Cosmological Aspects of the Particle Physics*

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Abstract. In this paper, the multi-component Higgs field is expressed in terms of one component of this field, which plays the role of an order parameter, following the ideas of H. Haken's synergetics. A mathematical model combining the theory of elementary particles and gravity is proposed. The energy-momentum tensor of the Hilbert – Einstein equation is expressed in terms of an order parameter. The Higgs field, in which the phase transition occurs, eventually plays the role of an inflaton, similar to inflation theories.

Keywords: inflaton, the Higgs field, phase transition, inflation, energy-momentum tensor.

1 Introduction

Research in particle physics and cosmology provides the most in-depth knowledge of our Universe. It is not surprising that one of the global problems facing modern physics is the problem of combining cosmology with elementary particle physics [1, 2].

In the 90s of the 20th century, an accelerated expansion of the Universe was discovered. As it was found [3], the universe consists of about 70% of dark energy, 26% of dark matter, and only 4% of ordinary (baryonic) matter. Currently, we do not have full knowledge of the nature of dark matter and dark energy. So far, it is only known that baryon matter itself does not have enough gravity to explain the structure of our Universe. The rapid rotation of our galaxy would cause its stars to be scattered everywhere. Everything we can see around us has only 10% of the gravity needed to keep stars in their orbits. The existence of the galaxies and superclusters we observe can be explained by the additional gravity of dark matter — a matter that does not emit or reflect light. However, its concentration bends the light passing nearby. Einstein predicted the expansion of the Universe at a rate called the Hubble constant, but modern measurements show a higher rate than Einstein predicted. This continuous acceleration of the Universe is due to dark energy — a repulsive force that acts oppositely to dark matter, causing the Universe to expand rather than merge into organized structures. Our universe is

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almost completely flat, which can only be possible due to the influx of new energy throughout space. The discovery of the accelerated expansion of the Universe contributed to the recognition of the Guth-Linde-Vilenkin theory of Eternal Inflation [4-7]. In this theory, the accelerated expansion of the Universe is caused by a scalar field-an inflaton, when this field is at the «top” of its potential energy – in a pseudo-vacuum state. The transition of an inflaton from a pseudo-vacuum state to a true vacuum state is interpreted as a Big Bang. In the vicinity of a true vacuum, the inflaton behaves like a wave, and its canonical quantization leads to the appearance of quasiparticles.

The interpretation of dark energy and dark matter as different States of the inflaton allows us to build simple and beautiful mathematical models of the dynamics of the Universe, taking into account only the curvature of space-time and the inflaton. Such models can be given the standard form of a Unified field theory. The inflaton energy-momentum tensor is included in the right side of the Hilbert-Einstein equation for the curvature of space-time and, accordingly, in cosmological equations whose solutions describe the accelerated expansion of the Universe [4].

In July 2012, scientists working at the Large Hadron Collider announced the discovery of a new fundamental particle, the Higgs boson. P. Higgs’ prediction of the existence of this fundamental particle — a necessary part of the family of fundamental particles in the Standard model of particle physics — was confirmed.

In the 1960s, Peter Higgs developed a theory explaining how particles carrying electromagnetic or weak interactions could get different masses during the gradual cooling of the Universe. He assumed that particles like protons, neutrons, and quarks gain mass by interacting with an invisible electromagnetic field known as the Higgs field [10]. The discovery of the Higgs boson marked the beginning of new research and a different understanding of reality.

2 The cosmological model

One of the most fruitful ideas of modern particle physics is the idea of P. Higgs that the Higgs boson is a multi-component scalar field, interacting with massless (true) neutrinos gives them masses and charges, thereby turning them into quarks and leptons, and interacting with a multi-component gauge field breaks it into components that are carriers of electro-weak and strong interaction [9-12]. Therefore, the Higgs boson can be part of any modern mathematical model that combines cosmology with particle physics, and its energy-momentum tensor must be included in the right side of the Hilbert – Einstein equation.

