Dynamic Model of Multiply Drive Conveyor for Automatic Controlling System

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Abstract. The article deals with the issue of starting a loaded multiply drive conveyor. The authors suggest using the simulation results to calculate launching targets. The authors prove that it is impossible to use current models in which the load is driven to the moment on the drum. However, a detailed investigation of the conveyor belt behaviour by the finite element method is unnecessarily elaborate. The authors carry out the analysis of the interaction system comprising the belt, rollers and drive drums of the conveyor. They develop a fairly simple finite-difference model of the conveyor. It allows for simulating starting modes. Integration and pull-out force from the drive drums along the conveyor belt are performed by a method of successive iteration, for which convergence is shown in this case. They perform a model as a type of software. The authors show simulation adequacy illustrated with test calculations in different modes of the conveyor operation. In particular, the launch mode is simulated, when the conveyor belt breaks, which should be excluded by matching controlling activities.

Keywords: Multiply Drive Conveyor, Modelling, Control.

1 Introduction

A belt conveyor is a transporting machine for moving solid bulk cargoes and break-bulk goods horizontally and with small obliquity. Conveyors provide a continuous
and non-stop flow of materials. The belt looped around pulleys is a pulling and a carry-
ing device. One of the pulleys is a tensioning device. The loading is carried on a top surface of the belt, a bottom part is idle. Rollers are supporting the top part of the belt, and the bottom part too in some conveyors. Conveyors have some well known advan-
tages: high capacity, great length (up to 3-4 km in coal mines, up to 1-2 km in potas-
sium mines etc.), simplicity and ease of operation. That is why they are used widely in mines, in metallurgical and chemical technology plants and in many other applica-
tions.

The belt is moved by one of more drive drums. Two of four drive drums are ap-
plied in long length conveyors. The entire load comes to the belt through one or more booting devices. An unloading is done from the end drum or from any place by scratchers. If the conveyor was halted with the cargo on them, it is a problem to calcu-
late an algorithm of starting. Wrong control commands are leads to break of the ir-
regular loaded belt. That happens because the load is given as a single moment on the drive drums usually when conveyors are modeling. All forces in the system are calcu-
lated with this assumption all over the belt. The moment determines forces on drums, power of electrical machines, required belt strength and an endurance of other ele-
ments of the conveyor. But as for long ones starting programs obtained by the method leads to breaks. Thus another model is need in this situation.

One of possible model is a finite element method using (as in [1], for example). It allows calculating all local forces, the tension of the belt in any place, interactions with the rollers [2] etc. But this classical method is not simple say the least of it. Ef-
fort outlays to apply it are inadequate in comparison with the final results. Some au-
thors ([3] and others) tried to simplify this method in application to conveyor model-
ing but their approaches are hard applied for conveyors with 2 or 4 drive drums.

Authors of [4] used Maxwellian and Kelvin bodies as the model basis to calculate the tension of the belt. But no each material of the belt has these properties. This dis-
advantage could be reduced by adding additional elements in the system, but this approach complicates the model and does not give us a guaranteed result. And the model obtained by this way gives us only belt tensions and deformations but no other properties and interactions with drums.

Any conveyor model must be dynamic and distributed. Else transition processes cannot be described by the model. They are most interesting as from scientific point of view so for determination of starting control sequences. The last cannot be calcu-
lated without taking in account wave processes, interaction between belt’s solid parts and distributed areas.

So this article is about a modification of listed approaches for multiply drive long conveyors modeling. The aim is to delete some restrictions of existing models without a massive growth of complexity. We have to obtain a method of starting control pro-
gram optimization for loaded conveyors after urgent stops. It determines some assu-
mptions and hypotheses about the moment of belt starting. Some of the results can-
not be published because an agreement with the customer. But the intermediate result (simplified finite elements model of the long multiply drive conveyor) is interesting enough by itself.
2 Materials and methods

A trivial physical model is the base of suggested approach. Their feature is integrating of all forces along the belt. A special modification of a simple iteration algorithm of parameters determination is used and described below. All reactions from drums and conditions of the belt sections also are taking into account.

