Formalization of Conceptual Classification Models of Knowledge

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Abstract. The paper explores the information (conceptual) systems in terms of the hierarchies of conceptual and material systems. Such systems are an object-oriented from the system-object approach side. The conceptual systems of which are external systems (systems-classes) that determine the properties of specific objects and objects are material or internal systems (facts systems) that implement real interactions. The paper also considers the results of a formal description of the hierarchy of conceptual systems. It takes into account their systemic relationships using a description logic for formalizing certain states of the system-object approach. The syntax and semantics of the *ALCOIQ* descriptive logic and its extension to the *SHOIQ* logic are described. Introduced and formally described the concepts of the volume and content of a conceptual system. It expands the system theory based on a system-object approach.

Keywords: information systems, system-object approach, conceptual systems, systems-classes, material systems, systems-facts, functional query, external determinant, description logic.

1 Introduction

Nowadays, information systems are widely used in various fields of human activity. Such systems are used to store, search and process information. They include general organizational resources (human, technical, financial, etc.) and information dissemination. In our study, the information system is understood as a material system (i.e., real, objectively existing), organizers, custodians and transforming data. Thus, information is a resource.

For example, a set of data under certain conditions, classification, knowledge model, including ontology. An important example for our research is conceptual systems as semantic networks associated with certain rules of concepts and concepts, or conceptual systems that consist of non-physical objects.

In [1, 2] substantiated, the importance of studying such systems and developing principles applicable to both material and conceptual systems for constructing general systems theories as well as for overcoming the omissions that separate the natural sciences and the humanities.

Proceedings of the of the XXIII International Conference "Enterprise Engineering and Knowledge Management" (EEKM 2020), Moscow, Russia, December 8-9, 2020.

The study and formalization of the structure of such systems as well as the concepts of a system-object approach system-class and property-class are described with the concepts of descriptive logic. Description logics are a family of languages for the formal description of knowledge [3]. In this sense, the use of DL allows you to describe the elements of the system using some logical expressions. The structure of the hierarchy of class systems is formally substantiated by describing the syntax and semantics of the *ALCOQ* and *SHIOQ* DLs. The mandatory implementation of the monocentrism principle for conceptual systems is shown. The concepts of the volume and content of system-classes are introduced and described by means of DL. These states expand the system theory based on the system-object approach. The results obtained in the future will allow us to improve existing and create new classifiers. From the point of view of the authors of this article the use of this approach is a promising direction for system-object approach.

2 Conceptual Systems in Terms of System-Object Approach

A feature of the system-object approach is the consideration of two fundamentally different types of systems: internal systems (material systems according to Ackoff) and external systems (conceptual systems according to Ackoff) [1, 4]. We adopted the terms “systems-fact” and “systems-classes” in accordance with [5] using system-object approach [6, 7]. Both ways of systems formation (internal and external) correspond to the basic dialectical principles of the system approach: integrity, systemicity, hierarchy and development presented in [8]. In addition, in [9] it was demonstrated that the main well-known system-wide regularities are fulfilled both for systems-facts and for systems-classes.

In connection with this feature and system-object approach, a system is considered as a functional object or class, a function or role of which is determined by the function or role of an object or class of a higher tier (a supersystem), which clarifies the definition of the system from [11].

A system’s function mentioned in this definition is considered a function of a supersystem as a functional request of a supersystem to a system with a specific function. It is called the external determinant of the system. The external determinant is the cause of the creating of the system the purpose of its existence and the main determinant of its structural, functional and substantial properties. Thus, the external determinant of the system is considered as a universal system-forming factor.

The process of functioning of a system is its internal determinant, since it directly determines its internal properties (properties of subsystems). Correspondence of the internal determinant of the system to its external determinant establishes between the system and the supersystem the relation of maintaining the functional ability of the whole.

In addition to the above-mentioned correspondence of conceptual systems representing systems-classes to systems-facts from the point of view of a system-object approach mentioned in [6, 7, 9]. Next, it's necessary to consider some features of conceptual systems as classes, which are systems (i.e., systems-classes).