The simplest cosmological model that combines cosmology with particle physics will be the one where the Higgs boson plays the role of an inflaton, that is, dark energy. An easy-to-calculate and elegant mathematical model of an inflaton are obtained from a single-component scalar field, whereas a multi-component Higgs field is needed to transform neutrinos into different types of quarks and leptons.

This work considers the possibility of reducing the multi-component Higgs field to an effective one-component inflaton and is based on the ideas of Haken’s synergetics
[13, 14]. In this case, one of the components of the Higgs field becomes an order parameter, and the other components are expressed through it. This leads to the fact that the energy-momentum tensor of the Hilbert – Einstein equation is expressed in terms of an order parameter, that is, it becomes a one-component inflaton. The components of the Higgs field interacting with a true neutrino are also expressed in terms of an order parameter, i.e. an inflaton.

Therefore, when a non-equilibrium phase transition from pseudo- to true vacuum occurs with an inflaton, neutrinos have masses and charges, and they turn into various quarks, leptons, and dark matter particles. Then the particles and antiparticles annihilate, heating the rest of the matter-there is a Big Explosion, which, thus, is part of the process of self-organization of the inflaton-neutrino system. The presence of this process of self-organization turns the cosmological (mathematical) time of the Universe into physical (entropic) time since the process of self-organization leads to a decrease in entropy, and after the Big Bang, it increases [15-26].

Let’s start our analysis of the possibility of reducing the dynamics of a multi-component Higgs field to the dynamics of a single component inflaton. We also study the dynamics of the Higgs scalar field in a certain multiverse. The dynamics of this system is described by the Cartan system of differential equations for the symplectic metric $\Omega$ [27]:

$$
\Omega = (da - bdt)\Lambda(db - \frac{x}{3}U(\phi_0, \phi_n)adt) + d\pi_0\Lambda(d\phi_0 + \frac{\partial U}{\partial \phi_0} dt) + \\
+ \sum d\pi_n\Lambda(d\phi_n + \frac{\partial U}{\partial \phi_n} dt) + \sum d\zeta_n \Lambda d\eta_n (i\gamma^\mu \partial_\mu - k\phi_n)\Psi_n
$$

and has the form [8]:

$$
0 = \frac{\partial \Omega}{\partial da} = \frac{\partial \Omega}{\partial db} = \frac{\partial \Omega}{\partial d\pi_0} = \frac{\partial \Omega}{\partial d\pi_n} = \frac{\partial \Omega}{\partial d\zeta_n} = \\
= db - \frac{x}{3}U(\phi_0, \phi_n)adt = da - bdt = \\
= d\phi_0 + \frac{\partial U}{\partial \phi_0} dt = d\phi_n + \frac{\partial U}{\partial \phi_n} dt = (i\gamma^\mu \partial_\mu - k\phi_n)\Psi_n d\eta_n,
$$
or:

\[
\begin{align*}
\dot{a} &= \frac{x}{3} U(\phi_0, \phi_n), \\
\dot{\phi}_0 &= -\frac{\partial U}{\partial \phi_0}, \phi_n = -\frac{\partial U}{\partial \phi_n}, n = 1, N \\
(i\gamma^\mu \gamma^\nu - k \phi_n)\Psi_n &= 0
\end{align*}
\] (3)

Here:
- \(a\) is the radius of curvature of the universe,
- \(b\) is its conjugate momentum,
- \(\phi_0, \phi_n\) are the components of the Higgs field,
- \(\Psi_n\) are the fields of the (true) massless neutrino interacting with the \(n\)-th component of the Higgs field \(\phi_n\).

System (3) is an approximate model system.

In the equation of the radius of curvature \(a\), from work [16], the potential energy of the Higgs fields is taken at the point of its minimum and a small term proportional to the square of the field change rate is dropped.