Let the next source data are known: the drum center coordinates and radiuses \((x, y), R\); the rotation resistive forces on rollers and passive drums \((\mu_{\text{roller}})\); the friction between the belt and drums \((\mu_{\text{drum}})\), the moments of drum’s inertia and all other. Most of them could be calculated by well known methodic.

The conveyor belt is divided into the elements. The equations of motion are solved for each of them further. Their number depends on a certain conveyor but ranges from ten-thousands to several million.

Let imagine the conveyor belt is divided on small parts or sections. For example, a real coal mine belt (4 kilometres) was divided on 40,000 sections of 0.1 meter. All sections are moving along an axis of the belt without any sagging. Dynamic equations of each section are integrated in projection on the axis. So we need to use different formulas for different positions of sections. For example, an integration of section on a drum is not the same when the section is between drums. In summa there are six possible section conditions A-F (Figure 1).

![Fig. 1. Configurations of the conveyor belt.](image)

A Figure 2 is about the forces between the drum and the belt section placed on them.
Fig. 2. The effect of force per the belt element on the drum (a) and belt (b).

Equations of forces balance in relation to a section in Figure 2a are (1). They are projected on geometrical axes ($X$, $Y$):

\[ \begin{cases} 
N \cdot \cos(\alpha) + F_{i+1} \cdot \sin \left( \alpha + \frac{\beta}{2} \right) - F_i \cdot \sin \left( \alpha + \frac{\beta}{2} \right) - mg \pm F_{it} \cdot \sin(\alpha) = -ma \cdot \sin(\alpha) \\
-N \cdot \sin(\alpha) + F_{i+1} \cdot \sin \left( \alpha + \frac{\beta}{2} \right) - F_i \cdot \cos \left( \alpha + \frac{\beta}{2} \right) \pm F_{it} \cdot \cos(\alpha) = -ma \cdot \cos(\alpha), 
\end{cases} \tag{1} \]

Where $m$ is the belt section weight, $a$ is the belt section acceleration, $N$ is the support reaction force, $\alpha$ and $\beta/2$ are shown in Fig. 2a, $F_{it}$ is the force of friction, $F_i$ and $F_{i+1}$ are forces from adjacent elements. The sign $\pm$ of the frictional force is `'+' on any drive drum and `'-' on passive drums because only drive drum friction accelerates the belt. Also $F_{it} = N \cdot \mu_{\text{drum}}$, where $\mu$ is the friction coefficient. The belt moves without slipping in normal mode so the coefficient does not depends on the belt speed.

The belt can slip on any drive drum only if all sections placed on drum are slipping. So the formula $F_{it} \leq N \cdot \mu_{\text{drum}}$ is summed for all sections to determinate a possible slipping. It gives us well known Euler formula $F_{it} = F_0 \left( \exp(\mu_{\text{drum}} \varphi) - 1 \right)$, if lengths of the sections are infinitive small, where $F_0$ is static belt tension, $\varphi$ is drum coverage angle. In this conditions $\sin(\alpha) \approx \alpha$, and $\cos(\alpha) \approx 1$.

Really the angle $\alpha$ is not infinitely small. It leads to growth of the error. But we take into account the weight of the section, but Euler formula is valid only for weightless belt. So our method to slip determination has disadvantages and advantages in comparison of Euler formula. Our method is better in terms of weight accounting.

The formula for support reaction forces is:
\[
\vec{N} = mg + \vec{F}_i - \vec{F}_{i-1}.
\] (2)

Where \( F_i \) and \( F_{i-1} \) are the forces acting on the belt section, \( m \) is the mass of the section and the load on it, \( m = m_{bs} + m_l \), where \( m_{bs} \) is the belt section mass and \( m_l \) is the load mass. The forces \( \vec{F}_{fr} = -\vec{F}_{drum} \) are directed perpendicularly to the supporting force and do not affect it.

