

SVD-Aided EKF with Process Noise Covariance Adaptation Applied to Satellite Attitude Dynamics

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Abstract

An attitude estimation scheme is designed with the process noise adaptation rule in the extended Kalman filter (EKF) algorithm for a spacecraft in low Earth orbit. The adaptations are designed for compensating the external disturbances by updating the process noise covariance. Satellite attitude estimation algorithm based on the Singular Value Decomposition (SVD) as a preprocessing step in EKF is proposed. The process noise covariance (Q) adaptation rule is incorporated into the previous filter design. The proposed filter has the capability to be robust against initialization errors and the dynamics modeling errors of the satellite. Numerical simulations based on several scenarios are employed to investigate the robustness of the filter.

Keywords 1

Singular value decomposition, extended Kalman filter, attitude estimation, adaptive filtering, rate gyro, small satellite.

1. Introduction

Attitude and angular rate of a satellite can be estimated using a conventional Kalman filter based on nonlinear measurements of reference directions (e.g. Earth magnetic field direction, Sun direction, etc.) [1–3]. On the other hand, linear measurements-based approach using vector observations in a preprocessing method can also be used for attitude estimation purposes. This approach is called nontraditional Kalman filter [3–7]. Here, a single-frame method determining the attitude angles as linear measurement inputs for the attitude filter such as Extended Kalman filter (EKF) or Unscented Kalman filter (UKF). The measurement model becomes linear as the state vector is composed of attitude angles that are directly observed by the preprocessing step. As an example, in [4], authors present an integrated satellite attitude determination system based on EKF and a single-frame estimator to estimate the attitude angles and angular velocities. The single-frame attitude estimator uses the TRIAD (Three-Axis Attitude Determination) method.

As the measurement model is linear in the nontraditional attitude filter, the overall computational load for the attitude estimation algorithm can be decreased at a great extent. In addition, if the filter is correctly tuned the estimation accuracy may increase compared to the estimates of a traditional filter. Because of these benefits the nontraditional approach for attitude filtering is a promising method for attitude estimation of small satellites including the nanosatellites [3, 5, 8, 9].

One of the difficulties for the non-traditional attitude filters that were met in previous investigations is the tuning of the filter in terms of the process noise covariance. Variations in the disturbance torques such as the residual magnetic torque, modeling errors in the satellite's dynamics and any fault in the actuators, they all cause changes in the process noise covariance of the filter. Despite being robust to any kind of variation in the measurements due to its nature, the nontraditional attitude filter is not capable of tuning itself against this kind of process noise covariance variations. The traditional Kalman filtering algorithms including the EKF, also have the same drawback when using any of them for attitude estimation: they are not robust against the environment changes. In case

CMIS-2021: The Fourth International Workshop on Computer Modeling and Intelligent Systems, April 27, 2021, Zaporizhzhia, Ukraine

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CEUR Workshop Proceedings (CEUR-WS.org)

of such changes, the estimation accuracy of the filter deteriorates and if the faults are long lasting, the filter may diverge. For any satellite, harsh operation conditions and anomalies in the geomagnetic field and sun are very likely in space environment. This situation leads us to a search for an Adaptive EKF (AEKF) algorithm that can be implemented in the nontraditional attitude filter.

The paper [10] proposes a robust adaptive UKF (RAUKF) to improve the accuracy and robustness of state estimation with having uncertain noise covariance. An online fault-detection mechanism is adopted to decide whether there is a need to update the current noise covariance or not. If necessary, innovation-based method and residual-based method are used to calculate the estimations of current noise covariance of process and measurement, respectively. The filter combines the last noise covariance matrices with the estimations for the new noise covariance matrices by using a weighting factor.

Like space anomalies the marine environment is also very complex; therefore, it is difficult to accurately obtain the system noise. The acoustic observations usually have gross errors, which affects the positioning accuracy of classical UKF. An RAUKF algorithm is proposed in [11] based on the Sage-Husa filter. The RAUKF can adaptively adjust the system noise to solve the divergence problem, and control the effects of gross measurement errors on dynamic state estimates.

In [12], the authors proposed an integrated algorithm which adapts both the measurement and process noise covariance matrices of the UKF for satellite attitude estimation. However, in this paper, the traditional approach is used for the attitude estimation.

In this study, we extend and add on to our previous researches about non-traditional filtering by proposing an adaptation method for tuning the Q matrix of the filter. The SVD method runs using the magnetometer and Sun sensor measurements as the first stage of the algorithm, and estimates the attitude of the satellite, giving one estimate at a single-frame. Then, these estimated attitude terms are used as inputs to the Q-adaptive EKF (AEKF). In different from the previous works of authors [28, 29], where SVD-Aided UKF adaptation is performed using single scale factors, in this study adapting the attitude filter is fulfilled using multiple scale factors.

