Mathematical model of intersecting cylindrical shells in Cartesian coordinates

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Abstract

Equations of elastic intersecting cylindrical shells in displacements are derived for the T-joint of pipes. Three-dimensional mathematical model is constructed within the framework of the membrane theory of shells, and the limitations of this approximation are found. Geometric and force conjugation conditions are set on the pipe intersection line, and boundary conditions are imposed on the end of the pipes. Complete three-dimensional mathematical model is presented in the Cartesian coordinate system, to achieve a unified approach to solving the problem without splitting into subdomains. Reduced statement of the boundary value problem with respect to only two independent variables is found. This result is obtained from the symmetry condition of the mechanical system with respect to the plane of the T-joint. The conjugation conditions are eliminated from the final formulation of the boundary value problem. Existence of singularity stress field in the vicinity of the junction and permissibility of using bushing connections in the formulation of the problem are illustrated by numerical example.

Keywords 1

boundary value problem, T-shaped shell joint, numerical methods, singularity

1. Introduction

Modern pipeline systems contain many intersection and pipe branches. An extensive review of the literature on shell compounds can be found in [1]. The engineering background of this problem is that the typical connecting fitting contains reentrant angle that generates stress concentration. The problem of stresses calculating in this intersection is close from a mathematical point of view to the Lame problem posed in L-shaped domain, the solution of which contains a singularity [2]. The presence of a peculiarity in the field of the reentrant angle and complex shape of the intersection line are main problems in the construction of analytical or numerical solution of the T-joint pipe problem.

The purpose of this work is to build a mathematical model of thin elastic shells intersecting at a right angle and to produce a reduced type of boundary problem.

The following tasks are solved in this article:

- boundary value problem in displacements is formulated for the T-joint of membrane cylindrical shells;
- original boundary value problem is written in Cartesian coordinates to obtain a single displacement vector for the entire domain;
- three-dimensional equations are transformed into two-dimensional ones using the symmetry of the mechanical system, necessary number of boundary conditions is imposed, and conjugation conditions are eliminated from the problem statement;
- possibility of using a bushing coupling is numerically investigated.

Numerical solution of the two-dimensional problem is planned by the weighted finite element method, proposed in [2–9].

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2. Problem statement for a membrane cylindrical shell of T-shaped joint in displacements

We consider two pipes with T-shaped joint. The radius of the branch is small compared to the radius of the main pipe, i.e. $r/R < 1/5$. The ratio between the thickness of the pipe and the radius for both pipes does not exceed $1/20$. In Figure 1, we denote: L, l – length of the large (1) and small (2) pipes, respectively, $(O; x, y, z)$ – Cartesian coordinate system, axis Ox coincides with the axis of large cylinder and the axis Oz coincides with axis of small cylinder. We introduce cylindrical coordinates (x, θ, ρ_1) and (z, φ, ρ_2) for both pipes.

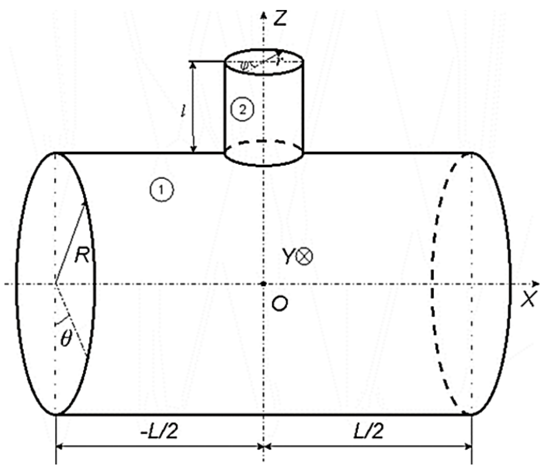

Figure 1: Coordinate system.

