A Technique for Structuring of Group Expert Judgments Formed Under Complex Forms of Ignorance

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Abstract
The main provisions of the technique of structuring the aggregated expert assessments in order to synthesize group opinion, which are formed under the influence of two or more types of ignorance, generated by uncertainty, inaccuracy, inconsistency, contradiction, has been proposed. To synthesize mathematical models for their implementation, the mathematical notation of the theory of evidence (DST) and the theory of plausible and paradoxical reasoning (DSmT) is used. Different quantified measure of uncertainty level in DST has been considered. A procedure for selecting the technique (rule) for aggregation expert judgments formed in the frame of Dempster-Shafer's model, based on quantitative measures of uncertainty, has been proposed. The proposed procedure allows to obtain the aggregated probability masses that provides the lowest achievable uncertainty level.

Keywords [1]
Information technology, ignorance, decision-making, expert evidence, combination rule

1. Introduction

The theory of choice and decision-making investigates mathematical techniques and models for organize the processes of synthesis of optimal strategic and managerial decisions in social, economic, technical, organizational and other systems. In practice, making effective decisions is impossible without the experience and knowledge of specialists (experts).

Current trends in the development of information technologies, striving to obtain the results of expert surveys, processing and analysis of expert assessments with a higher quality, while reducing the time allotted for making a decision, contribute to the complexity of the examination tasks. The choice and making of optimal and effective decisions for solving complex problems becomes much more complicated in situation of multi-criteria, multi-alternativeness, especially when solving semi-structured (mixed) and unstructured tasks, i.e. such tasks in which qualitative (not or partially formalized), little-known, uncertain factors prevail, especially if there is a tendency to increase their number. The situation is aggravated by the presence of ignorance of various nature, which has a negative impact on the processes associated with the acquisition and analysis of initial data (statistical, analytical, expert information). In such situation, a person cannot, at the heuristic level, guarantee an effective decision-making taking into account all the conflicting factors that affect the achievement of the goal of the problem under consideration.

This, in turn, creates the preconditions for the synthesis of a complex of formalized mathematical models focused on the intellectual support of the decision-making processes under complex forms of ignorance, multi-criteria and multi-alternativeness.

The purpose of the research is to present and formalize the main ideas of the technology for structuring expert knowledge under complex (combined) types of ignorance caused by uncertainty, inaccuracy, inconsistency or / and contradiction of expert knowledge. The basis of which is the procedure for identifying different forms of ignorance, their possible combinations, as the basis for the selection and application of methods for the analysis of expert assessments, which make it

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possible to correctly operate with the analyzed set of initial data in the conditions of the revealed type of ignorance.

2. Basics of Dempster-Shafer theory and Dezert-Smarandache theory

To solve the problem of group expert assessments structuring under complex (combined) types of ignorance such as uncertainty, inaccuracy, inconsistency and conflict, effective results can be obtained by using the DST [1, 2, 6, 22] and DSmT [26, 27] models and techniques.

The DST techniques allows to handle correctly with expert preferences formed under conditions of uncertainty, inconsistency (medium level) and allows to correctly operate with possible ways of intersection and union of expert evidence which can be formed in the processes of identifying and analyzing of expert date and knowledge. The DSmT is an extension of the notation of the DSmT, and allows to operate with deeper forms of ignorance, in particular with combinations of uncertainty and inaccuracy, uncertainty and non-specificity, etc.

Let a group of experts $\mathrm{E}=\{E_j \mid j=\overline{1,t}\}$, evaluating some initial set of objects $\mathrm{A}=\{A_j \mid j=\overline{1,m}\}$ frame of discernment), have formed profiles of expert preferences $\mathrm{B}=\{B_j \mid j=\overline{1,t}\}$, each $B_j=\{X_l \mid l=\overline{1,s}\}$, $X_l \subseteq \mathrm{A}$, reflects the preferences (choice) of the expert E_j and satisfies one of the following systems of rules.