The paper presents the mathematical models for a nanosatellite in Section 2. The SVD-aided EKF algorithm is introduced in Section 3 in detail for each sub-algorithm. The proposed filter adaptation method is also presented in this section. In Section 4, the simulation results of the proposed SVD-aided AEKF algorithm for a hypothetical nanosatellite are given. In Section 5, a brief summary and conclusions are given.

2. Singular Value Decomposition Method

Singular value decomposition (SVD) method is one of the single-frame methods (SFMs) used for minimization of Wahba's loss function [13]. At least two vectors need to be measured in order to determine the attitude using the optimal attitude matrix. The loss function is defined as,

$$L(\mathbf{A}) = \frac{1}{2} \sum_i a_i |\mathbf{b}_i - \mathbf{A}\mathbf{r}_i|^2 \quad (1)$$

$$\mathbf{B}_{\text{svd}} = \sum_i a_i \mathbf{b}_i \mathbf{r}_i^T \quad (2)$$

$$L(\mathbf{A}) = \lambda_0 - \text{tr}(\mathbf{A}\mathbf{B}^T) \quad (3)$$

where \mathbf{b}_i is the measurement vector in body coordinate system and \mathbf{r}_i is the vector in reference coordinate system, a_i is the non-negative weight with their sum of λ_0 . \mathbf{B}_{svd} can be seen in Equation (2) has singular value decomposition:

$$\mathbf{B}_{\text{svd}} = \mathbf{U}\mathbf{S}\mathbf{V}^T = \mathbf{U}\text{diag}[S_{11} \ S_{22} \ S_{33}]\mathbf{V}^T \quad (4)$$

$$\mathbf{A}_{\text{opt}} = \mathbf{U}\text{diag}[1 \ 1 \ \det(\mathbf{U})\det(\mathbf{V})]\mathbf{V}^T \quad (5)$$

where the left and right matrices are \mathbf{U} and \mathbf{V} , $\{S_{11}, S_{22}, S_{33}\}$ are the primary singular values and \mathbf{A}_{opt} is the optimal attitude transformation matrix. To examine the rotation angle error, covariance matrix can be defined as,

$$\mathbf{P}_{\text{svd}} = \mathbf{U}\text{diag}[(s_2 + s_3)^{-1} \ (s_3 + s_1)^{-1} \ (s_1 + s_2)^{-1}]\mathbf{U}^T \quad (6)$$

where $s_1 = S_{11}$, $s_2 = S_{22}$, $s_3 = \det(\mathbf{U})\det(\mathbf{V}) S_{33}$. In applications where only two vectors are available and one of them has faulty measurements, the SVD method cannot determine the attitude as at least two vector observations are required for the method to work properly.

2.1. Modeling of Earth's Magnetic Field Vector

International Geomagnetic Reference Field (IGRF) is used as the Earth's magnetic field model in this study [14]. Model needs two inputs, position of the near-Earth spacecraft and the time, for calculating the geomagnetic field vector as,

$$\mathbf{B}_o(k) = -\nabla \left\{ a \sum_{n=1}^N \sum_{m=0}^n \left(\frac{a}{r_{sc}} \right)^{n+1} \left[g_n^m(t) \cos(m\phi_{sc}) + h_n^m(t) \sin(m\phi_{sc}) \right] \times P_n^m(\cos\theta_{sc}) \right\} \quad (7)$$

where r_{sc} is the spacecraft's distance from Earth's center in km, $a = 6371.2$ km (magnetic reference spherical radius), θ_{sc} is colatitude in deg, ϕ_{sc} longitude in deg of the spacecraft at specified time, t . In the equation, the global variables $\{g, h\}$ are Gauss coefficients while P denotes the Legendre function of degree n and order m . The magnetometer measurements can be defined using the transformed magnetic field model using the attitude transformation matrix \mathbf{A} as,

$$\mathbf{B}_m(k) = \mathbf{A}(k)\mathbf{B}_o(k) + \mathbf{u}_B(k) \quad (8)$$

where \mathbf{B}_m is the magnetometer measurement vector and \mathbf{u}_B is the zero-mean Gaussian noise of the magnetometer measurements.

2.2. Modeling of Sun Direction Vector

The reference sun direction can be calculated in inertial frame at a specified time [15]. The sun sensor measurements can be defined using the transformed sun direction model using the attitude transformation matrix \mathbf{A} as,

$$\mathbf{S}_m(k) = \mathbf{A}(k)\mathbf{S}_o(k) + \mathbf{u}_S(k) \quad (9)$$

where \mathbf{S}_m is the sun sensor measurement vector, \mathbf{S}_o is the reference sun direction vector and \mathbf{u}_S is the zero-mean Gaussian noise of the sun sensor measurements.

