Numerical solution of the one-sided compressor thrust bearing dynamics equation

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Abstract

The aim of this work is to construct grid algorithms for solving nonstationary second-order partial differential equations that arise when modeling problems of the hydrodynamic theory of lubrication of thrust bearings. In constructing the grid schemes in the parts of the bearing, the finite element method and the version of discontinuous Galerkin method were used. To solve the pressure equation, the method of adder identities is used. To obtain a solution in a thrust bearing, a domain decomposition method is built based on the Lions method. Numerical experiments were performed demonstrating the convergence of the grid scheme of the Galerkin method on a sequence of condensing grids. A set of programs was built with the help of which it is possible to study the behavior of the bearing at various geometric and physical parameters. Determine lubricant consumption and bearing capacity over time.

Keywords 1

boundary value problem, thrust bearing, partial differential equations, heat equation, discontinuous Galerkin method, domain decomposition,

1. Introduction

Thrust plain bearings are an important design element for centrifugal and screw compressors. They are designed to take the axial load acting on the rotor, transfer it to the stator, and also to fix the rotor relative to the housing in the axial direction.

The lubricant flow in the lubricant layer of thrust bearing is mathematically described by a system of nonlinear differential equations. In this work, we use models of lubricant flow in bearings, proposed by Kazan mathematicians Sokolov, Khadiev and Maximov [1, 2].

Thrust bearings used in compressors have fixed pads and a rotating collar, between which lubricant flows. The surface of the pad is profiled and experiences thermal deformations. Therefore, the thickness of the gap of the lubricating layer is variable. Between the pads are channels through which lubricant is supplied. In the collar and bearing pads, it is assumed that the linear heat equation is satisfied. In the lubricating layer, the pressure distribution is described by the two-dimensional Reynolds equation, and heat transfer is described by the three-dimensional energy equation with dominant convection. In this case, there is no thermal conductivity in the radial direction in the lubricating layer.

The temperature distribution in the collar and pads is described by the linear heat conduction equation. In the lubricating layer, the pressure distribution is described by the two-dimensional Reynolds equation, and heat transfer is a three-dimensional nonlinear energy equation with dominant convection.

When modeling bearing dynamics, it is convenient to use a cylindrical coordinate system. We denote by the coordinate axes corresponding to the radius, angular coordinate, and thickness of the lubricant layer.

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2. Issue statement

The two-dimensional equation determining the pressure distribution has the form

$$r\frac{\partial}{\partial r}\left(rf_0\frac{\partial p}{\partial r}\right)+\frac{\partial}{\partial \varphi}\left(f_0\frac{\partial p}{\partial r}\right)=f, \quad x\in\Omega, \tag{1}$$

where p — pressure function, p and f_0, f — coefficients depending on the temperature of lubricant layer. At the boundaries of the region, the pressure is set.

The energy equation in the lubricating layer, collar and pad has the form

$$b\frac{\partial(\rho t)}{\partial \tau}+div(Vt-K\nabla t)=f, \quad x\in\Omega, \tag{2}$$

where the velocity vector and the thermal conductivity in the lubricating layer has the form

$$V=\begin{pmatrix} V_r \\ V_\varphi \\ V_y \end{pmatrix}, \quad K=\begin{pmatrix} 0 & 0 & 0 \\ 0 & K_{\varphi\varphi} & K_{\varphi y} \\ 0 & K_{y\varphi} & K_{yy} \end{pmatrix}.$$

The speed in the pad area is assumed to be zero, $V=(0,V_\varphi,0)^T$ in the collar area and K — diagonal matrix.

We supplement equation (2) with boundary conditions. By coordinate φ in the collar and lubricant layer we assume that the periodicity condition is fulfilled. In the y direction, at the boundaries between the lubricating layer and the solid bearing elements, we set the condition for equal temperatures and heat fluxes. Within the inter-air channel, at the conditional boundary $y=h(r,\varphi)$ of the lubricating layer, the temperature of the inflowing lubricant is set. At other boundaries of solid elements, a condition of the third kind of heat exchange with the environment is set. The equation in the lubricating layer has the following feature. By the variable r, there is only a convective term. It is assumed that the velocity V_r determines the outflowing flows on the external and internal radii of the region [1]. Therefore, boundary conditions are not set there. The temperature of the inflowing grease is considered set.

