problems of mechanics, then, in general relativity, Einstein and, finally, to field theory) showed the universality and convenience of its application to other problems of physics.

One of these problems is quantum field theory and, in particular, quantum electrodynamics.

In this paper, the mechanics of E. Cartan is used to formulate the quantum electrodynamics of the meson field.

Mesons are bound states of a quark and an antiquark. Mesons have a baryon number B = 0 and an integer (including zero) spin, i.e., they are bosons. The masses and quantum numbers of mesons are determined by the types of quark and antiquark that make up the meson, their radial quantum numbers, the relative orientation of their spins, and the values of isospins and orbital moments. The interaction caused by the meson field of nuclear forces is carried out using virtual particles.

The quark model allows one to qualitatively describe the structure of mesons and to obtain their quantum numbers.

The dynamics of quantized fields are written in the form of Cartan mechanics. One of the Cartan equations - the Schrödinger equation is solved by the perturbation theory method. As a result, the processes of photon boson emission and photon boson absorption are studied.

2 Main content. E. Cartan mechanics in quantum electrodynamics of a meson field

The Lagrangian of the meson field L has the form [10]:

$$\mathsf{L} = \int dV \left\{ \frac{1}{c^2} \frac{\partial \Psi^*}{\partial t} \frac{\partial \Psi}{\partial t} - \left(\vec{\nabla}\Psi^*\right) \cdot \left(\vec{\nabla}\Psi\right) - \frac{m^2c^2}{\hbar^2}\Psi^*\Psi \right\} \tag{1}$$

The Lagrange equation has the form:

$$\frac{\partial}{\partial t} \frac{\delta \mathsf{L}}{\delta \frac{\partial \Psi^*}{\partial t}} - \frac{\delta \mathsf{L}}{\delta \Psi^*} = 0 = \frac{1}{c^2} \frac{\partial^2 \Psi}{\partial t^2} - \Delta\Psi + \frac{m^2c^2}{\hbar^2}\Psi \tag{2}$$

By rewriting equation (2) in the form:

$$\frac{1}{c^2} \frac{\partial^2 \Psi}{\partial t^2} = \left\{ \Delta - \frac{m^2c^2}{\hbar^2} \right\} \Psi, \tag{3}$$

see that it follows from the equation::

$$i\hbar \frac{\partial \Psi}{\partial t} = c\sqrt{-\hbar^2\Delta + m^2c^2}\Psi \tag{4}$$

Equation (4) can be obtained from Lagrangian L_1:

$$\mathsf{L}_1 = \int dV \Psi^* \left\{ i\hbar \frac{\partial \Psi}{\partial t} - c\sqrt{-\hbar^2\Delta + m^2c^2}\Psi \right\}, \tag{5}$$

really:

$$\frac{\partial}{\partial t}\frac{\delta \mathsf{L}_1}{\delta\frac{\partial \Psi^*}{\partial t}} - \frac{\delta \mathsf{L}_1}{\delta \Psi^*} = -\left\{i\hbar\frac{\partial \Psi}{\partial t} - c\sqrt{-\hbar^2\Delta + m^2c^2}\Psi\right\}.$$

The Hamiltonian corresponding to the Lagrangian L_1 has the form:

$$H = \int dV \Pi_\Psi \frac{\partial \Psi}{\partial t} + \Pi_{\Psi^*}\frac{\partial \Psi^*}{\partial t} - \mathcal{L},$$

$$\text{where } \Pi_\Psi = \frac{\delta \mathsf{L}_1}{\delta\frac{\partial \Psi}{\partial t}} = i\hbar\Psi^*,\ \Pi_{\Psi^*} = \frac{\delta \mathsf{L}_1}{\delta\frac{\partial \Psi^*}{\partial t}} = 0.$$

For the second quantization of mesons, the corresponding ideas of Haken [10] were used:

$$H = \int dV i\hbar\Psi^*\frac{\partial \Psi}{\partial t} - \mathcal{L} = \int dV \Psi^* c\sqrt{-\hbar^2\Delta + m^2c^2}\Psi \tag{6}$$

when by expanding the operators Ψ и Ψ^+ in the creation and annihilation operators of quanta of this field - mesons, we reduce the corresponding Hamiltonian (6) to the second quantization representation (7).

To pass to the representation of secondary quantization, we expand Ψ by the annihilation operators:

$$\Psi = \sum_{\vec{K}}\frac{e^{-i\vec{K}\cdot\vec{r}}}{(2\pi)^{\frac{3}{2}}}a_{\vec{K}},$$

and $\Psi^+ = \sum_{\vec{K}}\frac{e^{-i\vec{K}\cdot\vec{r}}}{(2\pi)^{\frac{3}{2}}}a^+{}_{\vec{K}}$ concerning the creation operators.