We obtain equations in displacements for two intersecting cylinders subjected to an internal pressure from the differential equilibrium equations of the membrane shell theory [10] for values of the curvature radii and Lame coefficients $R_1^{(1)} = R_1^{(2)} = \infty$, $R_2^{(1)} = A_2^{(1)} = R$, $A_1^{(1)} = A_1^{(2)} = 1$, $R_2^{(2)} = A_2^{(2)} = r$:

$$
\begin{cases}
\dfrac{\partial^2 u^{(1)}}{\partial x^2} + \dfrac{1-\nu}{2R^2}\dfrac{\partial^2 u^{(1)}}{\partial \theta^2} + \dfrac{1+\nu}{2R}\dfrac{\partial^2 v^{(1)}}{\partial x \partial \theta} + \dfrac{\nu}{R}\dfrac{\partial w^{(1)}}{\partial x} = 0, \\
\dfrac{1+\nu}{2R}\dfrac{\partial^2 u^{(1)}}{\partial x \partial \theta} + \dfrac{1-\nu}{2}\dfrac{\partial^2 v^{(1)}}{\partial x^2} + \dfrac{1}{R^2}\dfrac{\partial^2 v^{(1)}}{\partial \theta^2} + \dfrac{1}{R^2}\dfrac{\partial w^{(1)}}{\partial \theta} = 0, \\
\dfrac{\nu}{R}\dfrac{\partial u^{(1)}}{\partial x} + \dfrac{1}{R^2}\dfrac{\partial v^{(1)}}{\partial \theta} + \dfrac{w^{(1)}}{R^2} = \dfrac{1-\nu^2}{EH}p, \\
\dfrac{\partial^2 u^{(2)}}{\partial z^2} + \dfrac{1-\nu}{2r^2}\dfrac{\partial^2 u^{(2)}}{\partial \varphi^2} + \dfrac{1+\nu}{2r}\dfrac{\partial^2 v^{(2)}}{\partial z \partial \varphi} + \dfrac{\nu}{r}\dfrac{\partial w^{(2)}}{\partial z} = 0, \\
\dfrac{1+\nu}{2r}\dfrac{\partial^2 u^{(2)}}{\partial z \partial \varphi} + \dfrac{1-\nu}{2}\dfrac{\partial^2 v^{(2)}}{\partial z^2} + \dfrac{1}{r^2}\dfrac{\partial^2 v^{(2)}}{\partial \varphi^2} + \dfrac{1}{r^2}\dfrac{\partial w^{(2)}}{\partial \varphi} = 0, \\
\dfrac{\nu}{r}\dfrac{\partial u^{(2)}}{\partial z} + \dfrac{1}{r^2}\dfrac{\partial v^{(2)}}{\partial \varphi} + \dfrac{w^{(2)}}{r^2} = \dfrac{1-\nu^2}{Eh}p,
\end{cases}
\tag{1}
$$

where ν – Poisson ratio, E – modulus of elasticity, H, h – thickness of large and small cylinders, $u^{(1)}, v^{(1)}, w^{(1)}$ and $u^{(2)}, v^{(2)}, w^{(2)}$ – components of the displacement vector, where the index (1) denotes belonging to a large cylinder, and index (2) denotes belonging to a small cylinder, p – uniform internal pressure.

Set the boundary conditions at the cylinders ends. We restrict movements of large cylinder along coordinate x, and we equate shear force to zero:

$$\begin{aligned} x=-\frac{L}{2}: \quad & u^{(1)}=0,\ \frac{1}{R}\frac{\partial u^{(1)}}{\partial\theta}+\frac{\partial v^{(1)}}{\partial x}=0; \\ x=\frac{L}{2}: \quad & u^{(1)}=0,\ \frac{1}{R}\frac{\partial u^{(1)}}{\partial\theta}+\frac{\partial v^{(1)}}{\partial x}=0. \end{aligned} \tag{2}$$

In a small cylinder, we restrict movements along angular coordinate and equate normal force to zero:

$$z=R+l: \quad v^{(2)}=0,\ \frac{\partial u^{(2)}}{\partial z}+\frac{\nu}{r}\frac{\partial v^{(2)}}{\partial\varphi}+\frac{\nu}{r}w^{(2)}=0. \tag{3}$$

To complete the statement of the boundary value problem, it is necessary to impose interface conditions on intersection lines. Due to smallness of the relationship $r/R<1/5$ we consider intersection line of two cylinders middle shells surfaces as a circle. Its equation in vector form is as follows:

$$\mathbf{r}_s=r\cos\varphi\,\mathbf{i}+r\sin\varphi\,\mathbf{j}+R\mathbf{k}. \tag{4}$$