3. Solution methods

The thickness of the lubricating layer h is a variable in connection with the profiling of the pad, as well as thermal deformations. Therefore, we first make a change of variables $r=\bar{r},\varphi=\bar{\varphi},y=h(r,\varphi)\bar{y}$, that translates the original computational domain into a rectangular one. A change of variables was performed, preserving the divergent form of the equation.

To numerically solve the obtained boundary value problem in the lubricating layer, a scheme of the discontinuous Galerkin method with rectangular elements is constructed. The choice is determined by the local conservatism of the grid schemes built on its basis, low circuit viscosity, and also their stability for a wide class of problems with dominant convection in a wide range of grid parameters [3]. In the constructed scheme, piecewise constant inside the computational domain $\bar{\Omega}_h$ and piecewise linear near the boundary Γ_y of the space of approximating functions were used. The choice of this type of approximating functions makes it possible to significantly reduce the amount of resources required for the calculation, without a tangible loss in the quality of the solution. A method of constructing schemes of this kind is given in [4].

The grid scheme allows us to reduce issue to a grid equation, which in operator notation has the form

$$B\left(\frac{\partial(\rho u_h)}{\partial \tau}\right)+(A_v+A_q+A_\gamma)u_h=F+F_g, \tag{3}$$

where u_h is a grid approximation of the temperature function t.

The operators in equation (3) are defined by the following forms

$$A_v u\cdot w_h=\int\limits_{\bar\Omega} A_v u w_h dx=\sum_{K\in\Im_h}\int\limits_K\left(-u\bar V\cdot\bar\nabla w_h\right)dx+\sum\nolimits_{\gamma\backslash\Gamma_y}\int\limits_\gamma\left[u_{+p}\left(\bar V\cdot\vec p\right)^- -u_{-p}\left(\bar V\cdot\vec p\right)^+\right]\left(w_{h,+p}-w_{h,-p}\right)dx,$$

$$A_q u\cdot w_h=\sum_{K\in\Im_h}\int\limits_K\left(q_h\cdot\bar\nabla w_h\right)dx+\sum\nolimits_{\gamma\backslash\Gamma_y}\int\limits_\gamma\left(w_{h,+p}-w_{h,-p}\right)q_{h,+p}\cdot pdx,$$

$$A_\gamma u\cdot w_h=\sum_{\gamma\in\Gamma_y}\int\limits_\gamma\omega_\alpha u w_h dx,\quad Bu\cdot w_h=\sum_{K\in\Im_h}\int\limits_K buw_h dx,$$

$$F\cdot w_h=\sum_{K\in\Im_h}\int\limits_K\bar f w_h dx,\quad F_g\cdot w_h=\sum_{\gamma\in\Gamma_y}\int\limits_\gamma g w_h dx$$

for any w_h from the space of approximating functions. Here $\Im_h$ is the set of finite elements of the region $\bar\Omega$, $K\in\Im_h$ — finite element; p — unit normal vector to the boundaries of the elements of the partition area, oriented so that $e\cdot p>0$, $e=(1,1,1)/\sqrt{3}$, $w^\pm$ — positive or negative part of the function w. The spaces of approximating functions contain discontinuous functions. Here, the symbols $w_{\pm p}$ denote the limit values of the functions of the partitioning elements adjacent to the boundary from the side $\pm p$. Equation (3) is approximated by an implicit scheme for which an iterative process is constructed with lowering ρ to the lower time layer.

For the numerical solution of equations in solids, the finite element method was used. To approximate equation (1), the method of adder identities is used. For solution obtained by the grid method, the upper relaxation method is used.

In order to consider heat transfer between regions, a method based on the Lyons method of decomposition of regions is constructed [5, p. 59]. Iterations of the decomposition method for two regions, with different speeds and thermal conductivity coefficients, are given by the equations:

$$\begin{aligned}
&div(V_1u_1^{k+1}-K_1\nabla u_1^{k+1})=f, && div(V_2u_2^{k+1}-K_2\nabla u_2^{k+1})=f,\\
&-(V_1u_1^{k+1}-K_1\nabla u_1^{k+1})\cdot n_1+\omega_u u_1^{k+1}=\lambda_1^k, && -(V_2u_2^{k+1}-K_2\nabla u_2^{k+1})\cdot n_2+\omega_u u_2^{k+1}=\lambda_2^k,\\
&\lambda_1^{k+1}=-\lambda_2^k+2\omega_u u_2^{k+1}, && \lambda_2^{k+1}=-\lambda_1^k+2\omega_u u_1^{k+1},
\end{aligned}\tag{4}$$

where n_1, n_2 — normal vectors to the boundary of the first and second regions, V_i,K_i,u_i — velocity vector, thermal conductivity coefficient and temperature of the i-th region.