Hamiltonian (6) in the second quantization representation will take the form:

$$H_1 = \int dV \Psi^+ c\sqrt{-\hbar^2\Delta + m^2c^2}\Psi = \sum_{\vec{K}'\vec{K}} c\sqrt{\hbar^2K^2 + m^2c^2}\int dV\frac{e^{i\left(\vec{K}'\cdot\vec{r} - \vec{K}\cdot\vec{r}\right)}}{(2\pi)^3}a^+{}_{\vec{K}'}a_{\vec{K}} =$$
$$= \sum_{\vec{K}} c\sqrt{\hbar^2K^2 + m^2c^2}\,a^+{}_{\vec{K}}a_{\vec{K}} \tag{7}$$

To quantize the electromagnetic field, we use the exposition method in [23].

The electric field strength $\vec{E}$ and the magnetic field induction $\vec{H}$ can be represented in the form:

$$\vec{E} = \sum_{\vec{K}\alpha}\omega_{\vec{K}}q_{\vec{K}\alpha}\vec{e}_{\vec{K}\alpha}\sqrt{4\pi}, \tag{8}$$

$$\vec{H} = \sum_{\vec{K}\alpha}P_{\vec{K}\alpha}\vec{h}_{\vec{K}\alpha}\sqrt{4\pi}. \tag{9}$$

Here $\hbar\vec{K} = \vec{P}$ – is the photon momentum, α – indices of 2 directions perpendicular to it. We substitute the expressions (8) and (9) into the Maxwell equations for the electromagnetic field in a vacuum: $\vec{\nabla}\cdot\vec{E} = \vec{\nabla}\cdot\vec{H} = 0$, this gives the equation:

$$\vec{\nabla} \cdot \vec{e}_{\vec{K}\alpha} = \vec{\nabla} \cdot \vec{h}_{\vec{K}\alpha} = 0,$$

$$\vec{\nabla} \times \vec{E} = -\frac{1}{c}\frac{\partial \vec{H}}{\partial t} = \sum_{\vec{K}\alpha} \omega_{\vec{K}} q_{\vec{K}\alpha}(t) \vec{\nabla} \times \vec{e}_{\vec{K}\alpha} = -\sum_{\vec{K}\alpha} \frac{dP_{\vec{K}\alpha}(t)}{dt} \frac{1}{c} \vec{h}_{\vec{K}\alpha}, \text{ gives}$$

$$\frac{dP_{\vec{K}\alpha}(t)}{dt} = -\omega^2{}_{\vec{K}} q_{\vec{K}\alpha} \text{ и } \vec{\nabla} \times \vec{e}_{\vec{K}\alpha} = \frac{\omega_{\vec{K}}}{c} \vec{h}_{\vec{K}\alpha} \tag{10}$$

The equation:

$$\vec{\nabla} \times \vec{H} = \frac{1}{c}\frac{\partial \vec{E}}{\partial t} = \sum_{\vec{K}\alpha} \vec{P}_{\vec{K}} \vec{\nabla} \times \vec{h}_{\vec{K}\alpha} = \sum_{\vec{K}\alpha} \frac{1}{c} \omega_{\vec{K}} \frac{dq_{\vec{K}\alpha}}{dt} \vec{e}_{\vec{K}\alpha},$$

leads to:

$$P_{\vec{K}\alpha}(t) = \frac{dq_{\vec{K}\alpha}(t)}{dt} \text{ и } \vec{\nabla} \times \vec{h}_{\vec{K}\alpha} = \frac{\omega_{\vec{K}}}{c} \vec{e}_{\vec{K}\alpha}. \tag{11}$$

Calculating the rotor from the expressions (10) and (11) we get::

$$\vec{\nabla} \times \left(\vec{\nabla} \times \vec{e}_{\vec{K}\alpha}\right) = \vec{\nabla}\vec{\nabla} \cdot \vec{e}_{\vec{K}\alpha} - \Delta \vec{e}_{\vec{K}\alpha} = \frac{\omega_{\vec{K}}}{c} \vec{\nabla} \times \vec{h}_{\vec{K}\alpha},$$

what gives

$$\Delta \vec{e}_{\vec{K}\alpha} = -\left(\frac{\omega_{\vec{K}}}{c}\right)^2 \vec{e}_{\vec{K}\alpha}. \tag{12}$$

Similarly:

$$\vec{\nabla} \times \left(\vec{\nabla} \times \vec{h}_{\vec{K}\alpha}\right) = \vec{\nabla}\vec{\nabla} \cdot \vec{h}_{\vec{K}\alpha} - \Delta \vec{h}_{\vec{K}\alpha} = \frac{\omega_{\vec{K}}}{c} \vec{\nabla} \times \vec{e}_{\vec{K}\alpha},$$

what gives

$$\Delta \vec{h}_{\vec{K}\alpha} = -\left(\frac{\omega_{\vec{K}}}{c}\right)^2 \vec{h}_{\vec{K}\alpha}. \tag{13}$$

Expressions (12) and (13) show that $\vec{e}_{\vec{K}\alpha}$ и $\vec{h}_{\vec{K}\alpha}$ – are eigenvectors of the d'Alembert operator Δ with eigenvalues $\left(\frac{\omega_{\vec{K}}}{c}\right)^2$, therefore $\vec{e}_{\vec{K}\alpha}$ are orthogonal $\vec{e}_{\vec{K}'_{\alpha'}}$ $\vec{K} \neq \vec{K}'$ or $\alpha \neq \alpha'$, similarly $\vec{h}_{\vec{K}\alpha}$ are orthogonal $\vec{h}_{\vec{K}'\alpha'}$ $\vec{K} \neq \vec{K}'$ or $\alpha \neq \alpha'$.