We can find three mutually perpendicular characteristic vectors [11] of tangent $\mathbf{t}$, normal $\mathbf{n}$, and binormal $\mathbf{b}$ to junction line (4):

$$\mathbf{t}=-\sin\varphi\,\mathbf{i}+\cos\varphi\,\mathbf{j}, \qquad \mathbf{n}=\cos\varphi\,\mathbf{i}+\sin\varphi\,\mathbf{j}, \qquad \mathbf{b}=\mathbf{k}.$$

We define geometric condition of displacements continuity on the intersection line as [12]:

$$\left(\mathbf{u}^{(1)}\cdot\mathbf{t}\right)=\left(\mathbf{u}^{(2)}\cdot\mathbf{t}\right),\quad \left(\mathbf{u}^{(1)}\cdot\mathbf{n}\right)=\left(\mathbf{u}^{(2)}\cdot\mathbf{n}\right),\quad \left(\mathbf{u}^{(1)}\cdot\mathbf{b}\right)=\left(\mathbf{u}^{(2)}\cdot\mathbf{b}\right). \tag{5}$$

For shear forces, the coupling conditions have the form [10]:

$$S^{(1)}=-S^{(2)}. \tag{6}$$

Proposition 1. On the intersection line of the two middle surfaces of cylindrical shells, described by equation (4), the following relations are satisfied:

$$\begin{aligned} & -u^{(1)}\sin\varphi-v^{(1)}\sin\theta\cos\varphi+w^{(1)}\frac{r}{R}\cos\theta\cos\varphi=v^{(2)}, \\ & u^{(1)}\cos\varphi-v^{(1)}\sin\theta\sin\varphi+w^{(1)}\cos\theta\sin\varphi=w^{(2)}, \\ & v^{(1)}\cos\theta+w^{(1)}\sin\theta=u^{(2)}, \\ & \frac{H}{R}\frac{\partial u^{(1)}}{\partial\theta}+H\frac{\partial v^{(1)}}{\partial x}=-\frac{h}{r}\frac{\partial u^{(2)}}{\partial\varphi}-h\frac{\partial v^{(2)}}{\partial z}. \end{aligned} \tag{7}$$

Proof. We represent the displacement vector of the middle surface of each cylinder as:

$$\mathbf{u}^{(1)}=u^{(1)}\mathbf{i}+\left(-v^{(1)}\sin\theta+w^{(1)}\cos\theta\right)\mathbf{j}+\left(v^{(1)}\cos\theta+w^{(1)}\sin\theta\right)\mathbf{k};$$

$$\mathbf{u}^{(2)}=\left(-v^{(2)}\sin\varphi+w^{(2)}\cos\varphi\right)\mathbf{i}+\left(v^{(2)}\cos\varphi+w^{(2)}\sin\varphi\right)\mathbf{j}+u^{(2)}\mathbf{k}.$$

We open scalar products (5) and obtain three kinematic conjugation conditions. We obtain the fourth conjugation condition from equation (6), written in displacements. □

We complete problem statement for cylindrical shells of a T-shaped joint by the formulation of the equations (1), boundary conditions (2), (3), and conjugation conditions (7).

3. Boundary value problem for two rectangles in Cartesian coordinates

3.1. Converting equations in two-dimensional form

To reduce a number of required functions in equations (1), we express

$$w^{(1)}=-R\nu\frac{\partial u^{(1)}}{\partial x}-\frac{\partial v^{(1)}}{\partial\theta}+\frac{1-\nu^2}{EH}pR^2,\quad w^{(2)}=-r\nu\frac{\partial u^{(2)}}{\partial z}-\frac{\partial v^{(2)}}{\partial\varphi}+\frac{1-\nu^2}{Eh}pr^2. \tag{8}$$

Then the boundary value problem will take the following form:

$$\begin{cases} 2R\left(1+\nu\right)\dfrac{\partial^2 u^{(1)}}{\partial x^2}+\dfrac{1}{R}\dfrac{\partial^2 u^{(1)}}{\partial \theta^2}+\dfrac{\partial^2 v^{(1)}}{\partial x \partial \theta}=0, \\ \dfrac{\partial^2 u^{(1)}}{\partial x \partial \theta}+R\dfrac{\partial^2 v^{(1)}}{\partial x^2}=0, \\ 2r\left(1+\nu\right)\dfrac{\partial^2 u^{(2)}}{\partial z^2}+\dfrac{1}{r}\dfrac{\partial^2 u^{(2)}}{\partial \varphi^2}+\dfrac{\partial^2 v^{(2)}}{\partial z \partial \varphi}=0, \\ \dfrac{\partial^2 u^{(2)}}{\partial z \partial \varphi}+r\dfrac{\partial^2 v^{(2)}}{\partial z^2}=0. \end{cases} \tag{9}$$