Figure 2b shows the forces when the section is not on the drum. The forces balance equations are the follow:

\[
\begin{align*}
N \cdot \cos(\alpha) + F_i \cdot \sin(\alpha) - F_{i\pm1} \cdot \sin(\alpha) \pm \delta \cdot F_{fr} \cdot \sin(\alpha) - mg &= ma \cdot \sin(\alpha) \\
-N \cdot \sin(\alpha) - F_i \cdot \cos(\alpha) + F_{i\pm1} \cdot \cos(\alpha) \pm \delta \cdot F_{fr} \cdot \cos(\alpha) &= -ma \cdot \cos(\alpha),
\end{align*}
\] (3)

Where \( F_{fr} = N \cdot \mu_{roller} \cdot \cos(\alpha) \) is the force of friction when the section is on the roller (\( \delta = 1 \)).

We can neglect the rollers frictional forces when the conveyor starts because a summary load and it’s inertia are huge. And, \( \mu_{roller} \) is not a constant, it depends on a belt speed because bearing’s drag forces and companion processes. This coefficient is very hard to determine precisely. That is why usually it is determines empirically as a constant in dependence of belt type, loads and roller’s construction [10].

We have to project all forces to the straight, non-stretched belt. So, the formula (1) will be the follow:

\[
F_{i+1} \cdot \cos\left(\frac{\beta}{2}\right) \pm mg \cdot \sin(\alpha) + F_{drum} - F_{i-1} \cdot \cos\left(\frac{\beta}{2}\right) = ma.
\] (4)

The sign \( \pm \) in (4) depends on a direction of movement through the drum. For example, a gravity makes the sign ‘–’ when a section lifts on the drum.

Because the force is transmitted from the drum to the section only by friction:

\[
F_{drum} \leq F_{fr} = N \mu_{drum},
\] (5)

Where \( F_{drum} \) is a drive drum circumferential force. The frictional force is summed for all sections placed on the drum as explained above. In other words:

\[
F_{drum} = \min(N \mu_{drum} ; \frac{M}{R}),
\] (6)

Where \( M \) is the known drum torque, and \( R \) is the drum radius. If the minimum (6) is \( N \mu_{drum} \), there are no slipping of the belt. Consequently there are no belt ignition possibilities.

In formula (4), all the forces can be calculated as:

\[
F_i = C \cdot \Delta L_i,
\] (7)
where $F_i$ is the force from the section numbered $i$, $C$ is the belt tension coefficient, $\Delta L_i$ is the elongation of the section.

The support reaction force is always directed perpendicularly to the surface. It must be taken into account (2):

$$N = mg \cdot \cos(\alpha) + (F_{i+1} + F_{i-1}) \cdot \sin\left(\frac{\beta}{2}\right).$$  \hfill (8)

From expressions (6), (8) and (4), we can obtain:

$$F_{i+1} \cdot \cos\left(\frac{\beta}{2}\right) \pm mg \cdot \sin(\alpha) + \left[ \min\left( N \mu_{\text{drum}} ; \frac{M}{R} \right) \right] - F_{i-1} \cdot \cos\left(\frac{\beta}{2}\right) = ma.$$  \hfill (9)

A same expression for the element placed not on the drive drum is:

$$F_i - F_{i \pm 1} \pm \delta \cdot (mg + F_i - F_{i \pm 1}) \cdot \cos(\alpha) \cdot \mu_{\text{roller}} - mg \cdot \sin(\alpha) = ma.$$  \hfill (9a)

The supporting forces are reduced from (3) and (4) because they don’t influence to the belt acceleration. The roller friction forces are small in comparison with the load inertia at the belt starting moment, and can be reduced too. So:

$$F_i - F_{i \pm 1} - mg \cdot \sin(\alpha) \approx ma.$$  \hfill (9b)