Thus, the energy of the electromagnetic field H_2 is equal to:

$$H_2 = \frac{1}{8\pi} \int dV (H^2 + E^2) = \frac{1}{8\pi} \cdot 4\pi \int dV \left(\sum_{\vec{K}\alpha\vec{K}'\alpha'} P_{\vec{K}\alpha} P_{\vec{K}'\alpha'} \cdot \vec{h}_{\vec{K}\alpha} \cdot \vec{h}_{\vec{K}'\alpha'} + \right.$$
$$\left. + \sum_{\vec{K}\alpha\vec{K}'\alpha'} \omega_{\vec{K}} \omega_{\vec{K}'} q_{\vec{K}\alpha} q_{\vec{K}'\alpha'} P_{\vec{K}\alpha} P_{\vec{K}'\alpha'} \vec{e}_{\vec{K}\alpha} \cdot \vec{e}_{\vec{K}'\alpha'}\right) = \frac{1}{2} \sum_{\vec{K}\alpha} \left(P_{\vec{K}\alpha}{}^2 + \omega_{\vec{K}}{}^2 q_{\vec{K}\alpha}{}^2\right) =$$

$$= \sum_{\vec{K}\alpha} \hbar\omega_{\vec{K}} \left(\frac{P_{\vec{K}\alpha}{}^2}{2\hbar\omega_{\vec{K}}} + \frac{\omega_{\vec{K}}}{2\hbar} q_{\vec{K}\alpha}{}^2 \right) = \sum_{\vec{K}\alpha} \hbar\omega_{\vec{K}} \left[\left(\frac{P_{\vec{K}\alpha}}{\sqrt{2\hbar\omega_{\vec{K}}}} + i\sqrt{\frac{\omega_{\vec{K}}}{2\hbar}} q_{\vec{K}\alpha} \right) \left(\frac{P_{\vec{K}\alpha}}{\sqrt{2\hbar\omega_{\vec{K}}}} - \right.\right.$$

$$\left.\left. - i\sqrt{\frac{\omega_{\vec{K}}}{2\hbar}} q_{\vec{K}\alpha} \right) + \frac{1}{2} \right] = \sum_{\vec{K}\alpha} \hbar\omega_{\vec{K}} \left(a^{+}_{\vec{K}\alpha} a_{\vec{K}\alpha} + \frac{1}{2} \right) \tag{14}$$

We have obtained the Hamiltonian of the electromagnetic field in the secondary quantization representation.

The Lagrangian of the meson field (1) is invariant concerning the transformations:

$$\Psi \to e^{i\alpha}\Psi \text{ и } \Psi^* \to e^{-i\alpha}\Psi^*.$$

Therefore [3]:

$$\delta\mathcal{L} = \frac{\partial\mathcal{L}}{\partial\Psi}\delta\Psi + \frac{\partial\mathcal{L}}{\partial\Psi^*}\delta\Psi^* + \frac{\partial\mathcal{L}}{\partial\frac{\partial\Psi}{\partial x^\mu}}\delta\frac{\partial\Psi}{\partial x^\mu} + \frac{\partial\mathcal{L}}{\partial\frac{\partial\Psi^*}{\partial x^\mu}}\delta\frac{\partial\Psi^*}{\partial x^\mu} = \left(\frac{\partial\mathcal{L}}{\partial\Psi} - \frac{\partial}{\partial x^\mu}\frac{\partial\mathcal{L}}{\partial\frac{\partial\Psi}{\partial x^\mu}} \right)\delta\Psi + \left(\frac{\partial\mathcal{L}}{\partial\Psi^*} - \right.$$

$$\left. - \frac{\partial}{\partial x^\mu}\frac{\partial\mathcal{L}}{\partial\frac{\partial\Psi^*}{\partial x^\mu}} \right)\delta\Psi^* + \frac{\partial}{\partial x^\mu}\left(\frac{\partial\mathcal{L}}{\partial\frac{\partial\Psi}{\partial x^\mu}}\delta\Psi \right) + \frac{\partial}{\partial x^\mu}\left(\frac{\partial\mathcal{L}}{\partial\frac{\partial\Psi^*}{\partial x^\mu}}\delta\Psi^* \right) \tag{15}$$