4. Results

To solve the constructed grid circuits, a set of C ++ programs was created using the Eigen 3.0 class library [6, 7]. With its help, numerical experiments were carried out, demonstrating the convergence of schemes on condensing grids. In numerical experiments, the approximate solution u_h was compared with the exact solution t of the issue test model. Figure 1 shows a graph of the solution error depending on the grid step and number of points of partition of space and time n. The figure shows that there is a convergence of the constructed method with linear velocity with increasing n.

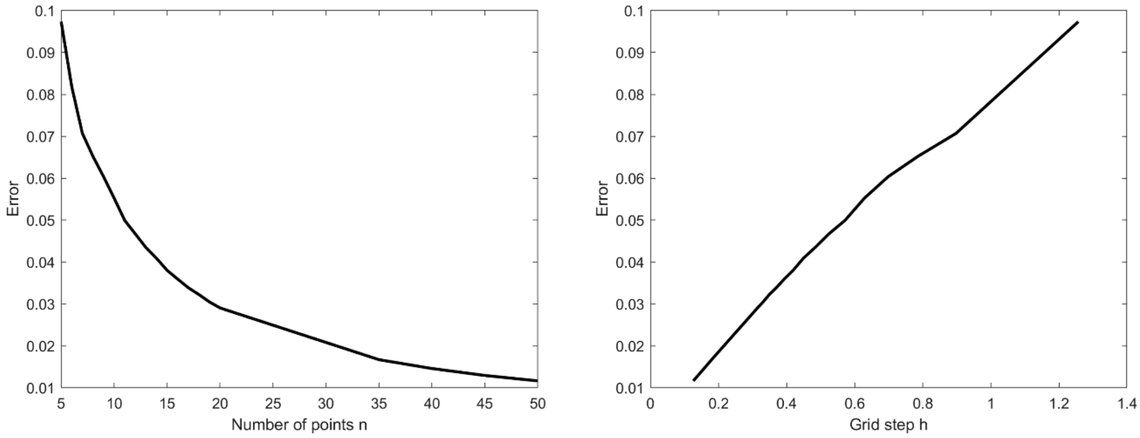

Figure 1: Graph of the solution error depending on the number of points n and grid step h.

The sought quantities in the problem under consideration are temperature and pressure. To find them with real parameters, the methods and program complexes described above are used. Figure 2 shows a graph of isobars, and Figure 3 shows a graph of isotherms in the lubricating layer of a thrust bearing when considering the stationary version of the problem.