3. Mathematical Models

3.1. Satellite's Rotational Motion

The kinematics equation of motion of the satellite using Euler angle representation (roll (ϕ), pitch (θ), and yaw (ψ)) is given as [16],

$$\begin{bmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix} = \begin{bmatrix} 1 & \sin(\phi)\tan(\theta) & \cos(\phi)\tan(\theta) \\ 0 & \cos(\phi) & -\sin(\phi) \\ 0 & \sin(\phi)/\cos(\theta) & \cos(\phi)/\cos(\theta) \end{bmatrix} \begin{bmatrix} p \\ q \\ r \end{bmatrix} \quad (10)$$

where the components of the vector $\boldsymbol{\omega}_{BR}$ in body frame with respect to the reference frame are given as p, q, r . The body angular rate with respect to the inertial frame is expressed as,

$$\boldsymbol{\omega}_{BI} = [\omega_x \quad \omega_y \quad \omega_z]^T \quad (11)$$

with the relationship

$$\boldsymbol{\omega}_{BR} = \boldsymbol{\omega}_{BI} - \mathbf{A}[0 \quad -\omega_o \quad 0]^T \quad (12)$$

where ω_o defines the orbital angular velocity with respect to the inertial frame, \mathbf{A} is the attitude transformation matrix composed of Euler angles.

For estimating the attitude and attitude rates together, rate gyro driven kinematics or both the dynamics and the kinematics of the satellite are utilized. We use dynamic equations in this study. The satellite's rotational motion dynamics are derived based on the Euler's equations as,

$$J_x \frac{d\omega_x}{dt} = N_x + (J_y - J_z) \omega_y \omega_z, \quad (13a)$$

$$J_y \frac{d\omega_y}{dt} = N_y + (J_z - J_x) \omega_z \omega_x, \quad (13b)$$

$$J_z \frac{d\omega_z}{dt} = N_z + (J_x - J_y) \omega_x \omega_y, \quad (13c)$$

where J_x , J_y and J_z are the principal moments of inertia and N_x , N_y and N_z are the terms of the external disturbance torque.

3.2. Measurement Models

Single-frame method aided filters use the SFM estimates and the noise covariance calculations. Since the SFM determines the coarse attitude angles, the three elements of the measurements for the filter are directly composed of these angles as,

$$\begin{aligned} z_\phi(k) &= \phi_{svd}(k) + v_\phi(k) \\ z_\theta(k) &= \theta_{svd}(k) + v_\theta(k) \\ z_\psi(k) &= \psi_{svd}(k) + v_\psi(k) \end{aligned} \quad (14)$$

where $v_{(\cdot)}(k)$ is the measurement noise and $\mathbf{z}_\Phi = [z_\phi \ z_\theta \ z_\psi]^T$. The rate gyros are used as the measurement input to the filter as well,

$$\mathbf{z}_\omega(k) = \boldsymbol{\omega}_{Bl}(k) + \mathbf{u}_\omega(k) \quad (15)$$

where \mathbf{u}_ω is the zero-mean Gaussian noise of the rate gyro measurements and $\mathbf{z}_\omega = [z_{\omega_x} \ z_{\omega_y} \ z_{\omega_z}]^T$. The variances and the expectations of the measurement noises are,

$$\begin{aligned} E[v_{(\cdot)}(k)] &= 0, \quad E[v_\phi^2(k)] = \text{Var}(v_\phi(k)), \quad E[v_\theta^2(k)] = \text{Var}(v_\theta(k)), \quad E[v_\psi^2(k)] = \text{Var}(v_\psi(k)), \\ E[v_{\omega_x}^2(k)] &= \text{Var}(v_{\omega_x}(k)), \quad E[v_{\omega_y}^2(k)] = \text{Var}(v_{\omega_y}(k)), \quad E[v_{\omega_z}^2(k)] = \text{Var}(v_{\omega_z}(k)). \end{aligned} \quad (16)$$

4. SVD-Aided EKF for Attitude and Rate Estimation

For the full-state estimation, the state vector (\mathbf{x}) is composed of attitude angles and angular rates as,

$$\mathbf{x} = [\phi \ \theta \ \psi \ \omega_x \ \omega_y \ \omega_z]^T \quad (17)$$

The process and measurement models can be expressed in the discrete-time as,

$$\hat{\mathbf{x}}(k+1) = \mathbf{f}[\hat{\mathbf{x}}(k), k] + \mathbf{w}(k) \quad (18)$$

$$\mathbf{z}(k) = \mathbf{H} \hat{\mathbf{x}}(k) + \mathbf{u}(k) \quad (19)$$

where $\mathbf{f}[\cdot]$ is the state transition matrix, \mathbf{w} is Gaussian process noise, \mathbf{u} is gaussian measurement noise, \mathbf{z} is the measurement vector, \mathbf{H} is the measurement matrix.