We can calculate an acceleration of each section and determine a new speed value and a new position using (9, 9a and 9b) for matching sections. The speed and positions we can obtain from simple Euler method:

$$\begin{align*}
\Delta v_i &= a_i \cdot \Delta t \\
v_i^* &= v_i + \Delta v_i \\
I_{\text{new}} &= I_{\text{old}} + \bar{v} \cdot \Delta t + \bar{a} \cdot \left(\frac{\Delta t^2}{2}\right), \\
\Delta l_i &= \left(v_i^* + v_i\right) \frac{\Delta t}{2}
\end{align*}$$  \hfill (10)

Where (everything for the section numbered $i$): $\Delta v_i$ is a change of the speed, $a_i$ is the acceleration, $\Delta t$ is a time step, $v_i^*$ is a preliminary new speed, $I_{\text{new}}$ is the new and $I_{\text{old}}$ is the old position (at the previous time step), $\bar{v}$ is the speed in the preceding step, $\bar{a}$ is the acceleration in the preceding step, $\Delta l_i$ is an elongation because a difference between $I_{\text{new}}$ and $I_{\text{old}}$.

Before starting the modelling, we have to construct a general simulation scheme, such as positions and diameters of the drums, directions and values of their moments, bearing and idle areas of the belt etc. We also have to specify loading and unloading...
zones, and zone of reloading if exist, where the loading is pouring from one area to another.

The formula for splitting the belt into \( n \) sections is applies depending on the section position: on the belt or the drum. If the section centre is on the drum, we assume that the entire section is on the drum.

For straight segments, coordinates of section centres \((x_i, y_i)\) are:

\[
\begin{align*}
x_i &= x_1 + i \Delta l \cdot \cos(\alpha) - \frac{1}{2} \Delta l \cdot \cos(\alpha)
y_i &= y_1 + i \Delta l \cdot \sin(\alpha) - \frac{1}{2} \Delta l \cdot \sin(\alpha)
\end{align*}
\]

And for segments on the drum are:

\[
\begin{align*}
x_i &= (x_1 - x_c) \cdot \cos\left(\Delta \alpha_i - \frac{\Delta \alpha}{2}\right) + (y_1 - y_c) \cdot \sin\left(\Delta \alpha_i - \frac{\Delta \alpha}{2}\right) + x_1
y_i &= (x_1 - x_c) \cdot \sin\left(\Delta \alpha_i - \frac{\Delta \alpha}{2}\right) - (y_1 - y_c) \cdot \cos\left(\Delta \alpha_i - \frac{\Delta \alpha}{2}\right) + y_1
\end{align*}
\]

Where \( x_c, y_c \) are coordinates of the drum centres, \( x_1, y_1 \) are the coordinates of reference point and \( \alpha = \arctg\left(\frac{y_2 - y_1}{x_2 - x_1}\right) \).

Because all the sections are the same length, we assume that the belt mass is distributed evenly, and a mass of one belt section without the loading is:

\[
m_{bi} = \frac{m_p}{n}.
\]

If there is a load on the belt at the beginning of the simulation, it is necessary to evenly distribute its mass to the loaded sections. Next, we have to take into account additional masses during loading, reloading and unloading.

After calculating the masses of the elements, according to formulas \((4), (5)\) and \((6)\), it is necessary to calculate the accelerations of each section by a follow simple iteration method.

The circuit of the belt is continuous and looped, so we can use a simple iteration method to calculate all the forces in \((9)\). At the beginning, we have three unknowns for each section: two forces and the acceleration. At the first step, we can use some initial value of the forces \(F_i\), and leave the second force remains unknown. The accelerations must be initialized as zero, the velocity of all the belt is known or zero. Then we can calculate \(F_2\) and move to the next section with the same unknowns. We will get all the forces after going consequently through all the section of the looped belt. After that we can refine \(F_1\) at the second step. Knowing \(F_1^{(2)}\) (the first force at the second step) and \(F_N^{(1)}\) (the last force of the first step) we can calculate the acceler-
tion at the second step \((a_i^{(2)})\). Then we can calculate all the forces and refine the accelerations of the sections, repeating until:

\[
\forall i : \left| F_i^{(k)} - F_i^{(k-1)} \right| > \varepsilon,
\]

(14)

Where \(\varepsilon\) is a previously specified final error.