The scalar field equation in a homogeneous and isotropic universe has the form:

\[
\ddot{\phi} + 3\frac{\dot{a}}{a} \dot{\phi} + \ddot{U}(\phi) = 0
\] (4)

Since the solution of \(a\), from the first equation (3), has the form

\[
a = \exp\left\{\frac{x}{3} U(\phi_0, \phi_n) t\right\} a_0,
\]

which means:

\[
\frac{\dot{a}}{a} = H = \sqrt{\frac{x}{3} U(\phi_0, \phi_n)} >> 1
\]

\(|U'(\phi)| >> 1\) and \(|\phi| \sim 1\) if \(\phi \neq \phi_j, |\phi| << H\phi\), therefore, we can roughly replace equation (4) with the equation:

\[
\ddot{\phi}_j \approx -\frac{a}{3\dot{a}} \ddot{U}\phi_j(\phi_k) = -\frac{1}{3H} \ddot{U}\phi_j(\phi_k) = -U'_\phi(\phi_k)
\] (5)
Equations of this type are included in the system (3). In the lower of the equations (3), we neglected the interaction of the neutrino with the gravitational field (the curvature of the space-time of the universe).

In our mathematical model, we select the potential energy of the Higgs field $U$ as:

$$U = -\lambda_0 \frac{V_0^2}{2} + \sum_n (\frac{\phi_n^2}{2} + \frac{\lambda_n \phi_n^2}{2}) + V_0,$$  \hspace{1cm} (6)

where

$$V_0 = \frac{U}{\phi_i} = \text{const.}$$

Then the Higgs fields equations from the system (3) will take the form:

$$\begin{cases}
\dot{\phi}_0 = -U' \phi_0 = \lambda_0 \phi_0 - \sum_n \phi_n \phi_0 \\
\dot{\phi}_n = -U' \phi_n = -\lambda_n \phi_n - \frac{\phi_n^2}{2}
\end{cases}$$  \hspace{1cm} (7)

We will look for the solution of the second equation of the system (7) in the form [29]:

$$\phi_n = C_n(t) e^{-\lambda_n t}.$$  

Then we get the equation for $C_n(t)$:

$$\begin{align*}
\dot{C}_n(t) &= -e^{-\lambda_n t} \frac{\phi_0^2}{2} \\
C_n(t) &= -\int_{-\infty}^{t} e^{-\lambda_n \theta} \frac{\phi_0^2}{2} d\theta
\end{align*}$$
This gives an expression for $\varphi_n$ in the form:

$$\varphi_n(t) = - \int e^{-\lambda_n (\theta - t)} \frac{\varphi_0^2(\theta)}{2} d\theta = - \int e^{\lambda_n (\theta - t)} \left( \frac{\varphi_0^2(\theta)}{2} \right) d\theta =$$

$$= - \frac{e^{\lambda_n (\theta - t)}}{\lambda_n} \frac{\varphi_n^2(\theta)}{2} \bigg|_{\theta=-\infty}^{\theta=0} + \int_{-\infty}^{0} e^{\lambda_n (\theta - t)} \varphi_0(\theta) \varphi'_0(\theta) \approx$$

$$\approx - \frac{\varphi_0^2(t)}{2\lambda_n} + \frac{\lambda_0}{\lambda_n} \int_{-\infty}^{0} e^{\lambda_n (\theta - t)} \varphi_0^2(\theta) d\theta.$$

Assuming $\left| \frac{\lambda_0}{\lambda_n} \right| << 1$, we get

$$\varphi_n \approx - \frac{\varphi_0^2}{2\lambda_n}$$

(8)

(the principle of subordination of G. Haken, [12, 27]).

Substituting this relation into equation (7), we get:

$$\dot{\varphi}_0 = \lambda_0 \varphi_0 + \sum_n \frac{1}{2\lambda_n} \varphi_0^3,$$

(9)

its stationary solution:

$$\tilde{\varphi}_0' = 0, \quad \tilde{\varphi}_0^2 = - \frac{2\lambda_0}{\sum_n \frac{1}{\lambda_n}},$$

(10)

exists only when $\lambda_0 < 0$.