1. The initial data set A is a set of the mutual exclusion and exhaustive elements (DS model). In this case, B_j is a 2^{A}-dimensional vector, each element of which is obeyed the next system of rules [1, 2, 6, 22]:
   1. $X_l=\{\varnothing\}$;
   2. $X_l=\{A_i\}$;
   3. $X_l=\{A_i \mid i=\overline{1,k}\}$, $k<n$; (1)
   4. $X_l=\mathrm{A}=\{A_i \mid i=\overline{1,n}\}$.
2. The initial data set A is a set of the mutual exhaustive elements (DSm model). In this case, B_j is a D^{A}-dimensional vector, each element of which is obeyed the next system of rules [26, 27]:
   1. Conditions, that match (1).
   2. If $X_l, X_k \subset D^{\mathrm{A}}$, than $X_l \cap X_k \in D^{\mathrm{A}}$ and $X_l \cup X_k \in D^{\mathrm{A}}$. (2)

For each B_j, $j=\overline{1,t}$, a vector $\mathrm{m}_j=\{m_l \mid l=\overline{1,s}\}$ will be obtained whose elements (basic probability assignment, *bpa*) satisfy the conditions:

$$0 \le m(X_l) \le 1, \quad \forall(X_l \in \Lambda), \quad m(\varnothing)=0, \quad \sum_{X_l \in \Lambda} m(X_j)=1, \tag{3}$$

where Λ corresponds to 2^{A}, or D^{A} respectively.

3. Measures of uncertainty in DST

There are two main types of uncertainty: non-specificity or imprecision, and conflict in the theory of evidence.

The first of them (non-specificity) allows to determine how the *bpa's* of corresponding focal elements are imprecise, and is directly related to the cardinality of the formed focal elements. The non-specificity manifests itself in a situation when several elements of the frame of discernment are not defined (not specified).

Hartley's weighted entropy allows to quantify the degree of non-specificity [7]:

$$N(m)=\sum_{X_j \subseteq \mathrm{A}, X_j \neq \varnothing} m(X_j)\log_2\left(\left|X_j\right|\right), \; 0 \le N(m) \le \log_2(|\Omega|). \tag{4}$$

F. Smarandache, A. Martin and C. Osswald [28] are introduced the concept of the "degree of specificity" $\delta_S(m) \in [0,1]$:

$$\delta_S(m) = 1 - d(m, m_S), \ \forall m \quad d(m, m_X) = d(m, m_Y), \quad m(X) = m(Y), \tag{5}$$

where as a metric $d(m, m_S)$ can be used any measure that characterizes the distance between the selected groups of evidence; the values of *ms* satisfy the conditions:

$$m_S(X_{\max}) = 1, \ X_{\max} = \arg\max\left(\frac{m(X)}{|X|}\right), X \in 2^A, X \neq \varnothing \tag{6}$$

The second type of uncertainty (conflict) allows to identify and quantify the discrepancy (contradiction) both within the group of evidence and between several groups of evidence.

There are two types of conflict in DST: internal or auto-conflict and general or global conflict. Auto-conflict is a type of conflict that occurs only within a group of evidence [8, 20]. Global conflict indicates inconsistency among selected groups of evidence (represented by m_1 and m_2), and includes both inconsistency within individual evidence (related to individual m-functions) and inconsistency between selected groups of evidence (between m_1 and m_2) [9].

Conflict is characterized by differences in the selection and evaluation of the elements of the frame of discernment, and may be the result of the confusion, dissonance, discord and strife [10, 14, 15, 31]:

$$Conf(m) = -\sum_{X_j \in 2^{\mathrm{A}}} m(X_j)\log_2\left(f(X_j)\right), \tag{7}$$

where the function $f(X_j)$ can take the values of the functions $Bel(C) = \sum_{X_j \subseteq C,\, X_j \in 2^{\mathrm{A}}} m(X_j)$, $Pl(C) = \sum_{X_j \cap C \neq \varnothing,\, X_j \in 2^{\mathrm{A}}} m(X_j)$,

$betP(C) = \sum_{X_j \in 2^{\mathrm{A}}, X_j \neq \varnothing} \left(|C \cap X_j| / |X_j|\right) m(X_j)$, or $Con(X_j) = \sum_{X_i \in 2^{\mathrm{A}}} m(X_i)\dfrac{|X_j - X_i|}{|X_j|}$.

Global uncertainty by G. Klir and B. Parviz [14] is the sum of its components: conflict and non-specificity:

$$T(m) = Conf(m) + N(m), \tag{8}$$

where $N(m)$ is Hartley's weighted entropy (4); $Conf(m)$ is a conflict measure (7).