The systemicity of such systems-classes is due to the fact that each class depends on the functional ability of a class of a higher tier. For clarity, we give an example: the classes “car” and “truck” functionally support the class “vehicle” as types of road transport i.e. are systems (subsystems of the “vehicle” system). For example, classes “green car” and “blue car” are formally also species of the same class, but functionally does not depend from class “vehicle”, therefore, they are not systems (subsystems). Similarly, for systems-facts: the engine as part of the car functionally depends on the car and is its subsystem. At the same time, a piece cut from a car is also a part of the car, but does not support it functionally and, therefore, is not a subsystem of it.

From the point of view of this study, it is important to emphasize that systems-classes’ hierarchical structure has some differences from the hierarchy of systems-facts. This feature consists in the fact that the hierarchy of system-facts formed by the part-whole relation. It does not have an upper boundary in accordance with the well-known principle of infinity. The hierarchy of system-classes formed by the genus-species relation has an upper boundary in accordance with a known logical law the inverse relationship between the volume and content of concepts (classes) [5, 15]. The fact is that the said law requires a reduction in content, i.e. reducing the amount of information, which corresponds to the number of features describing the content of the class, with an increase in the volume of the class, i.e. the number of subclasses that make up the class. Moreover, the content, of course, can only decrease to zero. This determines the upper boundary of the hierarchy of systems-classes (conceptual systems).

These features are essential for our study for the reason that the basic properties of any system (including a system-class) are determined by a supersystem (in this case, a supersystem-class). The reason for the existence of a system in accordance with the system-object approach is functional request of the supersystem. Those the reason for the presence of certain properties of the system is determined by the hierarchy. Moreover, the analysis of the hierarchy of systems-facts, due to its infinity, does not allow us to determine the final reason for the presence of system properties, which contradicts the principle of determinism. An analysis of the hierarchy of systems-classes allows you to determine the final reason for the presence of system properties due to the finiteness of this hierarchy. Thus, the hierarchy of systems-classes, which does not contradict the provision on the infinity of the world (in terms of the volume of classes), does not contradict the principle of determinism.

3 Application of Description Logic

3.1 Description logic basis

Descriptive logic (DL) is a knowledge representation language for describing the concepts of a subject area in an unambiguous, formalized form. Any DL has syntax and semantics. The basic syntactic elements of the language of descriptive logic are the atomic concept and role, corresponding to predicates of the language of mathematical logic. Concepts are used to describe classes, roles – to describe the relationship between concepts. Concepts and roles allow you to describe classes and their properties [21]. One of the basic descriptive logics is *ALC* [3, 22]. The syntax of the *ALC* logic is presented below in short form:

$$\{\top; \bot; A; A \sqsubseteq C; \neg C; C \sqcap D; C \sqcup D; \exists R.C; \forall R.C\}$$

The symbols $\top$ and $\bot$ are concepts (called true and false). *A* – atomic concept, *C*, *D* – arbitrary concepts. *R* is the atomic role.

The semantics of DL is described using the concept of interpretation. Interpretation is a pair $I = (\Delta, .^I)$, consisting of a nonempty set, called the domain of this interpretation, and an interpreting function I that compares: for each atomic concept, $A \in CN$ is an arbitrary subset of $A^I \subseteq \Delta, CN$ is the set of all concepts; each atomic role $R \in RN$ is an arbitrary subset of $R^I \subseteq \Delta \times \Delta, RN$ is the set of all roles.

Theories that describe knowledge bases distinguish common knowledge about concepts and their relationships, which are expressed using general statements – terminologies, or axioms, as well as knowledge about individual objects, their properties and relationships with other objects – statements about individuals. The DL distinguishes a set of terminological axioms called *TBox*, and a set of statements about the relationships and properties of individuals – *ABox*. Together they form a knowledge base, or ontology $K = TBox \cup ABox$.