Using infinitesimal transformations:

$$\Psi' = (1 + i\alpha)\Psi \text{ и } \Psi^{*'} = (1 - i\alpha)\Psi^* \quad \Psi' - \Psi = \delta\Psi = i\alpha\Psi \quad \Psi^{*'} - \Psi^* =$$

$$= \delta\Psi^* = -i\alpha\Psi^*$$

and the Lagrange equations we bring (15) to the form:

$$0 = \delta\mathcal{L} = \left\{ i\frac{\partial}{\partial x^\mu}\left(\frac{\partial\mathcal{L}}{\partial\frac{\partial\Psi}{\partial x^\mu}}\Psi \right) - i\frac{\partial}{\partial x^\mu}\left(\frac{\partial\mathcal{L}}{\partial\frac{\partial\Psi^*}{\partial x^\mu}}\Psi^* \right) \right\}\alpha.$$

So, a consequence of the Lagrangian invariance is the existence of the 4th vector j^μ:

$$j^\mu = i\frac{\partial\mathcal{L}}{\partial\frac{\partial\Psi}{\partial x^\mu}}\Psi - i\frac{\partial\mathcal{L}}{\partial\frac{\partial\Psi^*}{\partial x^\mu}}\Psi^*. \tag{16}$$

For which the conservation law holds

$$\partial_\mu \tilde{j}^\mu = 0 \tag{17}$$

Using (1) we obtain the explicit form of the probability current:

$$\tilde{j}^\mu = i\left(\frac{\partial\Psi}{\partial x^\nu}\Psi^* - \frac{\partial\Psi}{\partial x^\nu}\Psi \right) g^{\mu\nu}.$$

Electricity:

$$j^\mu = e\tilde{j}^\mu \tag{18}$$

Thus, the Hamiltonian of the interaction of an electromagnetic field with a field having an electric charge [16]

$$H_{вз} = \frac{1}{c}\int dV j^{\mu} A_{\mu}. \tag{19}$$

From the expression for j^{μ} and A_{μ} it can be seen that $H_{вз}$ has the form:

$$H_{вз} = \sum_{\vec{K}\alpha\vec{q}} \left(g_{\vec{q}} a^{+}_{\vec{K}-\vec{q}} a_{\vec{K}} b^{+}_{\vec{q}\alpha} + g^{*}_{\vec{q}} a^{+}_{\vec{K}} a_{\vec{K}-\vec{q}} b_{\vec{q}\alpha} \right). \tag{20}$$

Using the results of the second quantization of the meson and electromagnetic fields and their interaction, one can formulate quantum electrodynamics in the form of the mechanics of E. Cartan [5].

To do this, take the 2 Cartan form Ω in the form:

$$\begin{aligned} \Omega = \Big\{ i\hbar d|\, 1t\rangle - \Big[\textstyle\sum_i \sqrt{\hbar^2 K_i^2 + mc^2}\, a_i^{+} a_i + \sum_{\vec{K}\alpha} \hbar\omega_{\vec{K}} \left(b^{+}_{\vec{K}\alpha} b_{\vec{K}\alpha} + \frac{1}{2} \right) + \\ + \textstyle\sum_{\vec{K}\alpha\vec{q}} \left(g a^{+}_{\vec{K}-\vec{q}} a_{\vec{K}} b^{+}_{\vec{q}\alpha} + g^{*} a^{+}_{\vec{K}} a_{\vec{K}-\vec{q}} b_{\vec{q}\alpha} \right) \Big] |\, 1t\rangle dt \Big\} \wedge d\xi + \\ + \textstyle\sum_{\vec{K}\vec{K}'} \left([b_{\vec{K}}, b^{+}_{\vec{K}'}] - \delta_{\vec{K}\vec{K}'} \right) d\xi^{\vec{K}} \wedge d\eta^{\vec{K}'} + \sum_{\vec{K}\vec{K}'} [b_{\vec{K}}, b_{\vec{K}'}] d\mu^{\vec{K}} \wedge d\theta^{\vec{K}'} + \\ + \textstyle\sum_{\vec{K}\vec{K}'} [b^{+}_{\vec{K}}, b^{+}_{\vec{K}'}] d\nu^{\vec{K}} \wedge d\varepsilon^{\vec{K}'} + \sum_{\vec{K}\vec{K}'} \left([a_{\vec{K}}, a^{+}_{\vec{K}'}] - \delta_{\vec{K}\vec{K}'} \right) d\alpha^{\vec{K}} \wedge d\beta^{\vec{K}'} + \\ + \textstyle\sum_{\vec{K}\vec{K}'} [a_{\vec{K}}, a_{\vec{K}'}] d\gamma^{\vec{K}} \wedge d\delta^{\vec{K}'} + \sum_{\vec{K}\vec{K}'} [a^{+}_{\vec{K}}, a^{+}_{\vec{K}'}] d\omega^{\vec{K}} \wedge d\mathfrak{z}^{\vec{K}'}. \end{aligned} \tag{21}$$