The contradiction of the group of evidence can be defined as a weighted contradiction of all focal elements of the group of evidence [28], $Contr_m \in [0,1]$:

$$Contr_m = \sum_{X_j \in 2^{\mathrm{A}}} m(X_j) d(m, m_{X_j}), \tag{9}$$

where $\forall X_i, X_j \subseteq \mathrm{A}$: $m_{X_j}(X_i) = \begin{cases} 1, & i = j, i = \overline{1,k}; \\ 0, & i \neq j. \end{cases}$

4. The technique of structuring the aggregated expert assessments formed under complex forms of ignorance

The proposed technology is designed to solve the problem of analysis (ranking, clustering, ranking clusters) of group expert assessments under multi-criteria, multi-alternativeness and complex forms of ignorance (uncertainty, inaccuracy, inconsistency, conflict) in order to synthesis a final (aggregated) expert assessment.

The generalized scheme of the proposed information technology of structuring the expert date and knowledge in the context of complex forms of ignorance and synthesis of group decisions is shown in Fig. 1.

Let's consider the main ideas of the proposed technology. Let $\mathrm{A} = \{A_i \mid i = \overline{1,n}\}$ be a set of alternatives, on which certain limitations can be imposed: mutually exclusive and / or mutually exhaustive elements, which determines the type of model in frame of which expert evidence will be formed.

Let a group of experts $\mathrm{E} = \{E_j \mid j = \overline{1,t}\}$ have formed profiles of expert preferences $\mathrm{B} = \{B_j \mid j = \overline{1,t}\}$ on the set $\mathrm{A} = \{A_i \mid i = \overline{1,n}\}$. The profile B_j formed by the expert E_j reflects his preferences regarding all analyzed elements of the set A, and corresponds to one of the systems of rules (1) or (2), respectively (depending on the chosen analysis model).