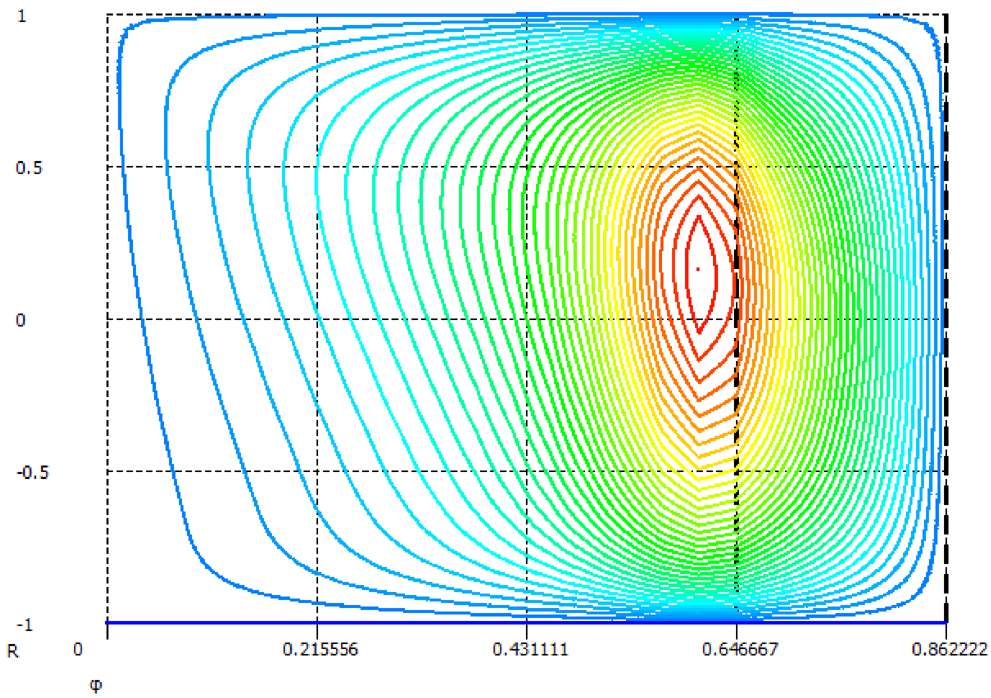

Figure 2: Graph of isobars.

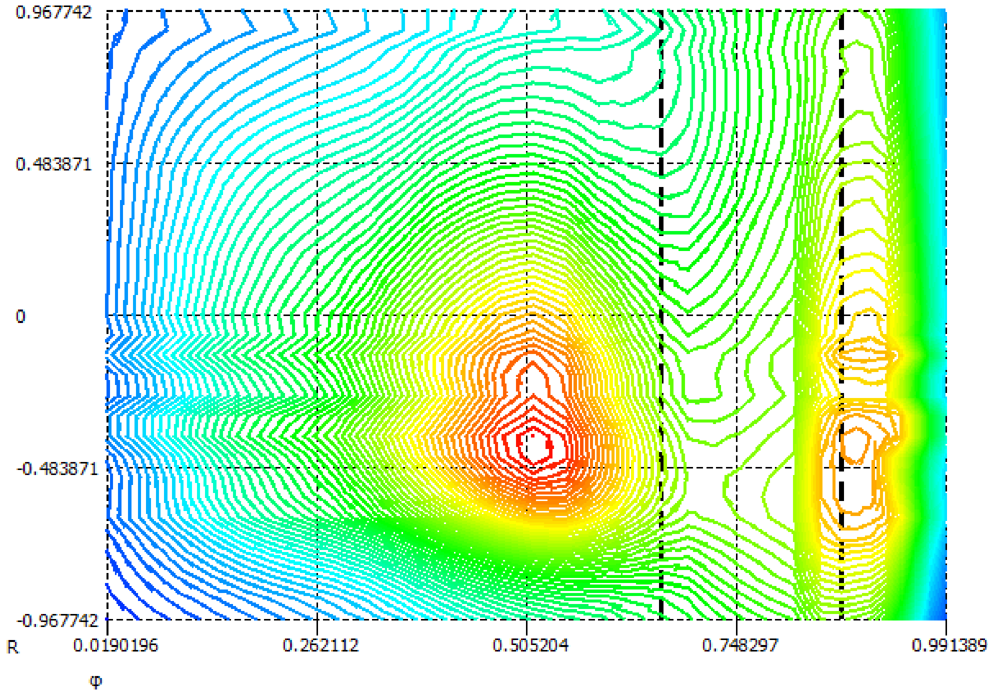

Figure 3: Graph of isotherms.

The decomposition method (4) is used to solve the problem. Figure 4 shows a graph of the temperature distribution at fixed r and φ at the centers of the calculated areas of the pad, collar and lubricant layer.