Knowing the acceleration, we can compute the velocity changes, tensions, and new velocities of each section by (10). Also we have to determine the belt slipping on the drive drums according to the formula (6). If the condition:

\[
F_{fr} < \frac{M}{R},
\]

(15)

Is true, where \(F_{fr}\) is the sum of the frictional forces of all elements that are currently on the drum, it means the slipping of the belt on the drive drum.

We can calculate all the tensioning \(L_i = F_i / C\) from (7). Then we can check is there a belt breakup by comparing the tensioning with a known maximum value. If \(L_i > L_{max}\) is true, the belt is broken. A total tensioning is a summa of the section’s tensions, so we can calculate the belt sag.

As a result, the belt conveyor simulation algorithm is follow:

- determine runs, drums, mark the loading, reloading and unloading zones;
- split the belt into the same length sections;
- calculate the masses of sections without load (13);
- distribute the weight of the load between sections;
- calculate the accelerations and forces of each section using (4) and (8);
- calculate each section tensioning;
- determine the possible breakings of the belt;
- calculate the sag and each run tensioning;
- calculate the new speeds and positions of the sections by (10);
- determine the possible belt slipping on the drive drums by (15);
- repeat the algorithm for the next time step until end of modeling.

The model and the algorithm are realized in special software designed in JavaScript. It also includes an elementary scheme editor (Figure 3). The scale of the drums and the vertical scale are differs from the horizontal one for better visualise.

The software allows to view wave effects when the loaded belt is starting. So the software is different from traditional methods of computing when only integral propagation velocities are used. Also it is different from complex finite element methods, which usually allow one to study the wave in detail in a very limited zone of the belt (at least, not on a kilometric one).

We have constructed the scheme of mine belt conveyor ‘4KL1400A-1000-4A-U5’ from a South Yakut coal basin. This conveyor contains four drive drums; its belt length is about 2 km. We have divided the belt into 4000 sections of 0.481 m each, as
described above. Initial forces are determined as zero and are refilled by our algorithm.

![Fig. 3. Test multiply-drive conveyor diagram.](image)

The carrier run of the conveyor is divided into two parts: a short one, which ends with the reloading point, and a long one, which ends with the unloading point as shown at Figure 3. All the points are so small in comparison of the conveyor length, that we have assumed their belonging to one section each. We also have supposed that the load appears in the loading point regularly.

### 3 Results and Discussion

A first discussed result is a scenario of normal conveyor working in state mode. Here are no breakings and slipping on the drive drums. That is why a trend of tensions should be similar to Figure 4 (a dependence between number of the section and the tension forces).

![Fig. 4. Force chart for normal operation.](image)

We can see two peaks where the sections are running over the drum. The load on the second drive is greater than on the first, because the second part is longer, and the
second drum is more affects here. These results are adequate and correspond to the analysis of the conveyor belt tension in a static mode according to [1].

As a second example, we have explored the behaviour of the conveyor at starting with the loading exceeded the transportation capacity (Figure 5) in the same axes.

![Fig. 5. Force chart for normal operation.](image)

We can see a huge difference between the first and second drive forces. Some forces have negative values. This means that there was an excess of the maximum tension and the belt breaking. The software displays only the first breaking point. In a similar chart, we can see sums of section forces, there are unusually small and negative sums of forces on the drums and very small forces on the idle run of the conveyor (Figure 6).

![Fig. 6. Chart of the sum of forces on the elements at the moment after the belt breakup.](image)
The segments of the chart with negative values show the most probably belt breakings, for example, the section 1427 and all. The breaking leads to negative rotation of idle drums.

The software also allows one to simulate the belt slip on the drive drums with insufficient tension. It allows one to study the belt wearing capacity. Many scientists emphasize the importance of this issue.

4 Conclusion

The designed software allows us to explore attractive operation modes of long conveyors due to the use of larger processing power of a newer personal computer than that of earlier authors. The model is not complicated; it can also be used to a conveyor controlling system design for emergency situations.

References