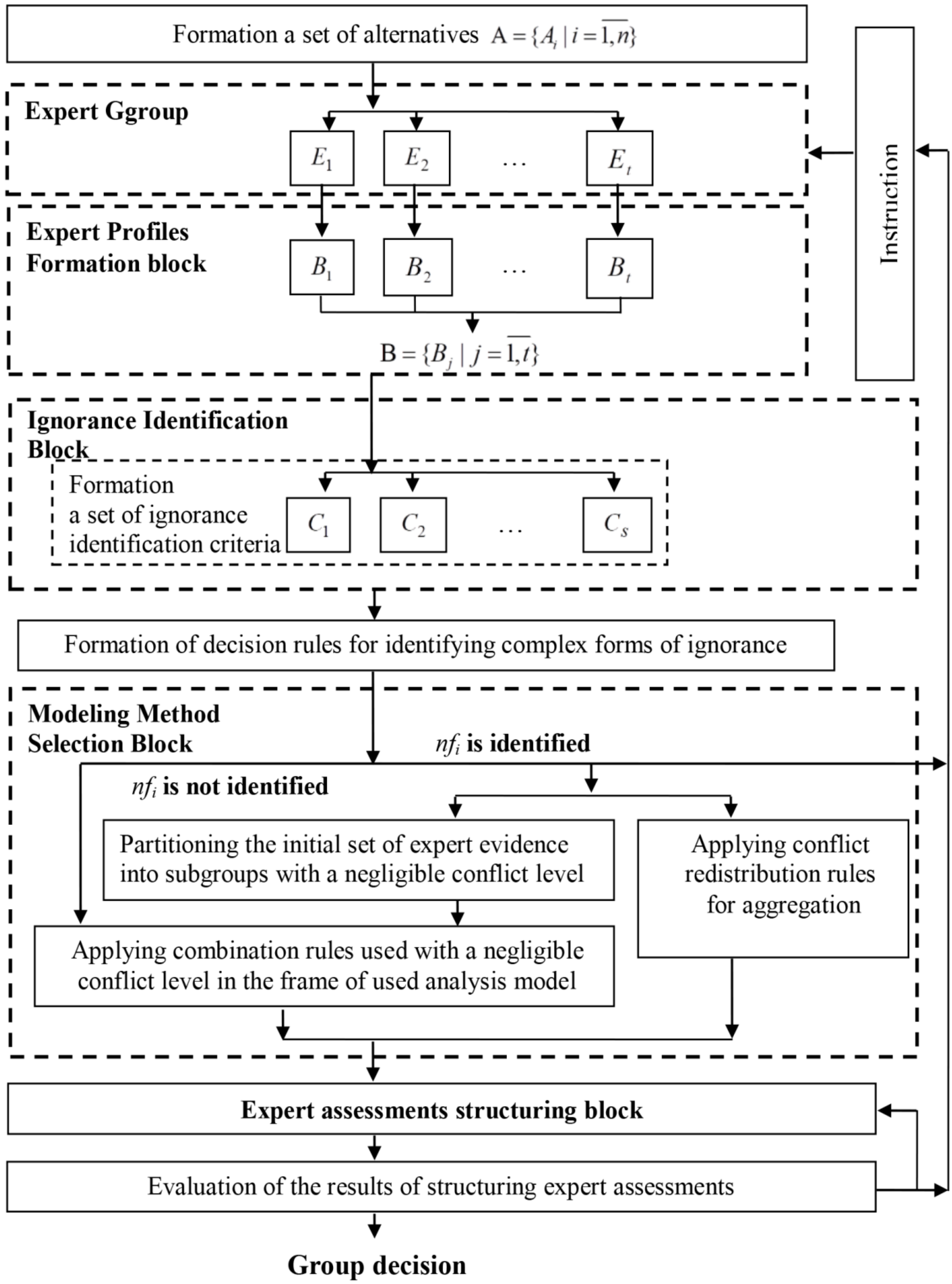

Figure 1: A structure of information technology for the analysis of expert assessments formed under complex forms of ignorance

For each expert the same instruction has been presented, which prescribes what they should do with the set A. The instruction contains information about a scale measurement type, within which experts express their preferences, which in turn affects the information received from experts (words, conditional gradations, numbers, rankings, binary relations or other objects of non-numerical nature).

The profile B_j formed by the expert E_j reflects his preferences, expressed within a given scale, with respect to the elements of the set A. The expert himself decides which elements (or selected groups of elements) of the set A will be evaluated. Thus, the profile of preferences B_j formed by E_j may contain: estimates expressed with respect to all elements of the set A; the assessments expressed regarding the preferred elements of the set A; the estimates expressed regarding the selected groups of preferred elements of the set A.

Next, the set of expert assessments $\mathrm{B} = \{B_i \mid i = \overline{1,n}\}$ is fed to the input of the block of ignorance nf_i identification, $nf_i \in NF$, in this case we are talking about such types of ignorance nf_i as uncertainty, inaccuracy, inconsistency, conflict or their combinations that can be simultaneously present in the knowledge system. In the ignorance identification block, a system of identification criteria $C_i = \{c_j^{(i)} \mid j = \overline{1,z}\}$, $i = \overline{1,p}$, of the analyzed forms of ignorance $NF = \{nf_i \mid i = \overline{1,p}\}$ is formed. On the basis of formed $C_i = \{c_j^{(i)} \mid j = \overline{1,z}\}$ a system of decision rules $SR_i = \{R_l^{(i)} \mid l = \overline{1,h}\}$ for nf_i identification is developed. For nf_i identification it can be used one or combination of features, which allows to unambiguously establish the presence of nf_i in the initial data (knowledge) set. The absence of nf_i is recognized if for all set of proposed ignorance identification criteria C_i confirmed absence of nf_i ($\forall j : c_j^{(i)} \to \text{absence of } nf_i;$); the presence of nf_i is recognized if there is at least 1 criterion $c_j^{(i)}$ (from a given set C_i) signals the presence of nf_i ($\exists j : c_j^{(i)} \to nf_i$).

First, it is necessary to form a set of criteria $C_i = \{c_j^{(i)} \mid j = \overline{1,z}\}$, which, in turn, are considered as indicators of the presence of ignorance in the analyzed data (knowledge) set.

For identification the above forms of ignorance (and their combinations), it is proposed to use the following features:

1. The structure of expert evidence.
2. The level of conflict.
3. Indicators of the quality of the received evidence: level of auto-conflict; the degree of specificity of the generated evidence, etc.
4. The degree of inconsistency of the formed expert evidence.
5. Limitations which are imposed on the frame of discernment A.

The next step is to form a system of decision rules $SR_i = \{R_l^{(i)} \mid l = \overline{1,h}\}$ for analyzed forms of ignorance identification.