The boundary conditions for the large cylinder are the same, while for the small cylinder they take the form:

$$z=R+l: \quad v^{(2)}=0, \; \frac{\partial u^{(2)}}{\partial z}=-\frac{\nu}{Eh}pr. \tag{10}$$

The conjugation conditions are:

$$\begin{gathered} -u^{(1)}\sin\varphi-v^{(1)}\sin\theta\cos\varphi-\nu r\cos\theta\cos\varphi\frac{\partial u^{(1)}}{\partial x}-\frac{r}{R}\cos\theta\cos\varphi\frac{\partial v^{(1)}}{\partial \theta}+ \\ +r\cos\theta\cos\varphi\frac{1-\nu^2}{EH}pR=v^{(2)}, \\ u^{(1)}\cos\varphi-v^{(1)}\sin\theta\sin\varphi-R\nu\cos\theta\sin\varphi\frac{\partial u^{(1)}}{\partial x}-\cos\theta\sin\varphi\frac{\partial v^{(1)}}{\partial \theta}+ \\ +\cos\theta\sin\varphi\frac{1-\nu^2}{EH}pR^2=-r\nu\frac{\partial u^{(2)}}{\partial z}-\frac{\partial v^{(2)}}{\partial \varphi}+\frac{1-\nu^2}{Eh}pr^2, \\ v^{(1)}\cos\theta-R\nu\sin\theta\frac{\partial u^{(1)}}{\partial x}-\sin\theta\frac{\partial v^{(1)}}{\partial \theta}+\sin\theta\frac{1-\nu^2}{EH}pR^2=u^{(2)}, \\ \frac{H}{R}\frac{\partial u^{(1)}}{\partial \theta}+H\frac{\partial v^{(1)}}{\partial x}=-\frac{h}{r}\frac{\partial u^{(2)}}{\partial \varphi}-h\frac{\partial v^{(2)}}{\partial z}. \end{gathered} \tag{11}$$

In order to obtain a single displacement vector for the entire domain, we present complete three-dimensional problem of shell theory in the alternative formulation (8), (9) in Cartesian coordinate system. To do this, we use expression of Cartesian coordinates through of cylindrical coordinates and formulas for replacing independent variables [13] for large and small pipes, respectively.

Then, we transform the three-dimensional equations in Cartesian coordinates into equations that depend on independent variables x and z. To do this, we use the symmetry of the mechanical system with respect to the plane xOz.

Proposition 2. Two-dimensional equations of intersecting cylindrical shells in a Cartesian coordinate system have the form:

$$\begin{cases} 2R^2\left(1+\nu\right)\dfrac{\partial^2 u_x}{\partial x^2}+4\left(R^2-z^2\right)\dfrac{\partial^2 u_x}{\partial z^2}-2z\dfrac{\partial u_x}{\partial z}-2z\sqrt{R^2-z^2}\dfrac{\partial^2 v_y}{\partial x \partial z}- \\ \qquad -\sqrt{R^2-z^2}\dfrac{\partial v_y}{\partial x}+2\left(R^2-z^2\right)\dfrac{\partial^2 w_z}{\partial x \partial z}-z\dfrac{\partial w_z}{\partial x}=0, \\ 2\sqrt{R^2-z^2}\dfrac{\partial^2 u_x}{\partial x \partial z}-z\dfrac{\partial^2 v_y}{\partial x^2}+\sqrt{R^2-z^2}\dfrac{\partial^2 w_z}{\partial x^2}=0, \\ R\nu\dfrac{\partial u_x}{\partial x}-2z\dfrac{\sqrt{R^2-z^2}}{R}\dfrac{\partial v_y}{\partial z}+2\dfrac{R^2-z^2}{R}\dfrac{\partial w_z}{\partial z}=\dfrac{1-\nu^2}{EH}pR^2. \end{cases} \tag{12.1}$$