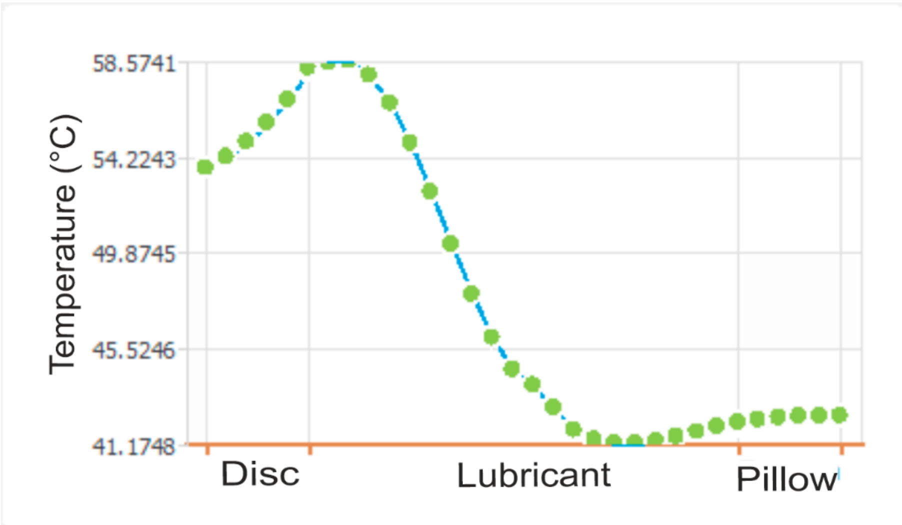

Figure 4: Graph of the temperature distribution at fixed r and φ.

During bearing operation, the disc moves in a vertical direction. A set of programs allows to study the behavior of the bearing at various geometric and physical parameters over time. The Figure 5 shows graphs of changes in bearing characteristics during sinusoidal collar motion. The dots mark the characteristics under consideration, and the line shows the thickness of the lubricating layer during the movement of the disk.

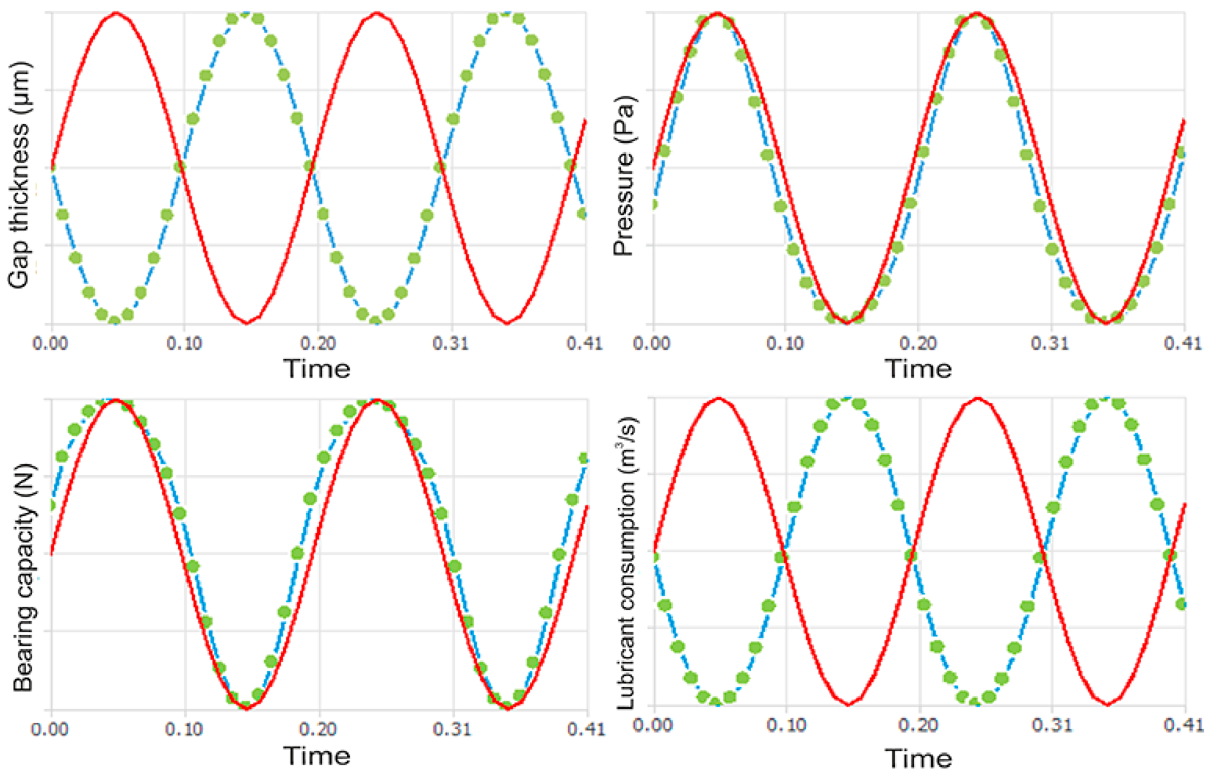

Figure 5: Graphs of changes in bearing characteristics during collar motion.

5. Acknowledgements

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6. References

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