Based on the formed decision rules $R_l^{(i)}$, a rule for choosing a method for modeling the above forms of ignorance (and their combinations) can be obtained:

$$B_j \in \begin{cases} P_1, & \text{if} \quad \forall l : R_l^{(i)} \to \text{absence } nf_i; \\ P_2, & \text{if} \quad \exists l : R_l^{(i)} \to nf_i; \end{cases}$$

where P_1 indicates that expert evidence are no contradict, have a high (acceptable) quality, and consistent; P_2 indicates that expert evidence have a high (not acceptable) level of conflict.

If $B_j \in P_1$, then it is assumed that the expert evidence are consistent (they are characterized by close evidence, the presence of a low / insignificant level of conflict), and may indicate a high (acceptable) quality of expert evidence.

In this case, combination techniques (rules) can be used to find the aggregated expert judgments formed in frame of DST or DSmT data-model [1, 13, 19, 21, 24-27, 29, 32, 33]. The algorithm for the complex use of the combination techniques (rules) for finding group solution by combining expert assessments formed in frame of DST model has been proposed. A schematic generalized algorithm for choosing a combination rule is shown in Fig. 2.

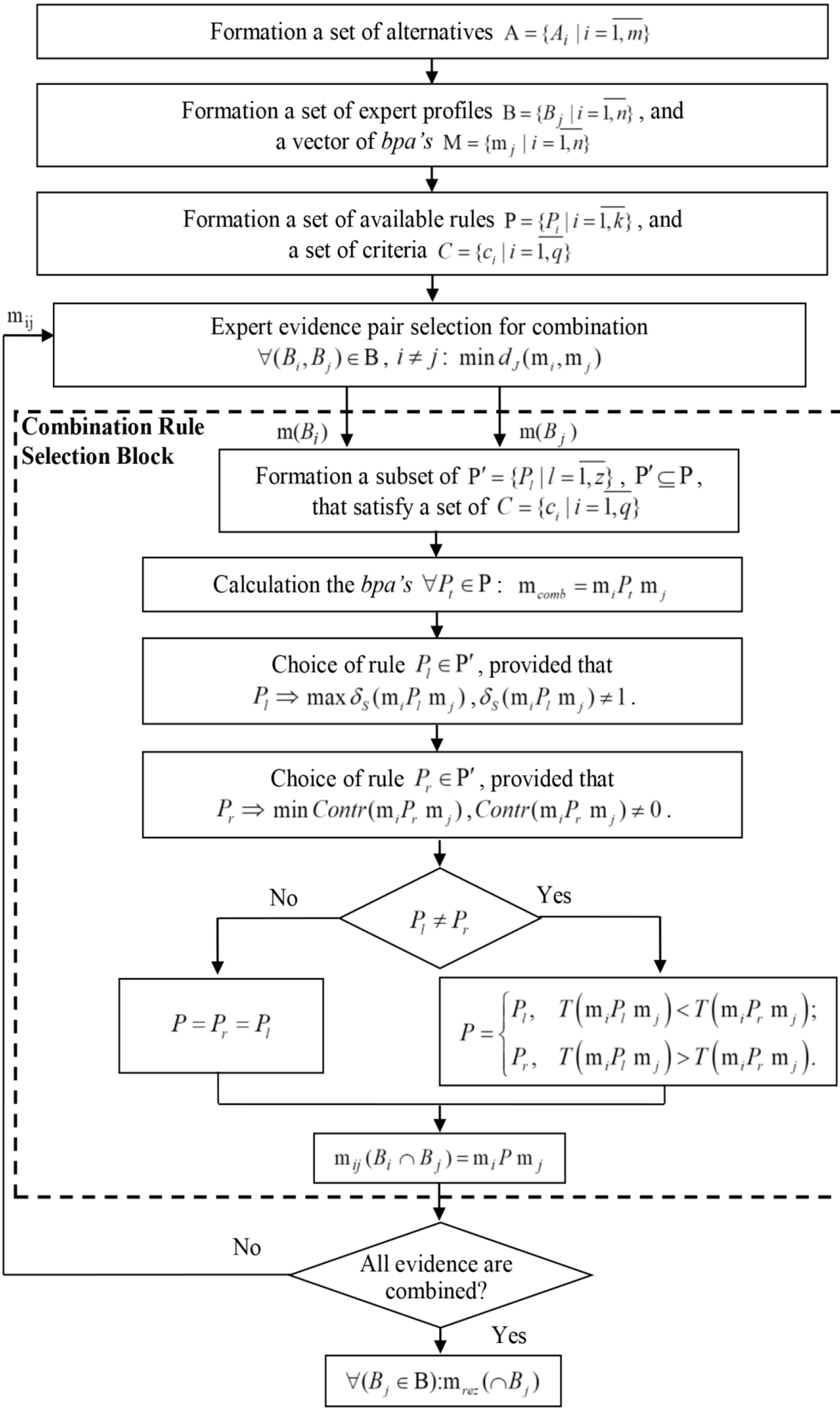

Figure 2: A general algorithm of combination techniques (rules) choice

Let a set of $\mathrm{P} = \{P_i \mid i = \overline{1,k}\}$ potential combination rules be given. It is proposed to choose a rule $P \in \mathrm{P}$, $\mathrm{m}_{combP} = \mathrm{m}_i P\ \mathrm{m}_j$, that minimizes the value of the total uncertainty of the combined *bpa's* $\min\left(T(\mathrm{m}_{combP})\right)$. Formally, the procedure for choosing a combination rule can be represented as next successive stages. At the first stage, from the set of available combination rules $\mathrm{P} = \{P_i \mid i = \overline{1,k}\}$, a subset $\mathrm{P}' \subseteq \mathrm{P}$ is selected that satisfies a set of specified criteria $C = \{c_i \mid i = \overline{1,q}\}$.

The data model (DST or DSmT model), expert's competence coefficients, conflict level information, ignorance level, degree of interaction and the structure of expert evidence can be considered as criteria for the choice of combination techniques. Recommendations for the choice of combination rules based on the analysis of a number of criteria are given in [1, 21, 24, 25]. As a result, the initial set $\mathrm{P} = \{P_i \mid i = \overline{1,k}\}$ will be narrowed down to subset $\mathrm{P}' = \{P_i \mid i = \overline{1,z}\}$, $z \le k$, that obtained by excluding from the set P, rules that do not satisfy the formed set of criteria