$$
\begin{cases}
2\left(r^2-x^2\right)\dfrac{\partial^2 u_x}{\partial x\partial z}-x\dfrac{\partial u_x}{\partial z}-x\sqrt{r^2-x^2}\,\dfrac{\partial^2 v_y}{\partial x\partial z}+x^2\dfrac{\partial^2 v_y}{\partial z^2}-\sqrt{r^2-x^2}\,\dfrac{\partial v_y}{\partial z}+ \\
\qquad\qquad +4\left(r^2-x^2\right)\dfrac{\partial^2 w_z}{\partial x^2}+2r^2\left(1+\nu\right)\dfrac{\partial^2 w_z}{\partial z^2}-2x\dfrac{\partial w_z}{\partial x}=0, \\
\sqrt{r^2-x^2}\,\dfrac{\partial^2 u_x}{\partial z^2}+\sqrt{r^2-x^2}\,\dfrac{\partial^2 v_y}{\partial x\partial z}+2\sqrt{r^2-x^2}\,\dfrac{\partial^2 w_z}{\partial x\partial z}=0, \\
2\dfrac{r^2-x^2}{r}\dfrac{\partial u_x}{\partial x}-2x\dfrac{\sqrt{r^2-x^2}}{r}\dfrac{\partial v_y}{\partial x}+r\nu\dfrac{\partial w_z}{\partial z}=\dfrac{1-\nu^2}{Eh}pr^2.
\end{cases}
\tag{12.2}
$$

Proof. To prove this proposition, we use equation of the corresponding cylinder. We consider only the positive part of the cylinder and replace variables using formulas:

$$
\begin{array}{ll}
y=\sqrt{R^2-z^2}; & y=\sqrt{r^2-x^2}; \\
\dfrac{\partial}{\partial x}=\dfrac{\partial}{\partial x},\quad \dfrac{\partial}{\partial y}=-\dfrac{\sqrt{R^2-z^2}}{z}\dfrac{\partial}{\partial z}, & \dfrac{\partial}{\partial x}=\dfrac{\partial}{\partial x},\quad \dfrac{\partial}{\partial y}=-\dfrac{\sqrt{r^2-x^2}}{x}\dfrac{\partial}{\partial x}, \\
\dfrac{\partial}{\partial z}=\dfrac{\partial}{\partial z},\quad \dfrac{\partial^2}{\partial x\partial y}=-\dfrac{\sqrt{R^2-z^2}}{z}\dfrac{\partial^2}{\partial x\partial z}, & \dfrac{\partial}{\partial z}=\dfrac{\partial}{\partial z},\quad \dfrac{\partial^2}{\partial y\partial z}=-\dfrac{\sqrt{r^2-x^2}}{x}\dfrac{\partial^2}{\partial x\partial z}, \\
\dfrac{\partial^2}{\partial x^2}=\dfrac{\partial^2}{\partial x^2},\quad \dfrac{\partial^2}{\partial x\partial z}=\dfrac{\partial^2}{\partial x\partial z}, & \dfrac{\partial^2}{\partial x^2}=\dfrac{\partial^2}{\partial x^2},\quad \dfrac{\partial^2}{\partial x\partial z}=\dfrac{\partial^2}{\partial x\partial z}, \\
\dfrac{\partial^2}{\partial y^2}=\dfrac{R^2-z^2}{z^2}\dfrac{\partial^2}{\partial z^2}-\dfrac{R^2}{z^3}\dfrac{\partial}{\partial z},\quad \dfrac{\partial^2}{\partial z^2}=\dfrac{\partial^2}{\partial z^2}. & \dfrac{\partial^2}{\partial y^2}=\dfrac{r^2-x^2}{x^2}\dfrac{\partial^2}{\partial x^2}-\dfrac{r^2}{x^3}\dfrac{\partial}{\partial x},\quad \dfrac{\partial^2}{\partial z^2}=\dfrac{\partial^2}{\partial z^2}.
\end{array}
\tag{13}
$$

After addition of similar terms, we get equations (12), as required. □

The resulting equations depend on two independent variables. In this case, equations (12.1) are applicable in the domain, which is projection of a large cylinder on the symmetry plane, and equations (12.2) are applicable in the domain, which is the projection of a small cylinder. In this regard, the system is divided into two subsystems, which can be considered independently.