The process and measurement noises $\mathbf{u}(k) = [v_\phi(k) \ v_\theta(k) \ v_\psi(k) \ v_{\omega_x}(k) \ v_{\omega_y}(k) \ v_{\omega_z}(k)]^T$ and $\mathbf{w}(k)$ are assumed linearly additive Gaussian, temporally uncorrelated with zero mean. Their corresponding covariances are,

$$E[\mathbf{w}(i)\mathbf{w}^T(j)] = \mathbf{Q}(i)\delta(ij), \quad E[\mathbf{u}(i)\mathbf{u}^T(j)] = \mathbf{R}(i)\delta(ij) \quad (20)$$

Here, $\delta(ij)$ is the Kronecker symbol. The process and measurement noises are assumed to be uncorrelated as,

$$E[\mathbf{w}(i)\mathbf{u}^T(j)] = \mathbf{0}, \forall i, j \quad (21)$$

The state vector estimate is,

$$\hat{\mathbf{x}}(k+1) = \hat{\mathbf{x}}(k+1/k) + \mathbf{K}(k+1) \times \{\mathbf{z}(k+1) - \mathbf{H} \hat{\mathbf{x}}(k+1/k)\} \quad (22)$$

where the measurement vector is presented as $\mathbf{z}(k+1) = [\mathbf{z}_\phi(k+1) \quad \mathbf{z}_\omega(k+1)]^T$ with the measurement matrix \mathbf{H} . For our case, \mathbf{H} is a 6x6 unit matrix because of the linearity. The extrapolation value is,

$$\hat{\mathbf{x}}(k+1/k) = \mathbf{f}[\hat{\mathbf{x}}(k), k] \quad (23)$$

The gain of the filter is,

$$\mathbf{K}(k+1) = \mathbf{P}(k+1/k) \mathbf{H}^T \times [\mathbf{H} \mathbf{P}(k+1/k) \mathbf{H}^T + \mathbf{R}(k)]^{-1} \quad (24)$$

The extrapolation error covariance matrix is,

$$\mathbf{P}(k+1/k) = \frac{\partial \mathbf{f}[\hat{\mathbf{x}}(k), k]}{\partial \hat{\mathbf{x}}(k)} \mathbf{P}(k/k) \frac{\partial \mathbf{f}^T[\hat{\mathbf{x}}(k), k]}{\partial \hat{\mathbf{x}}(k)} + \mathbf{Q}(k) \quad (25)$$

The estimation error covariance matrix is,

$$\mathbf{P}(k+1/k+1) = [\mathbf{I} - \mathbf{K}(k+1) \mathbf{H}] \mathbf{P}(k+1/k) \quad (26)$$

The filter is represented by the Equations (22) – (26).

One of the difficulties for the nontraditional attitude filters is the tuning of the filter's process noise covariance matrix, \mathbf{Q} [17]. Especially, the process noise covariance needs to be tuned when the environment changes [18]. Here, any condition change is referred as a change that may affect the process model of the filter. These might include uncertainties in the inertia parameters, disturbance torques (as it may happen when the satellite goes in/out of the eclipse) as well as the changes in the controller parameters (such as a malfunction) although a controller is not considered in this paper.

5. SVD-Aided Q-Adaptive EKF

The SVD method runs as the first stage of the algorithm and estimates the attitude of the satellite giving one estimate at a single-frame. Then, these estimated attitude terms are given as inputs to the EKF. This forms the SVD-Aided EKF scheme. An adaptation using process noise covariance matrix is also added to this filter called SVD-Aided AEKF. The algorithms in SVD-Aided AEKF is summarized in the block diagram in Figure 1.