Here

$$[a, b] = ab - ba \tag{22}$$

The equations of E. Cartan [5] give:

$$\begin{aligned} 0 = \frac{\delta\Omega}{\delta d\xi} = \frac{\delta^2\Omega}{\delta d\eta^{\vec{K}'} \delta d\xi^{\vec{K}}} = \frac{\delta^2\Omega}{\delta d\theta^{\vec{K}'} \delta d\mu^{\vec{K}}} = \frac{\delta^2\Omega}{\delta d\varepsilon^{\vec{K}'} \delta d\nu^{\vec{K}}} = \frac{\delta^2\Omega}{\delta d\beta^{\vec{K}'} \delta d\alpha^{\vec{K}}} = \frac{\delta^2\Omega}{\delta d\delta^{\vec{K}'} \delta d\gamma^{\vec{K}}} = \\ \frac{\delta^2\Omega}{\delta d\mathfrak{z}^{\vec{K}'} \delta d\omega^{\vec{K}}} = -\Big\{ i\hbar d|\, 1t\rangle - \Big[\textstyle\sum_i \sqrt{\hbar^2 K_i^2 + mc^2}\, a_i^{+} a_i + \sum_{\vec{K}\alpha} \hbar\omega_{\vec{K}} \left(b^{+}_{\vec{K}\alpha} b_{\vec{K}\alpha} + \frac{1}{2} \right) + \\ + \textstyle\sum_{\vec{K}\alpha\vec{q}} \left(g a^{+}_{\vec{K}-\vec{q}} a_{\vec{K}} b^{+}_{\vec{q}\alpha} + g^{*} a^{+}_{\vec{K}} a_{\vec{K}-\vec{q}} b_{\vec{q}\alpha} \right) \Big] |\, 1t\rangle dt \Big\} = [b_{\vec{K}}, b^{+}_{\vec{K}'}] - \delta_{\vec{K}\vec{K}'} = [b_{\vec{K}}, b_{\vec{K}'}] == \\ [b^{+}_{\vec{K}}, b^{+}_{\vec{K}'}] = [a_{\vec{K}}, a^{+}_{\vec{K}'}] - \delta_{\vec{K}\vec{K}'} = [a_{\vec{K}}, a_{\vec{K}'}] = [a^{+}_{\vec{K}}, a^{+}_{\vec{K}'}]. \end{aligned} \tag{23}$$

Equation (23) describes two interacting bosonic fields and their dynamics, which is described by the Schrödinger equation [10].

$$\begin{aligned} i\hbar \frac{d|1t\rangle}{dt} = \Big[\textstyle\sum_i \sqrt{\hbar^2 K_i^2 + mc^2}\, a_i^{+} a_i + \sum_{\vec{K}\alpha} \hbar\omega_{\vec{K}} \left(b^{+}_{\vec{K}\alpha} b_{\vec{K}\alpha} + \frac{1}{2} \right) + \\ + \textstyle\sum_{\vec{K}\alpha\vec{q}} \left(g a^{+}_{\vec{K}-\vec{q}} a_{\vec{K}} b^{+}_{\vec{q}\alpha} + g^{*} a^{+}_{\vec{K}} a_{\vec{K}-\vec{q}} b_{\vec{q}\alpha} \right) \Big] |\, 1t\rangle. \end{aligned} \tag{24}$$

Introducing the notation H_0 for the sum of the Hamiltonians of the free fields of the meson and electromagnetic fields:

$$H_0 = \textstyle\sum_i \sqrt{\hbar^2 K_i^2 + mc^2}\, a_i^{+} a_i + \sum_{\vec{K}\alpha} \hbar\omega_{\vec{K}} \left(b^{+}_{\vec{K}\alpha} b_{\vec{K}\alpha} + \frac{1}{2} \right) \tag{25}$$

and H_1 for the Hamiltonian of the interaction of the meson field with the electromagnetic field:

$$H_1 = \sum_{\vec{K}\alpha\vec{q}} \left(g a^{+}_{\vec{K}-\vec{q}} a_{\vec{K}} b^{+}_{\vec{q}\alpha} + g^{*} a^{+}_{\vec{K}} a_{\vec{K}-\vec{q}} b_{\vec{q}\alpha} \right) \tag{26}$$

we rewrite equation (24) in the form:

$$i\hbar \frac{d|1t\rangle}{dt} = (H_0 + H_1)|\,1t\rangle. \tag{27}$$

And let's move on to the description of the interaction [10]:

$$|\,1t\rangle = e^{\frac{-iH_0 t}{\hbar}}|\,t\rangle \tag{28}$$

$$H_0|\,1t\rangle + e^{\frac{-iH_0 t}{\hbar}} i\hbar \frac{d|t\rangle}{dt} = H_0|\,1t\rangle + H_1 e^{\frac{-iH_0 t}{\hbar}}|\,t\rangle,$$

what gives:

$$i\hbar \frac{d|t\rangle}{dt} = e^{\frac{iH_0 t}{\hbar}} H_1 e^{\frac{-iH_0 t}{\hbar}}|\,t\rangle = H_1(t)|\,t\rangle \tag{29}$$

For calculate

$$H_1(t) = e^{\frac{iH_0 t}{\hbar}} H_1 e^{\frac{-iH_0 t}{\hbar}}$$

we use [10]

$$b_{\vec{q}\alpha}(t) = e^{\frac{iH_0 t}{\hbar}} b_{\vec{q}\alpha} e^{\frac{-iH_0 t}{\hbar}} = e^{i\omega_{\vec{q}} b^{+}_{\vec{q}\alpha} b_{\vec{q}\alpha} t} b_{\vec{q}\alpha} e^{-i\omega_{\vec{q}} b^{+}_{\vec{q}\alpha} b_{\vec{q}\alpha} t}.$$