3.2. Boundary conditions for equations in Cartesian coordinates

We perform a similar replacement of variables under conditions (2), (10). We obtain the boundary conditions in sections 1, 4, 7, see Figure 2:

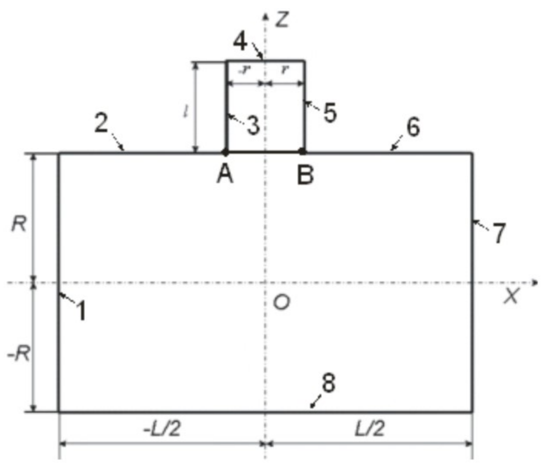

Figure 2: The middle surface projection of the cylinders on the symmetry plane.

$$
\begin{aligned}
&(1) x=-\frac{L}{2}, -R \le z < R: && u_x=0, && 2\sqrt{R^2-z^2}\frac{\partial u_x}{\partial z}-z\frac{\partial v_y}{\partial x}+\sqrt{R^2-z^2}\frac{\partial w_z}{\partial x}=0;\\
&(4) z=R+l, -r<x<r: && -\sqrt{r^2-x^2}u_x+xv_y=0, && \frac{\partial w_z}{\partial z}=-\frac{\nu}{Eh}pr;\\
&(7) x=\frac{L}{2}, -R<z<R: && u_x=0, && 2\sqrt{R^2-z^2}\frac{\partial u_x}{\partial z}-z\frac{\partial v_y}{\partial x}+\sqrt{R^2-z^2}\frac{\partial w_z}{\partial x}=0.
\end{aligned} \tag{14}
$$

After projecting of equations onto the symmetry plane xOz, additional boundaries appear on the flat T-shaped domain. It is necessary to impose boundary conditions, at these boundaries.

Proposition 3. The boundary conditions for the problem (12) in sections 2, 3, 5, 6, 8 of Figure 2 have the following form:

$$
\begin{aligned}
&(2) z=R, -\frac{L}{2}<x\le -r: && v_y=0, && \frac{\partial u_x}{\partial x}=\frac{1}{\nu}\frac{1-\nu^2}{EH}pR;\\
&(3) x=-r, R\le z<R+l: && v_y=0, && \frac{\partial w_z}{\partial z}=\frac{1}{\nu}\frac{1-\nu^2}{Eh}pr;\\
&(5) x=r, R<z<R+l: && v_y=0, && \frac{\partial w_z}{\partial z}=\frac{1}{\nu}\frac{1-\nu^2}{Eh}pr;\\
&(6) z=R, r\le x<\frac{L}{2}: && v_y=0, && \frac{\partial u_x}{\partial x}=\frac{1}{\nu}\frac{1-\nu^2}{EH}pR;\\
&(8) z=-R, -\frac{L}{2}<x<\frac{L}{2}: && v_y=0, && \frac{\partial u_x}{\partial x}=\frac{1}{\nu}\frac{1-\nu^2}{EH}pR.
\end{aligned} \tag{15}
$$

Proof. We use the dependence of the circumferential force in a cylindrical shell on the internal uniform pressure [10]:

$$
N_\theta = pR, \qquad N_\varphi = pr. \tag{16}
$$

We take into account the load application method relative to the plane xOz and symmetric nature of the displacement distribution v_y. Then, at the intersection points of the shells with the plane xOz, we can take $v_y = 0$. From this condition and (16) we obtain (15), as required. □

