Algorithm for Calculating the Törnqvist Index for Assessing Changes in Quantitative Indicators of Socio-Economic Systems and Processes (at the Macro and Meso Levels)*

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Abstract. The article examines the existing methodological problems associated with the use of the traditional (in the work referred to as simple) Törnqvist index in practical research in economics. To solve the problems associated with the existing methodological defects when using the simple Törnqvist index, the article proposes an algorithm for calculating the adaptive Törnqvist index with indications of the mechanisms by which these defects are eliminated. The proposed adaptive Törnqvist index is devoid of the drawbacks noted in the article, which increases the level of relevance in economic studies of the quantitative characteristics of socio-economic systems and processes using such an index. When calculating the adaptive Törnqvist index proposed in the article, differences in the mechanisms of changes in the dynamics of quotes of currency pairs and changes in the value of money over time are taken into account.

Keywords: Törnqvist Index, Törnqvist Adaptive Index, Quantitative Characteristics of the Socio-Economic System or Process.

1 Introduction

The Törnqvist index [1] has long established itself as an adequate tool in various economic studies at the macro and meso levels (for example see [2-8]). But in the works listed above, the adaptation of the formula for calculating the Törnqvist index to changes in the values of some parameters and variables included in this formula is not taken into account. The heterogeneity of these parameters and variables leads to the fact that their values can change (in economic dynamics) independently of each other according to algorithms that for each parameter and each variable included in the formula for calculating the Törnqvist index can be different.

The proposed study aims to eliminate this methodological defect in the use of the Törnqvist index in dynamic economic research at the macro and meso levels of the economy.

2 Some Useful Definitions

To reduce the level of subjectivity and increase the unambiguity of understanding the materials that will be presented below, we will define the terminology and algorithms for quantitative calculations used in the study.

Let, within the framework of the study, m socio-economic systems (SES: countries, regions, financial and industrial groups or other SES functioning at the macro or meso levels of the system hierarchy in the economy) are considered.

A basket is a set of n factors (or, in another terminology, determinants; we denote them by $A_{j,i}$, j=1,…,m; i=1,…,n) in the j-th SES, which the researcher - for some him essential features - chose for calculation from the total number of available factors. The list of factors is the same for all investigated m SES.

To concretize the process of presenting the material, let $A_{j,i}$ have selected items of expenses of the j-th SES. Let the value of the costs of the j-th SES by the time t-Δt for each i-th factor $A_{j,i,t-\Delta t}$ is equal to $p_{j,i,t-\Delta t}$, and the value of the costs of the j-th SES by the time t for each i of this factor $A_{j,i,t}$ is equal to $p_{j,i,t}$.

Then the total cost of the j-th SES ($\Sigma A_{j,t-\Delta t}$) for the analyzed basket by the time t-Δt for all $A_{j,i}$ is equal to:

$$\Sigma A_{j,t-\Delta t} = \sum_{i=1}^{n} p_{j,i,t-\Delta t}. \quad (1)$$

And the total cost of the j-th SES ($\Sigma A_{j,t}$) for the analyzed basket by the time t for all $A_{j,i,t}$ is equal to:

$$\Sigma A_{j,t} = \sum_{i=1}^{n} p_{j,i,t}. \quad (2)$$

In the example under consideration, the researcher is usually interested in the comparability of costs for each factor $A_{j,i,t}$ with the total costs of the j-th SES, where $T_{j,t-\Delta t}$ are the total costs of the j-th SES at the time t-Δt, $T_{j,t}$ are the total costs of the j-th SES by the time t. Then the weight coefficients of the costs of the j-th SES at the corresponding time point for each factor $A_{j,i,t}$ are calculated by the formulas:

$$w_{j,i,t-\Delta t} = \frac{p_{j,i,t-\Delta t}}{T_{j,t-\Delta t}}; \qquad w_{j,i,t} = \frac{p_{j,i,t}}{T_{j,t}}. \quad (3)$$

Here, the weighting factors reflect the specific weight of expenses for the factor $A_{j,i}$ in the total expenses of the j-th SES at the corresponding time.

Then the simple Törnqvist index (IT_j) for a period (t-Δt; t) for the j-th SES can be calculated as follows [1]:

$$IT_j = \prod_{i=1}^{n} \left(\frac{p_{j,i,t}}{p_{j,i,t-\Delta t}} \right)^{1/2 \left[\frac{p_{j,i,t-\Delta t} w_{j,i,t-\Delta t}}{\sum_{i=1}^{n} (p_{j,i,t-\Delta t} w_{j,i,t-\Delta t})} + \frac{p_{j,i,t} w_{j,i,t}}{\sum_{i=1}^{n} (p_{j,i,t} w_{j,i,t})} \right]}. \quad (4)$$

The simple Törnqvist index (IT_j) shows the relative change in costs for the analyzed basket for the j-th SES over a period (t-Δt; t) with the value of the index for the previous period, which ended at time t-Δt.