Substituting the expression (10) in (8), we get:
In this case, the momentum energy tensor of the Higgs boson, which is included in the Hilbert – Einstein equation, takes the form:

$$T_{\mu\nu} = -g_{\mu\nu} U(\varphi_0, \varphi_0) = g_{\mu\nu} \left( \frac{\lambda_0}{2} \varphi_0^2 + \sum_n \frac{1}{\lambda_n} \varphi_n^2 + \sum_{k<n} \frac{1}{\lambda_k} \varphi_k \varphi_n + V_0 \right) =$$

$$= g_{\mu\nu} \left( \frac{\lambda_0}{2} \varphi_0^2 + \frac{1}{8} \sum_n \frac{\varphi_0^4}{\lambda_n} + V_0 \right).$$

(12)

Thus, the principle of subordination of Haken leads to the transformation of the n + 1 component Higgs field in the energy-momentum tensor into effective one-component field $\varphi_0$ – inflaton.

During the period of inflationary expansion of the universe (before the Big Bang) when $\varphi_0 = 0$, $T_{\mu\nu} = g_{\mu\nu} V_0$, and after the phase transition of the field $\varphi_0$ to the true vacuum:

$$\tilde{\varphi}_0 = \pm \sqrt{\frac{2\lambda_0}{\sum_n \frac{1}{\lambda_n}}},$$

the energy-momentum tensor takes the form:

$$T_{\mu\nu} = g_{\mu\nu} \left( -\frac{\lambda_0^2}{\sum_n \frac{1}{\lambda_n}} + \frac{1}{8} \sum_n \frac{1}{\lambda_n} \left( \sum_k \frac{1}{\lambda_k} \right)^2 + V_0 \right) = g_{\mu\nu} \left( V_0 - \frac{1}{2} \sum_n \frac{1}{\lambda_n} \right).$$

(13)

This shows that after such a phase transition (the Big Bang), the accelerated expansion of the Universe slows down.

According to the Higgs representations [13-14], the interaction of massless neutrinos with Higgs bosons leads to the appearance of a mass in them. The equation of such a neutrino has the form:
\[ \left(i \gamma^\mu \partial_\mu + k \varphi_n \right) \Psi_n = 0. \quad (14) \]

In the period of inflationary expansion, \( \tilde{\varphi}_0 = 0 \), and therefore

\[ \tilde{\varphi}_n \approx -\frac{\tilde{\varphi}_n^2}{2 \lambda_n} = 0, \]

after the phase transition \( \varphi_0 \) to the true vacuum

\[ \tilde{\varphi}_0^2 = -\frac{2 \lambda_0}{\sum_n \lambda_n}, \quad \tilde{\varphi}_n = \frac{\lambda_n}{\sum_{k=1}^{n} \lambda_k}. \]

Thus, after the "Big Bang", equation (14) takes the form:

\[ \left(i \gamma^\mu \partial_\mu + \frac{k \lambda_0}{\sum_{k=1}^{n} \frac{1}{\lambda_k}} \right) \Psi_n = 0. \quad (15) \]

The mass appears in neutrinos, and they become quarks and leptons (taking into account the nature of the interaction with the electromagnetic field). In different local universes, \( \lambda_0 \) and \( \lambda_n \) are different [28-33], so the masses of elementary particles in them are also different.

### 3 Conclusion

This paper implements a model for reducing the multi-component Higgs field to effective one-component inflation. The model is based on the ideas of Haken's synergetics [15].

Thus, it is shown how cosmology and elementary particle physics can be combined within the framework of the Guth – Linde – Vilenkin theory of eternal inflation [37-39]. This result allows us to impose certain conditions on the mass spectrum of elementary particles in our Universe based on cosmological observations of the slowing down of the Universe. It is also seen that non-equilibrium phase transitions occur in universes, making the cosmological time of the universe physical [35].

The problem of the cosmological constant, the accelerated cosmic expansion in recent times, and the coinciding current energy densities of the cosmological constant and matter, remains a complex puzzle for cosmology [40].
References