In [29], an extension of the *ALC* logic to *ALCOIQ* is presented. The following extensions are introduced in this logic: nominals (*O*) – representation of the individual in the form of a concept. If a is an individual, then $\{a\}$ is a concept. Thus, individual names enclosed in braces become full-fledged concepts. inverse roles (*I*). If *R* is an atomic role, then *R* – is the inverse role; numerical limitations (*Q*).

Each new symbol of the designation of logic means some expansion of it. Using these extensions individually, they say that we get the $ALC \subseteq L \subseteq ALCOIQ$ family of logics. *L* – Logic lying in the interval, belonging to this family.

3.2 Formalization of system-class hierarchy

You can define concepts for objects-classes using the DL. Roles in the DL will correspond to the properties-classes. However, expressing the logic of *ALCOIQ* will not be enough to describe the hierarchy of roles. To solve the problem of constructing a hierarchy of conceptual systems, we use the *SHOIQ* DL [3]. This DL extends *ALCOIQ* and has axioms for *RBox* roles (similar to *TBox* and *ABox*), which allows us to describe the hierarchy of roles as class systems. *ALCOIQ* logic expands with the following points:

- hierarchy of roles (*H*) – axioms of the form $R \sqsubseteq S$ are allowed, where *R*, *S* are arbitrary roles;
- transitive roles (*S*): axioms of the form $Tr(R)$ or R^* are allowed, where *R* is an arbitrary role, R^* is a transitive role.

In *SHOIQ* DL, axioms for *RBox* *R* roles are added to *TBox* and *ABox*, i.e. knowledge base $K = TBox \cup ABox \cup RBox$. Moreover, it is said that *R* is a sub-role of *S*, and *S* is a super-role of *R*.

However, it is necessary to expand the *SHOIQ* logic to justify the structure of the hierarchy of conceptual systems. Will do it by formally introducing into it the concepts of “volume” and “content” of a class system.

The volume of the system-class (*Vol*) is the totality of the species class systems included in this system-class, which is generic to them.

The content of the system-class (*Cont*) includes the supersystem-class (generic class) as well as a set of distinctive features (roles in the supersystem) of this class system.

We describe these concepts in terms of DL. The content of the system-class is expressed through the role that supports the functional ability of the class super-system, as well as through the supersystem-class itself.

$$Cont(S_{ij}^{l}) = S_{i-1,l}^{n} \sqcap \exists RS_{i+1,p_j}^{l_j},$$

where $i = \overline{0,N}$, is the number of the tier of the hierarchy; l, j, l_j, p_j – numbers inside the same tier of the hierarchy.

The concept of the volume of a class system can be described using the operation of combining concepts:

$$Vol(S_{ij}^{l}) = S_{i+1,1}^{j} \sqcup S_{i+1,2}^{j} \sqcup \ldots \sqcup S_{i+1,\bar{N}_{i+1}}^{j},$$

where, $S_{i+1,p}^{j} \sqsubset S_{ij}^{l}$, $p = \overline{1, \bar{N}_l}$, $\bar{N}_i$– the number of nodes of the *i*-level hierarchy.

Roles are also class systems. Therefore, they also have content (properties-properties) and volume.

Next, will see the possibility of creating a formal model of the hierarchy of systems-classes (conceptual systems) using descriptive logic that describes the system relationships between classes. In accordance with the system (system-object) approach a system is considered both as a fact (material object) and as a class (conceptual system). The function or role of this class is determined by the function of the fact or the role of the class of a higher tier (i.e., the supersystem-fact or supersystem-class). A formalized description of this understanding of the system using the notation adopted in the DL is as follows:

$$S_i = [S_{i-1}; RS_i \sqsubset RS_{i-1}]. \tag{1}$$

In expression (1), a formal description of the system is presented in accordance with the Abadi-Kardeli calculus [19]. There are $\forall S_i\ \exists RS_i$ and S_{i-1} are a system-class to indicate a system-class (node) of a higher tier of the S_i hierarchy. Thus, $RS_i \sqsubset RS_{i-1}$ – a method corresponding to the role (function) of the system S_i in the supersystem S_{i-1}; RS_i is a functional role (property-class) that supports the functional ability of a supersystem-class (concept).