3 Some Adaptive Additions to the Simple Törnqvist Index Calculated by the Formula (4)

1. To compare the values of the IT_j-index for the analyzed SES, it will be necessary to introduce the concept of the basic SES - SES, with the indicators of which the indicators of the remaining SES of the selection will be compared.

Let the SES with the number jB (SES_{jB}) be basic (in ongoing research). The indicators of all other m-1 SES of the selection are compared with the SES_{jB} indicators.

The mechanism for calculating IT_j for each of *j-th* SES is undergoing some changes related to the dynamics of currency quotes.

Let $k_{jB,t}$ is the value of the SES_{jB} currency (for example, Russia) in US dollars at time t. Then $k_{jB,t}$ is measured in rubles/$. At the time t-Δt, the value of the SES_{jB} currency will be equal $k_{jB,t-\Delta t}$.

Let $k_{j,t}$ be the value of the currency of the j-th SES, which is compared with SES_{jB} (in US dollars), at time t. Let it be, for example, Great Britain. Then $k_{j,t}$ is measured in £/$. At the time t-Δt, the value of the currency of the j-th SES will be equal to $k_{j,t-\Delta t}$.

Then for the j-th SES IT_j (taking into account changes associated with the dynamics of currency quotes) is calculated by the following formula:

$$IT_j = \prod_{i=1}^{n} \left(\frac{k_{j,t} / k_{jB,t} p_{j,i,t}}{k_{j,t-\Delta t} / k_{jB,t-\Delta t} p_{j,i,t-\Delta t}} \right)^{1/2 \left[\frac{k_{j,t-\Delta t} / k_{jB,t-\Delta t} p_{j,i,t-\Delta t} w_{j,i,t-\Delta t}}{\sum_{i=1}^{n} (k_{j,t-\Delta t} / k_{jB,t-\Delta t} p_{j,i,t-\Delta t} w_{j,i,t-\Delta t})} + \frac{k_{j,t} / k_{jB,t} p_{j,i,t} w_{j,i,t}}{\sum_{i=1}^{n} (k_{j,t} / k_{jB,t} p_{j,i,t} w_{j,i,t})} \right]}. \quad (5)$$

Or - after simplification:

$$IT_j = \frac{k_{j,t} / k_{jB,t}}{k_{j,t-\Delta t} / k_{jB,t-\Delta t}} \prod_{i=1}^{n} \left(\frac{p_{j,i,t}}{p_{j,i,t-\Delta t}} \right)^{1/2 \left[\frac{p_{j,i,t-\Delta t} w_{j,i,t-\Delta t}}{\sum_{i=1}^{n} (p_{j,i,t-\Delta t} w_{j,i,t-\Delta t})} + \frac{p_{j,i,t} w_{j,i,t}}{\sum_{i=1}^{n} (p_{j,i,t} w_{j,i,t})} \right]}. \quad (6)$$

Formula (6) takes into account the adaptation of the Törnqvist index to differences in the mechanisms of changes in the dynamics of quotes of currency pairs.

2. Besides, it should be borne in mind that the value of money (their purchasing power) changes over time (including due to inflation). Therefore, a correction factor should be introduced into formula (4), which would take into account the indicated processes. Since we are talking about a mechanism for bringing the value of money to a single standard, we will assume that all indicators measured in monetary equivalent should be normalized to the value of money at time t-Δt.

Let $\Sigma B_{j,t-\Delta t}$ is the total costs of the j-th SES at time t-Δt, and $\Sigma B_{j,t}$ is the total costs of the j-th SES at time t. Then for the j-th SES (taking into account changes associated with the dynamics of currency quotes and changes in the value of money), IT_j is calculated by the following formula:

$$IT_j = \frac{\Sigma B_{j,t} k_{j,t} / k_{jB,t}}{\Sigma B_{j,t-\Delta t} k_{j,t-\Delta t} / k_{jB,t-\Delta t}} \prod_{i=1}^{n} \left(\frac{p_{j,i,t}}{p_{j,i,t-\Delta t}} \right)^{1/2 \left[\frac{p_{j,i,t-\Delta t} w_{i,j,t-\Delta t}}{\sum_{i=1}^{n} (p_{j,i,t-\Delta t} w_{j,i,t-\Delta t})} + \frac{p_{j,i,t} w_{j,i,t}}{\sum_{i=1}^{n} (p_{j,i,t} w_{i,j,t})} \right]}. \quad (7)$$

Formula (7) takes into account the mechanisms of adaptation of the Törnqvist index to its use for research in SES, not only with different currencies but also with different - in terms of the level of development and dominant direction - economy (taking into account changes in the value of money over time).