The choosing the combination rule based on the analysis of quantitative characteristics of uncertainty has done on the second stage.

At first, the combination rule $P_l \in \mathrm{P}'$ is selected that maximizes the value of measure (5) that reflects the degree of specificity of the combination result max($\delta_S(\mathrm{m}_i P_l\ \mathrm{m}_j)$),. $\delta_S(\mathrm{m}_i P_l\ \mathrm{m}_j) \neq 1$.

Next, the combination rule $P_r \in \mathrm{P}'$ is selected that minimizes the value of the measure (9) that reflects the degree of contradiction of the combination result min($Contr(\mathrm{m}_i P_r\ \mathrm{m}_j)$), $Contr(\mathrm{m}_i P_r\ \mathrm{m}_j) \neq 0$.

If $P_l \neq P_r$, then a combination rule is selected that satisfies the following condition:

$$P = \begin{cases} P_l, & T\left(\mathrm{m}_i P_l\ \mathrm{m}_j\right) < T\left(\mathrm{m}_i P_r\ \mathrm{m}_j\right); \\ P_r, & T\left(\mathrm{m}_i P_l\ \mathrm{m}_j\right) > T\left(\mathrm{m}_i P_r\ \mathrm{m}_j\right). \end{cases} \tag{10}$$

where T is a measure of global uncertainty calculated by eq. (8).

If $B_j \in P_2$, then it was revealed that there is inconsistency (conflict) of expert evidence, which indicates the presence of several subgroups of experts with similar assessments, or the presence of so-called dissident experts (one or more experts with estimates significantly different from those of the main group).

As a result, three tasks arise:

1. Identification and exclusion of conflicting (contradictory) evidence (experts).
2. Partitioning (clustering) of the original set of expert evidence into homogeneous (with an acceptable conflict level) subgroups.
3. Aggregation of conflicting (contradictory) evidence in order to find a group assessment.

The solution to the first task lies in the use of different approaches (techniques, measures) allowing to quantify similarities and differences of expert opinions. These techniques use different distance metrics [3, 4, 11, 12, 30] and allow to calculate the level of conflict between the focal elements of several groups of evidence [8-10, 14, 15, 20, 31], for example, the degree of conflict between an expert and the rest of t-1 experts [20]. In this case, both the nature of the selected subsets of frame of discernment (including singletons) and the values of the obtained *bpa's* are taken into account.

