Abstract. A new approach is proposed to identify the optimal set of financial instruments of the broker’s client. The aggregate client portfolio is analyzed, including stocks, foreign currencies, futures contracts. The optimization criterion is the minimum financial collateral of the client’s operations at the expense of borrowed funds. The restrictions are the requirements of the financial regulator. The analysis of the regulation of such lending in the world practice is given. For the Russian rules of the financial collateral regulation for the operations using borrowed funds, a mathematical apparatus is presented for determining the optimal distribution of portfolio component weights, which provides the minimum amount of financial collateral, taking into account the regulator’s restrictions. The quantitative example of optimal portfolio formation is given. The directions for further development of the proposed approach are discussed.

Keywords: The optimal set of financial instruments, The minimum of margin collateral, The operations using borrowed funds.

1 Introduction

Financial market participants with the help of a broker can make unsecured transactions. In such operations, the investor is not required to pay the full amount of the transaction on the client account at the time of the operations. This process is called margin lending. This tool is common and in-demand on the stock market.

Margin lending regulation is based on the requirement for the broker to maintain a predetermined ratio of the client’s portfolio value and the size of market risk. The market risk of the client’s portfolio is understood as the sum of losses that will not be exceeded with a probability equal to the 99% confidence level (Value at Risk, VaR). Until recently, this approach concerned only securities transactions. However, the portfolios of brokerage clients may include foreign currency and derivative financial instruments. Therefore, it is necessary to expand the model for calculating margin lending, taking into account the complex composition of the client's portfolio.
2 The international practice of margin lending regulation

The requirements introduced by supervisory authorities (regulators) are the restrictions for the optimization task of achieving the best financial performance.

The margin lending supervision practices are not uniform. The regulation of margin lending services in various international jurisdictions is varied, and there is no unified approach. The European Union and the United States prohibit uncovered short sales (the sale of a financial instrument that the seller does not own at the time of the transaction, without the required collateral for the transaction) [1-6]. In addition, the United States and Canada have established the requirements for marking short sales [6-8]. In Malaysia, short sales are regulated by a number of legal documents [9-13]. In this case, the regime of regulated short sales is established, as well as the regulation of short sales in the corporate bond market and in the money market.

The margin lending regulation went through several stages of transformation. In the US jurisdiction, according to Rule 204 of the SHO Regulation [6], the requirements are set for brokers/dealers to take the measures to close short positions (an investor sells a financial instrument borrowed from a broker in order to profit from a drop in the price of the sold instrument). In this case, the broker/dealer in case of failure to fulfill obligations to close a short position in a securities transaction must no later than the beginning of the trading session on the day following the day of settlement of the transaction, immediately close the position (make the opposite transaction in order to reduce the number of financial instruments included in composition of the client’s portfolio). In some cases, the rule provides for a longer deadline for closing a position on a security sale transaction. If the broker/dealer does not fulfill the obligation to deliver a security that is in the possession of the person who sent the short sale request, the short sale position of the security must be closed no later than the beginning of the trading session on the 35th day following the date of execution of the transaction.

In the European Union, The Regulation 236/2012 on short sales and certain aspects of credit default swaps (a financial instrument in the form of a credit derivative, or an agreement in which the buyer pays a premium to the seller and the seller assumes the risks of paying third-party obligations) has been in force since 2012 [1] (hereinafter referred to as the Regulation). This Regulation prohibits uncovered short sales of shares that are allowed to be traded. A person has the right to open a short position on the stock if one of the following conditions is met:

- the investor has borrowed shares or has taken alternative measures with similar legal consequences;
- the investor has entered into an agreement to borrow shares or has a different requirement in accordance with the agreement, according to which the ownership of the corresponding number of securities of the same class must be transferred to ensure timely settlement of the transaction;
- the investor has entered into an agreement with a third party, in accordance with which the third party confirms the placement of shares, as well as takes the necessary measures to ensure that settlements can be expected at the appropriate time.
In Malaysia, under the capital markets and services Act [9], a person has the right to take a short position on securities if the investor owns the securities or the investor has a binding and unconditional right to own the security by purchasing it.