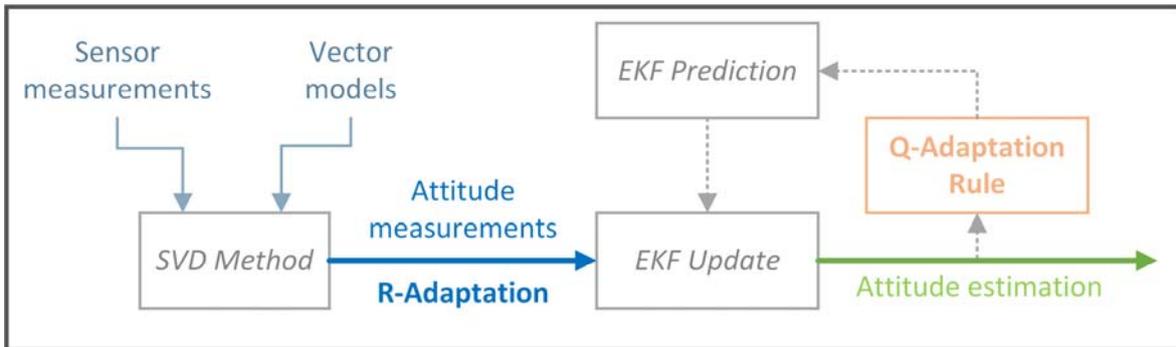


Figure 1: Relationships between the algorithms used for SVD-Aided AEKF

Here, there are two measurements used in SVD for attitude determination: magnetic field and sun direction. The determined attitude and corresponding covariance (\mathbf{R}) are the filter's inputs from the SVD. In addition to SVD's observations, rate gyro measurements are used as well in the full-state estimation process. Also, one more adaptation rule (Q-adaptation) is defined for making the filter adaptive to the environmental changes. This way, the filter is expected to be robust against process noises.

In case of process noise covariance change, the real error of the innovation covariance will exceed the theoretical one. Thus, the basic of the Q-adaptation is to obtain an appropriate multiplier matrix for the \mathbf{Q} matrix such that the real and theoretical values of the innovation covariance could match.

The innovation of the EKF in Equations (22) - (26) is,

$$\tilde{\mathbf{e}}(k) = \mathbf{z}(k+1) - \mathbf{H} \hat{\mathbf{x}}(k+1/k) = \mathbf{z}(k+1) - \mathbf{H} \mathbf{f}[\hat{\mathbf{x}}(k), k] \quad (27)$$

In order to adapt the covariance, an adaptive scaling factor is put into the procedure. Hence, if a scaling matrix ($\mathbf{\Lambda}$) built of multiple scaling factors, is added in to the algorithm as,

$$\mathbf{P}(k+1/k) = \frac{\partial \mathbf{f}[\hat{\mathbf{x}}(k), k]}{\partial \hat{\mathbf{x}}(k)} \mathbf{P}(k/k) \frac{\partial \mathbf{f}^T[\hat{\mathbf{x}}(k), k]}{\partial \hat{\mathbf{x}}(k)} + \mathbf{\Lambda}(k) \mathbf{Q}(k) \quad (28)$$

The scaling matrix $\mathbf{\Lambda}$ can be determined from equality,

$$\frac{1}{\mu} \sum_{j=k-\mu+1}^k \tilde{\mathbf{e}}(j) \tilde{\mathbf{e}}^T(j) = \mathbf{H} \mathbf{P}(k+1/k) \mathbf{H}^T + \mathbf{R}(k) \quad (29)$$

The left side of equality (29) represents the real, and the right side represents the theoretical value of the covariance of the innovation sequence.

The innovation covariance can be expressed in the form below by substituting (28) into (29),

$$\frac{1}{\mu} \sum_{j=k-\mu+1}^k \tilde{\mathbf{e}}(j) \tilde{\mathbf{e}}^T(j) = \mathbf{H} \left[\frac{\partial \mathbf{f}[\hat{\mathbf{x}}(k), k]}{\partial \hat{\mathbf{x}}(k)} \mathbf{P}(k/k) \frac{\partial \mathbf{f}^T[\hat{\mathbf{x}}(k), k]}{\partial \hat{\mathbf{x}}(k)} + \mathbf{\Lambda}(k) \mathbf{Q}(k) \right] \mathbf{H}^T + \mathbf{R}(k) \quad (30)$$

Here, μ is the width of the moving window.

From (30), it can be written,

$$\mathbf{H} \mathbf{\Lambda}(k) \mathbf{Q}(k) \mathbf{H}^T = \frac{1}{\mu} \sum_{j=k-\mu+1}^k \tilde{\mathbf{e}}(j) \tilde{\mathbf{e}}^T(j) - \mathbf{H} \left[\frac{\partial \mathbf{f}[\hat{\mathbf{x}}(k), k]}{\partial \hat{\mathbf{x}}(k)} \mathbf{P}(k/k) \frac{\partial \mathbf{f}^T[\hat{\mathbf{x}}(k), k]}{\partial \hat{\mathbf{x}}(k)} \right] \mathbf{H}^T - \mathbf{R}(k) \quad (31)$$

By multiplying (31) with \mathbf{H}^T on the left and with \mathbf{H} on the right, the scale matrix $\mathbf{\Lambda}$ can be determined as,

$$\mathbf{\Lambda}(k) = \left[\mathbf{H}^T \mathbf{H} \right]^{-1} \mathbf{H}^T \times \left\{ \frac{1}{\mu} \sum_{j=k-\mu+1}^k \tilde{\mathbf{e}}(j) \tilde{\mathbf{e}}^T(j) - \mathbf{H} \left[\frac{\partial \mathbf{f}[\hat{\mathbf{x}}(k), k]}{\partial \hat{\mathbf{x}}(k)} \mathbf{P}(k/k) \frac{\partial \mathbf{f}^T[\hat{\mathbf{x}}(k), k]}{\partial \hat{\mathbf{x}}(k)} \right] \mathbf{H}^T - \mathbf{R}(k) \right\} \times \mathbf{H} \left[\mathbf{Q}(k) \mathbf{H}^T \mathbf{H} \right]^{-1} \quad (32)$$