Therefore:

$$\frac{db_{\vec{q}\alpha}(t)}{dt} = i\omega_{\vec{q}} \left[b^{+}_{\vec{q}\alpha} b_{\vec{q}\alpha}, b_{\vec{q}\alpha} \right] = -i\omega_{\vec{q}} b_{\vec{q}\alpha}(t). \tag{30}$$

Solution (30) has the form:

$$b_{\vec{q}\alpha}(t) = e^{-i\omega_{\vec{q}} t} b_{\vec{q}\alpha} \tag{31}$$

$$\text{and } b^{+}_{\vec{q}\alpha}(t) = e^{i\omega_{\vec{q}} t} b^{+}_{\vec{q}\alpha}. \tag{32}$$

Similarly:

$$\frac{da_{\vec{K}}(t)}{dt} = -i\sqrt{\vec{K}^2 + \frac{m^2c^2}{\hbar^2}}\, a_{\vec{K}}(t)(\tag{33}$$

$$\text{and } a_{\vec{K}}(t) = e^{-i\sqrt{\vec{K}^2 + \frac{m^2c^2}{\hbar^2}}\, t} a_{\vec{K}},\ a^{+}_{\vec{K}}(t) = e^{i\sqrt{\vec{K}^2 + \frac{m^2c^2}{\hbar^2}}\, t} a^{+}_{\vec{K}}. \tag{34}$$

Therefore:

$$H_1(t) = \sum_{\vec{K}\alpha\vec{q}} \left(g e^{i\left(\sqrt{(\vec{K}-\vec{q})^2 + \frac{m^2c^2}{\hbar^2}} - \sqrt{\vec{K}^2 + \frac{m^2c^2}{\hbar^2}} + \omega_{\vec{q}}\right)\theta} a^+_{\vec{K}-\vec{q}} a_{\vec{K}} b^+_{\vec{q}\alpha} + \right.$$

$$\left. + g^* e^{-i\left(\sqrt{(\vec{K}-\vec{q})^2 + \frac{m^2c^2}{\hbar^2}} - \sqrt{\vec{K}^2 + \frac{m^2c^2}{\hbar^2}} + \omega_{\vec{q}}\right)\theta} a^+_{\vec{K}} a_{\vec{K}-\vec{q}} b_{\vec{q}\alpha} \right). \quad (35)$$

We pass from the differential equation (29) to the integral [10]:

$$| \, t\rangle = | \, 0\rangle + \frac{1}{i\hbar}\int_0^t d\theta H_1(\theta) \, | \, \theta\rangle. \quad (36)$$

In the first order of perturbation theory, integral equation (36) takes the form:

$$| \, t\rangle \approx | \, 0\rangle + \frac{1}{i\hbar}\int_0^t d\theta H_1(\theta) \, | \, 0\rangle = | \, 0\rangle +$$

$$+ \frac{1}{i\hbar}\int_0^t d\theta \sum_{\vec{K}\alpha\vec{q}} \begin{pmatrix} g e^{i\left(\sqrt{(\vec{K}-\vec{q})^2 + \frac{m^2c^2}{\hbar^2}} - \sqrt{\vec{K}^2 + \frac{m^2c^2}{\hbar^2}} + \omega_{\vec{q}}\right)\theta} a^+_{\vec{K}-\vec{q}} a_{\vec{K}} b^+_{\vec{q}\alpha} + \\ + g^* e^{-i\left(\sqrt{(\vec{K}-\vec{q})^2 + \frac{m^2c^2}{\hbar^2}} - \sqrt{\vec{K}^2 + \frac{m^2c^2}{\hbar^2}} + \omega_{\vec{q}}\right)\theta} a^+_{\vec{K}} a_{\vec{K}-\vec{q}} b_{\vec{q}\alpha} \end{pmatrix} | \, 0\rangle. \quad (37)$$

Suppose that in the initial state $| \, 0\rangle$ there is a charged boson with momentum

$$\hbar\vec{K}_1 \colon | \, 0\rangle = a^+_{\vec{K}_1} | \, \text{vacuum}\rangle. \quad (38)$$

Then:

$$| \, t\rangle \approx a^+_{\vec{K}_1} | \, \text{vacuum}\rangle + \frac{1}{i\hbar}\int_0^t d\theta \sum_{\vec{K}\alpha\vec{q}} g_{\vec{q}} e^{i\varepsilon_{\vec{K}\vec{q}}\theta} a^+_{\vec{K}-\vec{q}} a_{\vec{K}_1} b^+_{\vec{q}\alpha} | \, \text{vacuum}\rangle =$$

$$= a^+_{\vec{K}_1} | \, \text{vacuum}\rangle + \frac{1}{i\hbar}\sum_{\vec{q}\alpha} \frac{g_{\vec{q}}}{i\varepsilon_{\vec{K}_1\vec{q}}} \left(e^{i\varepsilon_{\vec{K}_1\vec{q}}t} - 1\right) a^+_{\vec{K}_1-\vec{q}} b^+_{\vec{q}\alpha} | \, \text{vacuum}\rangle \quad (39)$$