For example, let $\Omega = \{a,b,c,d\}$ be a frame of discernment, thus

Case 1: the evidence of experts

E_1 : $m\{a\} = 0.1$; $m\{b\} = 0.9$;
E_2 : $m\{a\} = 0.9$; $m\{b\} = 0.1$;

are contradictory (the same elements of the frame of discernment are evaluated, but they are assigned conflicting *bpa's*);

Case 2: the evidence of experts

$$E_1: \ m\{a\} = 0.4; \ m\{b\} = 0.6;$$
$$E_2: \ m\{c\} = 0.6; \ m\{d\} = 0.4;$$

are also contradictory (there are no common assessed elements of frame of discernment; when the evidence is combined, their intersections will give $\varnothing$).

To solve the second task in [16, 17, 23] has been proposed a procedure for structuring group experts evidence formed under uncertainty and inconsistency, which allows to split the original set of experts evidence into subgroups $\mathrm{E} \Rightarrow \{G_1\}, \{G_2\}, \ldots, \{G_q\}, \ldots, \{G_p\}$ ($G_q \subseteq \mathrm{E}$, $\{G_q\} = \{E_1, \ldots, E_r\}$, $t \geq r \geq 1$, $t \geq p \geq 1$), which are characterized by agreed expert evidence. And determine such E_j that do not belong to any group ($E_j \subseteq G_q$, provided that $|G_q| = 1$). The judgments of such experts E_j are significantly different from those of the main group. Further, within each of the formed subgroups, aggregate group estimates can be obtained. Provided that $p = 1$ (and, as a consequence, $t = r$), the opinions of the entire group E are considered consistent. If a tendency $p \rightarrow t$ and $r \rightarrow 1$ arises, further analysis is inappropriate.

To solve the third task, it is suggested to use one of the proportional conflict redistribution rules. Each of which (depending on the rule) is based on different approaches aimed at redistributing the local or general mass of *bpa's* on the subsets involved in the conflict. Thus, providing mechanisms for combining evidence with a very high level of conflict. In [24-27] notes that PCR5 is the only rule whereby the shares of each local conflict *bpa's* are redistributed to the subsets involved in the conflict in proportion to the relevant *bpa's* for these subsets. This technique makes it possible to achieve the most correct redistribution of the local conflict *bpa's*, but it entails certain computational difficulties.

The next step is the choice of the mathematical formalism for structuring expert data and knowledge formed under identified type of ignorance. If the absence of ignorance nf_i is confirmed, then the task of expert judgments structuring is reduced to solving the problem of finding the aggregated (generalized) expert judgments. If the analysis reveals the presence of nf_i (in this case the initial data set is characterized by inconsistency), then the task of expert judgments structuring is reduced to solving the problem of partition the initial data set into several clusters of experts with close (agreed, consistent, homogenous) evidence, for their subsequent analysis and search for an aggregated estimate within each of the selected subgroups.

In the case when the selection of subgroups of experts and the search for aggregated assessments within the selected subgroups is not acceptable, it is advisable to determine the reason for the spread of expert assessments, identify experts whose assessments violate the consistency of the general set of evidence, and conduct a second survey (possibly with adjustments to the composition of the expert group, changing the expert survey procedure, etc.) in order to obtain high quality expert evidence.

The result of the processes occurring in this block is the information prepared (structured) for decision making, which meets the set goals of the analysis.

The final stage is the interpretation of the results of structuring and the development of a group solution.

5. Conclusion

The main ideas of the technology of structuring the aggregated expert assessments formed under multi-criteria, multi-alternativeness and specific types of ignorance (caused by uncertainty, inaccuracy, inconsistency, conflict / contradiction) has been proposed.

The proposed technology is a set of systematized techniques and methods of intellectual support of decision making processes in various spheres of human activity.

This creates the basis for the development of integrated information technologies for intellectual support of the decision-making process, which allow solving typical decision-making problems in various subject areas, taking into account the type of expert measurements scale, the method of modeling the revealed type of ignorance, the form of presentation of expert judgments (crisp, interval, fuzzy) and the complex of modeled types of ignorance.

A procedure for evidence combination rule selection has been proposed. For each pair of combined expert judgments a rule is selected that minimizes the value of the degree of inconsistency and maximizes the value of the degree of specificity of the result of combination.

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