Earlier, one of the basic system-wide principles was mentioned, the principle of monocentrism, which directly determines that a stable system “will be characterized by a single center and if it is a complex then it has one higher, common center" [16]. This principle is a consequence of the hierarchical ordering of systems, in our case, the hierarchical structure of generic relations between systems-classes (conceptual systems).

The following are statements that substantiate this principle and the relationship structure of conceptual systems.

Statement 1. If the system-class is depending on of a system-class of a higher tier and the properties (properties-classes) of a class system are depending on a type of properties (properties-classes) of a system-class of a higher tier, then this hierarchy has one root.

Let there exist systems-classes S_{ij}^{l} and RS_{ij}^{l}; i is the number of the tier of the hierarchy; j is the ordinal number of the node in the tier and is the serial number of the supersystem in the tier. In terms of the SHOIQ: S_{ij}^{l} – concept, RS_{ij}^{l} – role (functional role). Suppose that there are systems-classes (descendants) $S_{i+1,p}^{j}$ in S_{ij}^{l}, i.e. $\exists S_{i+1p}^{j} \sqsubset S_{ij}^{l}: p = \overline{1,N}$. Let there exist systems-classes (properties-classes) $S_{i+1,p}^{j}$ occurring in RS_{ij}^{l}, $\exists RS_{i+1p}^{j} \sqsubset RS_{ij}^{l}: p = \overline{1,N}$. We describe the fragments *TBox* and *RBox* corresponding to the above in the form of expression (2) (see Figure 2):

$$TBox = \begin{Bmatrix} \cdots \\ S_{i+1,1}^{j} \sqsubset S_{ij}^{l} \\ \cdots \\ S_{i+1,N}^{j} \sqsubset S_{ij}^{l} \\ \cdots \end{Bmatrix}; RBox = \begin{Bmatrix} \cdots \\ RS_{i+1,1}^{j} \sqsubset RS_{ij}^{l} \\ \cdots \\ RS_{i+1,N}^{j} \sqsubset RS_{ij}^{l} \\ \cdots \end{Bmatrix}. \quad (2)$$

From (1) it is known that properties-classes (functional roles) support the functional ability of the super-system S_{ij}^{l}. Therefore, each class system must have supporting, functional roles that determine the purpose of this system. In turn, RS_{i+1p}^{j} is also a system-class and is a form of RS_{ij}^{l}. $RS_{ij}^{l}$is a supersystem-class i.e. it must have supporting features (properties-properties). We describe the system-class $S_{i,j}^{l}$ in terms of the logic SHOIQ. We get a concept that can be described using the intersection operation.

$$S_{i,j}^{l} \sqsubset S_{i-1,l}^{n} \sqcap \exists RS_{i+1,p_j}^{l_j},$$

where l_j is the serial number of the supersystem class (property-class), with respect to $S_{i,j}^{l}$; p_j is the serial number of the class system in the tier, with respect to $S_{i,j}^{l}$.

We refine the above expression (2) in accordance with the definition of the system in the form of expression (1) (see Figure 2):

$$TBox = \begin{Bmatrix} \cdots \\ S_{i,1}^{l} \sqsubset S_{i-1,l}^{n} \sqcap \exists RS_{i+1,p_1}^{l_1} \\ S_{i,2}^{l} \sqsubset S_{i-1,l}^{n} \sqcap \exists RS_{i+1,p_2}^{l_2} \\ \cdots \\ S_{i,N}^{l} \sqsubset S_{i-1,l}^{n} \sqcap \exists RS_{i+1,p_N}^{l_N} \\ \cdots \end{Bmatrix}; RBox = \begin{Bmatrix} \cdots \\ RS_{i,1}^{l} \sqsubset RS_{i-1,l}^{n} \sqcap \exists RS_{i+1,p_1}^{k_1} \\ RS_{i,2}^{l} \sqsubset RS_{i-1,l}^{n} \sqcap \exists RS_{i+1,p_2}^{k_2} \\ \cdots \\ RS_{i,N}^{l} \sqsubset RS_{i-1,l}^{n} \sqcap \exists RS_{i+1,p_N}^{k_N} \\ \cdots \end{Bmatrix}. \quad (3)$$