Here:

$$\varepsilon_{\vec{K}_1\vec{q}} = \sqrt{\left(\vec{K}-\vec{q}\right)^2 + \frac{m^2c^2}{\hbar^2}} - \sqrt{\vec{K}^2 + \frac{m^2c^2}{\hbar^2}} + \omega_{\vec{q}}. \quad (40)$$

Thus, we obtained a linear combination of the initial state $a^+_{\vec{K}_1} | \, \text{vacuum}\rangle$ of the existence of one charged boson and the state $\sum_{\vec{q}\alpha} C_{\vec{q}\alpha} a^+_{\vec{K}_1-\vec{q}} b^+_{\vec{q}\alpha} | \, \text{vacuum}\rangle$, in which this boson emitted one photon with momentum $\hbar\vec{q}$ and polarization α.

The probability of emitting a photon with $\hbar\vec{q}$ and α is $\left|C_{\vec{q}\alpha}\right|^2$:

$$\left|C_{\vec{q}\alpha}\right|^2 = \frac{1}{\hbar^2}\left|g_{\vec{q}}\right|^2 \frac{4}{\varepsilon^2_{\vec{K}_1\vec{q}}} \sin^2\frac{\varepsilon_{\vec{K}_1\vec{q}}t}{2}. \quad (41)$$

In the limiting case $t \to \infty$ it takes the form:

$$\left|C_{\vec{q}\alpha}\right|^2 \approx \frac{\pi^2}{\hbar^2}\left|g_{\vec{q}}\right|^2 \delta^2\left(\frac{\varepsilon_{\vec{K}_1\vec{q}}}{2}\right) = \frac{\pi^2}{\hbar^2}\left|g_{\vec{q}}\right|^2 \delta\left(\frac{\varepsilon_{\vec{K}_1\vec{q}}}{2}\right) \times \frac{1}{2\pi}\int_{-t}^{t} dt e^{\frac{\varepsilon_{\vec{K}_1\vec{q}}t}{2}} =$$

$$= \frac{2\pi}{\hbar^2}\left|g_{\vec{q}}\right|^2 t\delta\left(\varepsilon_{\vec{K}_1\vec{q}}\right). \quad (42)$$

Differentiating $\left|C_{\vec{q}\alpha}\right|^2$ in time, we find the probability of photon emission per unit time (per second):

$$W_{\vec{q}} = \frac{d\left|C_{\vec{q}\alpha}\right|^2}{dt} = \frac{2\pi}{\hbar^2}\left|g_{\vec{q}}\right|^2 \delta\left(\varepsilon_{\vec{K}_1\vec{q}}\right). \quad (43)$$

If in the initial state $\mid 0\rangle$ there is a charged boson with momentum $\hbar\vec{K}_1$ and an electromagnetic field quantum with momentum$\hbar\vec{K}_2$ and polarization α_1:

$$\mid 0\rangle = a^+_{\vec{K}_1} b^+_{\vec{K}_2\alpha_1} \mid \text{vacuum}\rangle. \quad (44)$$

Then approximate equality (37) takes the form:

$$\mid t\rangle \approx a^+_{\vec{K}_1} b^+_{\vec{K}_2\alpha_1} \mid \text{vacuum}\rangle +$$

$$+\frac{1}{i\hbar}\int_0^t d\theta \sum_{\vec{K}\alpha\vec{q}} g^* e^{-i\varepsilon_{\vec{K}\vec{q}}\theta} a^+_{\vec{K}} a_{\vec{K}-\vec{q}} b_{\vec{q}\alpha} a^+_{\vec{K}_1} b^+_{\vec{K}_2\alpha} \mid \text{vacuum}\rangle = a^+_{\vec{K}_1} b^+_{\vec{K}_2\alpha_1} \mid \text{vacuum}\rangle +$$
$$+\sum_{\vec{K}\alpha\vec{q}} \frac{e^{-i\varepsilon_{\vec{K}\vec{q}}t}-1}{\hbar\varepsilon_{\vec{K}\vec{q}}} g^* a^+_{\vec{K}} a_{\vec{K}-\vec{q}} a^+_{\vec{K}_1} \delta_{\vec{q}\vec{K}_2} \delta_{\alpha\alpha_1} \mid \text{vacuum}\rangle = a^+_{\vec{K}_1} b^+_{\vec{K}_2\alpha_1} \mid \text{vacuum}\rangle +$$
$$+\sum_{\vec{K}\alpha} \frac{e^{-i\varepsilon_{\vec{K}\vec{K}_2}t}-1}{\hbar\varepsilon_{\vec{K}\vec{K}_2}} g^* a^+_{\vec{K}} a_{\vec{K}-\vec{K}_2} a^+_{\vec{K}_1} \delta_{\alpha\alpha_1} \mid \text{vacuum}\rangle = a^+_{\vec{K}_1} b^+_{\vec{K}_2\alpha_1} \mid \text{vacuum}\rangle +$$
$$+\sum_{\vec{K}} \frac{e^{-i\varepsilon_{\vec{K}\vec{K}_2}t}-1}{\hbar\varepsilon_{\vec{K}\vec{K}_2}} g^* a^+_{\vec{K}} \delta_{\vec{K}-\vec{K}_2,\vec{K}_1} \mid \text{vacuum}\rangle = a^+_{\vec{K}_1} b^+_{\vec{K}_2\alpha_1} \mid \text{vacuum}\rangle +$$
$$+\frac{e^{-i\varepsilon_{\vec{K}_1+\vec{K}_2}t}-1}{\hbar\varepsilon_{\vec{K}_1+\vec{K}_2,\vec{K}_2}} g^* a^+_{\vec{K}_1+\vec{K}_2} \mid \text{vacuum}\rangle \quad (45)$$