Since the system (12) consists of two independent subsystems, the intersection line AB should be considered as a boundary, and therefore boundary conditions should be set on it. To do this, we transform the conjugation conditions (11) by expressing the cylindrical coordinates in terms of Cartesian coordinates and replacing the variables. Then, in the second condition, the terms from the 3rd to the 6th coincide with the left part of the first condition, therefore, they can be excluded from the condition. The conjugation conditions are set on the line AB, where $z = R$. In view of this, the first condition is satisfied identically. The last conjugation condition is the equality of the shear forces $S^{(1)} = -S^{(2)}$ on the intersection line. Two more boundary conditions can be obtained from it using Vekua bushing coupling [14, 15], according to which there are no tangential stresses at the boundary. Then we obtain the boundary conditions on the line AB for the large and small cylinders, respectively:

$$
\begin{gathered}
\frac{\partial v_y}{\partial x}=0, \quad -\nu R\frac{\partial u_x}{\partial x}+\frac{1-\nu^2}{EH}pR^2=0;\\
\frac{h}{r}\sqrt{r^2-x^2}\frac{\partial u_x}{\partial z}-\frac{h}{r}x\frac{\partial v_y}{\partial z}+2\frac{h}{r}\sqrt{r^2-x^2}\frac{\partial w_z}{\partial x}=0, \quad -r\nu\frac{\partial w_z}{\partial z}+\frac{1-\nu^2}{Eh}pr^2=0.
\end{gathered} \tag{17}
$$

3.3. Numerical example

To evaluate the possibility of using a bushing coupling at the junction of two pipes, we present results of numerical analysis in the FreeCAD application package. Two models were analyzed: 1) one-piece model of a T-joint of pipes considered as a three-dimensional body; 2) model of a large pipe under internal pressure with a small pipe inserted as a bushing. The following parameters are set:

$R = 37.5\,mm$, $r = 8.5\,mm$, $H = 5\,mm$, $h = 3\,mm$, $L = 100\,mm$, $l = 72\,mm$, steel S335JO, $\nu = 0.3$, $E = 2.1 \cdot 10^5\ MPa$, $\rho = 7800\,kg/m^3$, $[\sigma] = 510\ MPa$.

Figure 3 shows the stress distribution over the grid nodes for a one-piece model. It can be seen that the stress field changes rapidly in the vicinity of the connection line. The maximum value of von Mises's stresses is 256.29 MPa for the first model. When calculating the second model, the maximum value of von Mises's stresses is 283.13 MPa. The relative stress error is 9%, which is acceptable. When the thickness of the pipe wall reduces, the stress increases proportionally.

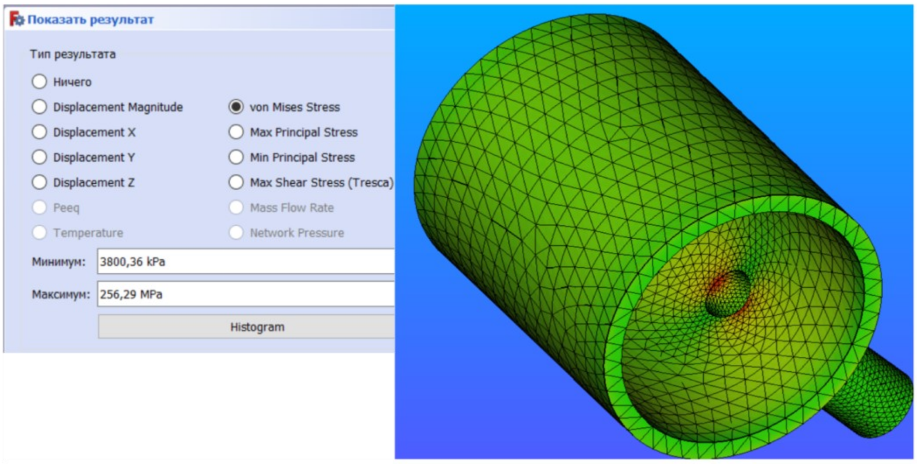

Figure 3: The stress distribution on the nodes of the mesh for one-piece model of T-shaped pipe intersection.

4. Conclusion

The equations of the cylindrical membrane shell theory in displacements are derived for the T-shaped domain. Boundary conditions and conjugation conditions are imposed on the intersection line. At that it is assumed that the radius of the branch pipe is small compared to the radius of the main pipe.

The original problem is represented as a three-dimensional boundary value problem in Cartesian coordinates. The reduced form of the equations is obtained due to the T-shaped region symmetry. The required number of boundary conditions is imposed. Thus, the resulting reduced problem regarding to the displacements u_x, v_y, w_z consists of equations (12.1), (12.2) and boundary conditions (14), (15), (17) in rectangular domains.

A numerical analysis is carried out, which showed that the bushing coupling can be used to transform the coupling conditions into boundary conditions.

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