As mentioned earlier, systems-classes must have species characteristics (properties- classes) that are different from the generic ones. This is necessary for constructing subsequent tiers of the hierarchy and correlates with the logical law of the inverse relationship of volume and content [19]. In accordance with this law a system-class inherited from the current system-class must have a large number of species characteristics, i.e. a large *content* $Cont\ (S_{ij}^{l}) \sqsubset Cont\ (S_{i+1,p}^{j})$, but a smaller volume $Vol\ (S_{ij}^{l}) \sqsupset Vol\ (S_{i+1,p}^{j})$. Applying this law in its entirety to the entire hierarchy of systems, the following relationships should be satisfied:

$$Cont(S_{0,p_0}) \sqsubset \cdots \sqsubset Cont\left(S_{i-1,p_k}^{p_{k-1}}\right) \sqsubset Cont\left(S_{i,p_{k+1}}^{p_k}\right) \sqsubset Cont\left(S_{i+1,p_{k+2}}^{p_{k+1}}\right) \sqsubset \cdots \quad (4)$$

$$Vol(S_{0,p_0}) \sqsupset \cdots \sqsupset Vol\left(S_{i-1,p_k}^{p_{k-1}}\right) \sqsupset Vol\left(S_{i,p_{k+1}}^{p_k}\right) \sqsupset Vol\left(S_{i+1,p_{k+2}}^{p_{k+1}}\right) \sqsupset \cdots \quad (5)$$

From (4) it follows that if you move along the tiers then each parent system-class should have fewer content than the current one, therefore, have a larger volume. It also follows from (5) that the number of features decreases to the limit state. This state will be the case when the content is the most complete, in this case $k = 0, p_0 = 0$. Moreover, we can talk about the root system-class S_0. This confirms the unity of the top of the classification scheme and statement 1 (see Figure 1).

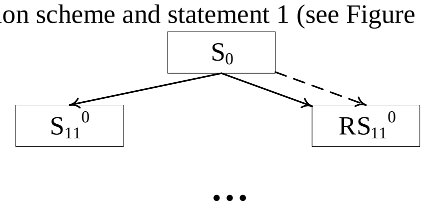

Fig. 1. The root of system-class's hierarchy

Statement 2. The root of the hierarchy of systems-classes is divided into systems-classes, which are objects-classes and properties-class.

Let there be a root system-class S_0, which has no parents. Suppose that it has two descendants (systems-classes) S_{11}^{0} and RS_{11}^{0}. The characteristics will be next $S_{11}^{0} \sqsubset S_0$; $RS_{11}^{0} \sqsubset S_0$; $Vol(S_0) = S_{11}^{0} \sqcup RS_{11}^{0}$; $Cont(S_0) = RS_{11}^{0}$.

In [18] – an extremely wide role, corresponding to the class “property”. In addition, this is consistent with the work of Melnikov [11], which describes the separation of properties into boundary and qualitative, which can be correlated with our reasoning. It is fair to say that in this case relations (4) and (5) will also take a place. This confirms the structure of the hierarchy of conceptual systems and statement 2.

Conclusion

The initial reason for the existence of systems and the presence of certain properties in them is due to the hierarchy of conceptual or external systems (systems-classes). Thus, reality is an object-oriented system. The classes of the system represent conceptual (external) systems-classes that determine the properties of objects. Objects – material (internal) systems-facts that carry out real interactions.

The concepts of the system-object approach "system-class" and "property-class" are unambiguously compared with the concepts of descriptive logic. The syntax and semantics of the descriptive logic *ALCOIQ* and its original extension *SHOIQ* allow us to justify the structure of the hierarchy of class systems and the mandatory implementation of the principle of monocentrism for conceptual systems. The introduction of the concepts of "volume" and "content" of class systems and their description by means of descriptive logic extends a system theory based on a system-object approach.

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