Thus, we obtained a linear combination of two initial states: a boson with momentum $\hbar\vec{K}_1$ and a photon with momentum $\hbar\vec{K}_2$ nd polarizationα_1 and the final state (after interaction): the state of the boson with momentum $\hbar\left(\vec{K}_1 + \vec{K}_2\right)$, which describes the absorption of the photon boson. The probability of this process is W':

$$\left|C'_{\vec{q}\alpha}\right|^2 = \frac{1}{\hbar^2}\frac{\left|g^*\right|^2}{\varepsilon^2_{\vec{K}_1+\vec{K}_2,\vec{K}_2}} 4\sin^2\frac{\varepsilon_{\vec{K}_1+\vec{K}_2,\vec{K}_2}}{2}t = \frac{\pi^2}{\hbar^2}\left|g^*\right|^2 \delta^2\left(\frac{\varepsilon_{\vec{K}_1+\vec{K}_2,\vec{K}_2}}{2}\right) =$$

$$= \frac{\pi^2}{\hbar^2} \left| g^* \right|^2 \delta^2 \left(\frac{\varepsilon_{\vec{K}_1+\vec{K}_2,\vec{K}_2}}{2} \right) \int_{-\infty}^{\infty} \frac{d\theta}{2\pi} e^{\frac{1}{2} i\theta \varepsilon_{\vec{K}_1+\vec{K}_2,\vec{K}_2}} = \frac{2\pi}{\hbar^2} \left| g^* \right|^2 \delta\left(\varepsilon_{\vec{K}_1+\vec{K}_2,\vec{K}_2} \right) t . \quad (46)$$

$$t \to \infty$$

Based on (46), we obtain the probability of photon absorption per unit time (per second):

$$W^{`}_{\vec{q}} = \frac{d \left| C^{`}_{\vec{q}\alpha} \right|^2}{dt} = \frac{2\pi}{\hbar^2} \left| g^* \right|^2 \delta\left(\varepsilon_{\vec{K}_1+\vec{K}_2,\vec{K}_2} \right) \quad (47)$$

So, we have shown that the mechanics of E. Cartan allows us to formulate quantum electrodynamics in a form convenient for calculations.

3 Conclusions

For the second quantization of mesons, ideas were used [10], which lead to equation (6), by expanding the operators Ψ и Ψ^+ in the creation and annihilation operators of quanta of this field - mesons we reduce the Hamiltonian (6) to the second quantization representation (7).

To quantize the electromagnetic field, we represent E $^{\rightarrow}$ and H $^{\rightarrow}$ in the form of (8) and (9) and substitute these expressions into Maxwell's equations. As a result, the Maxwell equations become the oscillation equations of the pendulums. And the energy of the electromagnetic field becomes the sum of the vibrational energies of the pendulums (14), which is easily quantized. Studying the invariance of the Lagrangian of the meson field, we find the shape of its current. The Landau and Lifshitz field theory suggests the type of interaction of the meson current with the electromagnetic field, which leads to the standard second-quantized form of this interaction. All this allows us to formulate the quantum electrodynamics of the meson field in the form of Eli Cartan mechanics (21) and (23). The Cartan equations give the Schrödinger equation (24) approximately (up to the first order of perturbation theory), solving which we obtain the probability of emission and absorption of a photon by a boson per unit time.

The modern use of the tools of Cartan mechanics for the formulation of all branches of theoretical physics: mechanics, electrodynamics, quantum mechanics [3], also involves the spread of Cartan mechanics in quantum electrodynamics asks.

This paper answers this question. To quantize the meson field, the Lagrangian and Hamiltonian formalism is used. Moreover, for quantization of the electromagnetic field, Maxwell's equations and the energy formula of the electromagnetic field are used. The type of electromagnetic current is derived from the Lagrangian invariance concerning the phase Ψ-operator of the meson field.

Also, the form of electromagnetic interaction of the electromagnetic field is A_μ with a meson current from electrodynamics.

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