4 Several Problems in the Practical Application of the Simple Adaptive Törnqvist Index (IT_j), Calculated by the Formula (7)

1) Since IT_j is a number from 0 to 1, its economic meaning is not yet clear within the framework of the research being conducted. This remains to be researched. It seems that there is no universal mechanism for defining such a meaning: it follows from the goals and content of each specific study.

2) Since IT_j is a number from 0 to 1, it is not yet clear what its “normal”, “bad” or “good” value is for each j-th SES - within the framework of the study. This also remains to be researched. It seems that there is no universal mechanism for defining such concepts: the content of these categories (“normal”, “bad” or “good” value of the IT_j index), as a rule, follows from the goals and content of each specific study.

3) It is not yet clear - even taking into account the refined formula (7) - how correct (or at least relevant) is the comparison of IT_j values for different SESs within the framework of the study. This also remains to be researched.

4) Here is a mechanism for calculating the values of a simple adaptive IT_j for one period Δt. Some researchers (see, for example, [6]) consider the chain version of the Törnqvist index. The essence of the difference between the chain version of the Törnqvist index and the simple adaptive Törnqvist index described here is that the IT_j value (in the chain version) at time t is calculated iteratively using the IT_j value at time

t-Δt (and intuitively, this looks plausible based on economic practice and mathematical content of IT_j). The extent to which the use of the Törnqvist chain index in economics is essential for increasing the relevance of results' research in dynamics for various types of SES is also still to be investigated.

5) The fact that the mechanisms of SES' behavior at the macro, meso, and micro levels of the system hierarchy in the economy are different is shown in [9-10]. But these differences (within the framework of the research conducted here) are not so important, since, for the application of the Törnqvist index to comparative studies of changes in aggregate indicators in economic dynamics, they are not significant from the point of view of their influence on the relevance of the values of the Törnqvist index obtained as a result of such studies and the formation of the conclusions, based on the obtained values. Another thing is important: to increase the relevance of studies conducted using the Törnqvist index, the authors recommend the use of initial data that would be tied to a specific level of the system hierarchy in the economy (macro or meso). Törnqvist index is of little use for the micro-level of the system hierarchy in the economy because problems at this level with the heterogeneity of comparable data (such as $p_{j,i,t}$) exist.

Notes:

1) The adaptive Törnqvist index can be used not only for a comparative analysis of the relative change in SES costs but also for a comparative analysis of the relative changes in other quantitative indicators of SES functioning (at the macro and meso levels of the system hierarchy in the economy).

2) If pj,i,t are not quantitative magnitudes of the same dimension (for example, rubles, percentages, pieces, etc.), instead of $p_{j,i,t}$ in formula (7), you can use the their relative change over the corresponding time (dimensionlessness):

$$q_{j,i,t-\Delta t} = \frac{p_{j,i,t-\Delta t} - p_{j,i,t-2\Delta t}}{p_{j,i,t-2\Delta t}}, \qquad q_{j,i,t} = \frac{p_{j,i,t} - p_{j,i,t-\Delta t}}{p_{j,i,t-\Delta t}}.$$

Then in formula (7) $p_{j,i,t-\Delta t}$ should be replaced by $q_{j,i,t-\Delta t}$, and $p_{j,i,t}$ should be replaced by $q_{j,i,t}$. True, here it is necessary to check that the root expression $\frac{p_{j,i,t}}{p_{j,i,t-\Delta t}}$ in formula (7) does not take on a negative value. In this case, the index IT_j (mathematically) will not make sense in the set of real numbers.

Although there are already scientific results using complex-valued numbers to socio-economic systems and processes (see, for example, [11]), these problems will not be considered here. This is the topic of further research on the Törnqvist index's application in the study of economic dynamics.

5 Final Remarks

The proposed algorithm for calculating the adaptive Törnqvist index is devoid of the above-mentioned defects, which increases the level of relevance of the application of this tool in studies at the macro and meso levels in economics. The dynamics of the adaptive Törnqvist index' values, although it because of the calculation algorithm incorporated in it - operates only with statistical data, allows relevant analysis of the

changes occurring in socio-economic systems of the most varied orientation and content. It should also be noted that the algorithm for calculating ITj, proposed by the authors, has already been implemented using Excel tools and applied for calculations at the macro-level of the economy.

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