For a specific case where the all states are measured ($\mathbf{H}=\mathbf{I}$) as in case, (32) reduces to

$$\mathbf{\Lambda}(k) = \left\{ \frac{1}{\mu} \sum_{j=k-\mu+1}^k \tilde{\mathbf{e}}(j) \tilde{\mathbf{e}}^T(j) - \left[\frac{\partial \mathbf{f}[\hat{\mathbf{x}}(k), k]}{\partial \hat{\mathbf{x}}(k)} \mathbf{P}(k/k) \frac{\partial \mathbf{f}^T[\hat{\mathbf{x}}(k), k]}{\partial \hat{\mathbf{x}}(k)} \right] - \mathbf{R}(k) \right\} \mathbf{Q}^{-1}(k) \quad (33)$$

If the process noise does not change, the scale matrix will be a unit matrix as $\mathbf{\Lambda}=\mathbf{I}$, since the innovation values already match. Here, \mathbf{I} represents the unit matrix.

The obtained scaling matrix should be diagonalized since the \mathbf{Q} matrix must be a diagonal, positive definite matrix. Remark that, as μ is a limited number because of the number of the measurements and the computations performed with the computer implies errors such as the approximation errors and the round off errors; $\mathbf{\Lambda}$ matrix, found by using Equations (32) or (33), may not be diagonal and may have diagonal elements which are “negative” or less than “one” (although the computer simulations may bring out such results that is physically impossible in reality). Therefore, in order to avoid such situation, composing the scale factor by the following rule is suggested by [19] as,

$$\mathbf{\Lambda}^* = \text{diag}(\lambda_1^*, \lambda_2^*, \dots, \lambda_n^*), \quad (34)$$

where,

$$\lambda_i^* = \max\{1, \Lambda_{ii}\} \quad i=1, \dots, n. \quad (35)$$

Here, Λ_{ii} represents the i^{th} diagonal element of the matrix Λ . Apart from that point, if there is an actuator fault in the system, Λ_k^* must be included in the estimation process as,

$$\mathbf{P}(k+1/k) = \frac{\partial \mathbf{f}[\hat{\mathbf{x}}(k), k]}{\partial \hat{\mathbf{x}}(k)} \mathbf{P}(k/k) \frac{\partial \mathbf{f}^T[\hat{\mathbf{x}}(k), k]}{\partial \hat{\mathbf{x}}(k)} + \Lambda^*(k) \mathbf{Q}(k) \quad (36)$$

Therefore, when process noise change, the related element of the scale matrix, which corresponds to the changed component of the state vector, increases and that brings out a smaller Kalman gain, which reduces the effect of the innovation on the state update process. As a result, the robustness of the filter against the process noise change is ensured and deterioration of the estimation procedure caused by the fault is prevented.

6. Analysis and Results

A low-Earth orbiting spacecraft with the principal moment of inertia $\mathbf{J} = \text{diag}(2.1 \times 10^{-3} \quad 2.0 \times 10^{-3} \quad 1.9 \times 10^{-3}) \text{ kg} \cdot \text{m}^2$ is considered in the simulations. The spacecraft is tumbling during the simulations with an initial state of $\mathbf{x}_0 = [0.03 \text{ rad} \quad 0.02 \text{ rad} \quad 0.01 \text{ rad} \quad 0.001 \text{ rad/s} \quad 0.0015 \text{ rad/s} \quad 0.002 \text{ rad/s}]$. The sun sensors are processed at 1 Hz and corrupted by Gaussian zero-mean noise with a standard deviation of 2% whereas the magnetometers are corrupted by 8%. The process noise covariance is selected as $\mathbf{Q} = 10^{-4} \mathbf{I}_{6 \times 6}$. There is an eclipse period along the orbit between 1500 and 2500 seconds, which causes the sun sensors to give zero-output.

This section analyzes the application of the process noise covariance adaptation on the filter. The Q-adaptive filter is called SVD-Aided AEKF in this study. The adaptation against process noise is investigated by applying noise increment on the system with a standard deviation of $\sigma_{1,2,3}^{\text{process}} = 0.01 \text{ rad}$ and $\sigma_{4,5,6}^{\text{process}} = 0.01 \text{ rad/s}$ between 3000 and 4000 seconds. The inherently R-adaptive filter called SVD-Aided EKF does not cope well with the process noise increment and deteriorates as seen from Figure 2 whereas simultaneous R- and Q-adaptive filter called SVD-Aided AEKF estimates more accurate attitude angles. On the other hand, angular velocity estimations deteriorate for both filters as seen in Figure 3 and are not improved as much as the attitude angles in Q-adaptation.

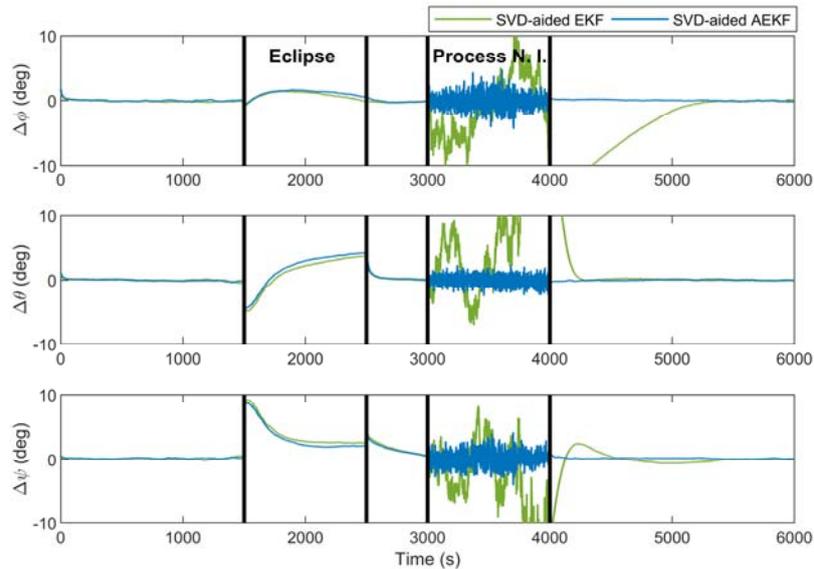


Figure 2: Attitude estimation errors for process noise increment (N.I.) case

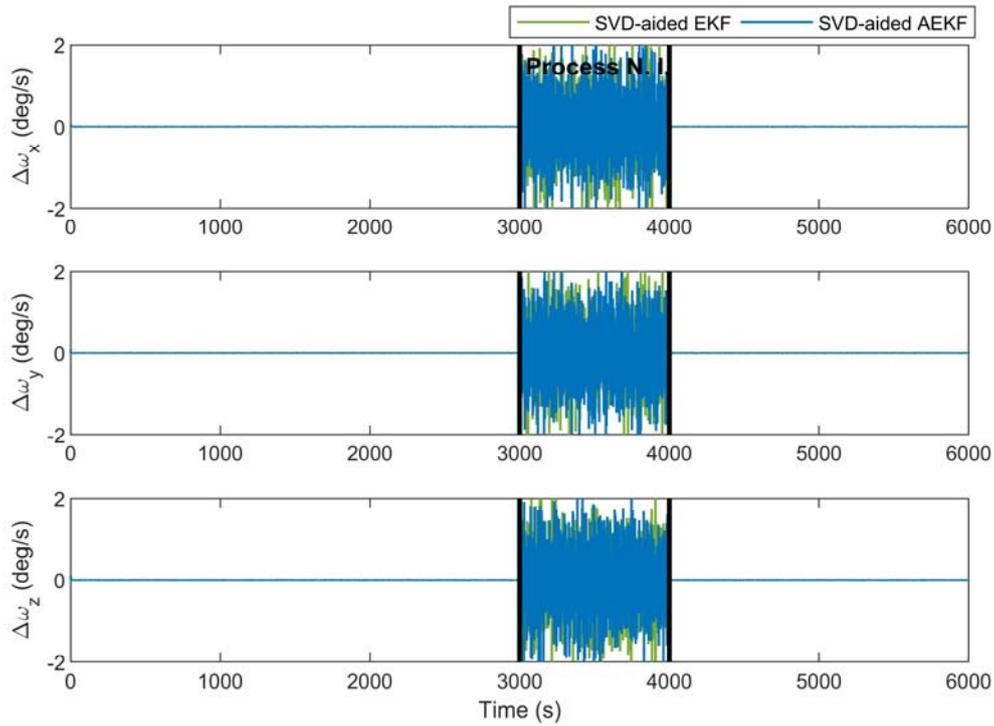


Figure 3: Angular velocity estimation errors for process noise increment (N.I.) case

Table 1

RMS errors of SVD-Aided EKF and AEKF during nominal mode and process noise increment (N.I.)

RMS Error	SVD-aided EKF		SVD-aided AEKF	
	Nominal	Process N.I.	Nominal	Process N.I.
ϕ (deg)	0.9479	4.8026	0.0971	1.2094
θ (deg)	0.0568	11.7243	0.0993	0.8061
ψ (deg)	0.4194	6.7010	0.3377	1.1692
ω_x (deg/s)	0.1545	0.8462	0.1546	0.7900
ω_y (deg/s)	0.0845	0.7890	0.0850	0.8197
ω_z (deg/s)	0.0055	0.7954	0.0057	0.7915

Table 1 is composed of root mean square (RMS) error values of the estimations of the filters considered in this study. The column ‘Nominal’ takes the period outside of the eclipse and outside of the process noise increment into account. The results do not differentiate much between method-to-method as there is no fault considered in this period. On the other hand, SVD-Aided AEKF improves the attitude results with respect to SVD-Aided EKF around 4 to 10 times for the process noise increment interval. Figure 4 demonstrates the adaptive factor used in the Q-adaptation process, which seem to be successful in compensating the noise on the process for estimating the attitude angles.

The adaptive algorithm performs the correction only when the real values of the process noise covariance does not match with the model used in the synthesis of the filter. Otherwise, the filter works in a regular manner.

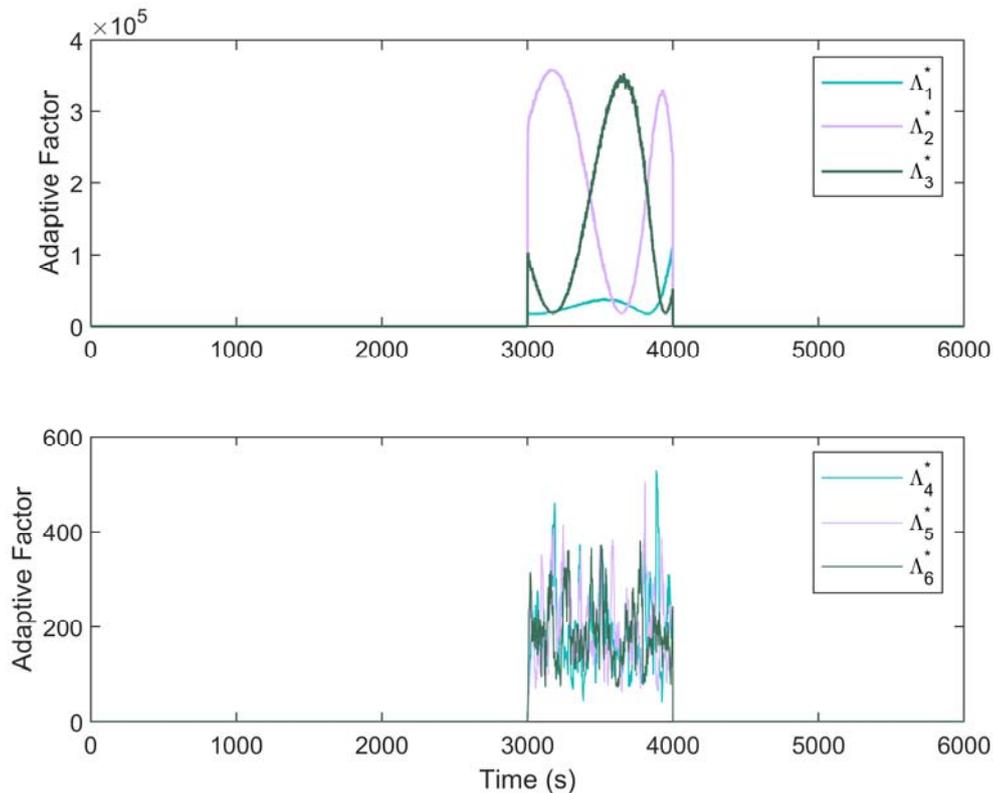


Figure 4: Adaptive factor of the SVD-aided AEKF

7. Conclusions

This paper presents the design and numerical analyses of Q- adaptive extended Kalman filtering algorithm. An attitude estimation algorithm is designed by integrating SVD and EKF methods to estimate the attitude angles and angular velocities. Processing of attitude sensors (magnetometer and sun sensor measurements in this case) within the SVD method computes the Euler angle measurements and their variances, and provides them to the filter as inputs. This filter also eliminates the possibility of poorly-chosen initial attitude as it initializes the attitude using SVD method.

Specifically, the Q adaptation method is proposed in this study. In case of process noise increment, which may be caused by the changes in the environment or satellite dynamics, performance of the Q- adaptive EKF is investigated. To adapt the EKF to the changing conditions its Q matrix is tuned by an adaptive method. Since the measurement model of the SVD-aided EKF algorithm is linear, the adaptation rule is similar to linear KF and easy to apply.

For investigating the system modelling errors, one more case is implemented by implementing a noise increment on the system. Here, the Q-adaptation rule is introduced for the EKF. Thus, the attitude and angular rates of the satellite can be estimated accurately in spite of any change in the process noise covariance.

8. References

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