Summarizing international experience in the field of margin lending regulation, we note that the mathematical apparatus, when introducing restrictions on the execution of uncovered transactions, is usually not used. It is generally accepted to use regulatory norms without using mathematically formalized dependencies.

Margin lending in the Russian jurisdiction was formed as a service in the early 90s. The demand for this service has led to a gradual complication in calculating the necessary collateral and expanding the client's portfolio: in addition to securities, it may also include foreign currencies and futures contracts.

On July 1, 2019, the Bank of Russia's Directive No. 4928-U of 08.10.2018 “On requirements for brokering when a broker makes certain transactions with securities and concludes contracts that are derivative financial instruments, liquidity criteria for securities provided as security for a client’s obligations to a broker when making such transactions by the broker and concluding such agreements, as well as on the mandatory standards of the broker making such transactions and concluding such agreements” came into force in Russia (hereinafter referred to as the Directive) [14].

The rules established by this Directive differ significantly from the rules for regulating margin collateral in other countries. At the same time, the Directive includes mathematical expressions of economic indicators and dependencies that allow formalizing the calculation of client margin support.

In this paper, on the basis of the Directive, we propose the mathematical model for determining the amount of client collateral (financial assets of the client that are blocked as a guarantee of transaction execution) for uncovered positions (the actual difference between the current value of the financial instrument and the available client collateral). The proposed model considers the definition of the minimum collateral that allows the client to comply with the regulatory standards for risk coverage. Risk coverage standards are defined and described in the next Chapter. Along with that, the collateral should be sufficient to maintain an uncovered position. This model will increase the protection of the broker's client from price fluctuations in various segments of the financial market (in addition to the stock market).

3 Minimization of margin collateral for a portfolio of heterogeneous financial instruments

3.1 The goal function

Let's consider the problem of determining the minimum margin collateral in a client's portfolio consisting of various financial instruments, such as shares, bonds, futures contracts, and foreign currency:
Here $M_0$ - the initial margin; $N_P$ - number of financial instruments in the client's portfolio; $\text{Rate}_j$ - the exchange rate of the jth foreign currency in relation to the national currency, determined on the basis of information on the latest exchange rate of the jth foreign currency to the national currency, formed during organized foreign exchange trading (if the jth currency is the national currency, $\text{Rate}_j=1$; $R_j$ - the parameter, which is calculated according to the methodology from the Directive [14] in two variants:

Variant 1: In this situation the broker does not divide the client portfolio into multiple sets with different types of property or futures contracts (hereinafter referred to as sets with dependent prices). In this case, the method of the Directive assumes the assignment of a base indicator in a set with dependent prices (the main financial instrument whose prices are the base for calculating the dependencies between other financial instruments in the client's portfolio). In this case, the broker does not calculate the reduced size of the initial margin associated with changes in the prices of the base indicator and other financial instruments in the portfolio.

In this case

$$R_j = R_{j,0} = \sum_{j=1}^{N_I} \min\{E_{i,j,0}(-D_{i,j}^+), E_{i,j,0}(D_{i,j}^-)\};$$  \hspace{1cm} (2)

Variant 2: In this situation, the broker divides the client portfolio into multiple sets with different types of property or futures contracts. At the same time, the broker assigns the base indicator in the set with dependent prices. In this case, the broker calculates the reduced size of the initial margin and

$$R_j = R_{j,0} + \sum_{n=1}^{N} R_{j,n}.$$  \hspace{1cm} (3)

Here

$$R_{j,n} = \max\{R_{j,n}^+, R_{j,n}^-\} + R_{j,n},$$  \hspace{1cm} (4)

$$R_{j,n}^+ = -\min\sum_{i=1}^{N_C} E_{i,j,n}(-\tilde{D}_{i,j}^+) \times U_{i,j,n}; 0,$$  \hspace{1cm} (5)

$$R_{j,n}^- = -\min\sum_{i=1}^{N_C} E_{i,j,n}(\tilde{D}_{i,j}^-) \times U_{i,j,n}; 0,$$  \hspace{1cm} (6)

$$R_{j,n}^* = \sum_{i=1}^{N_C} |E_{i,j,n}(d_{i,j,n})|.$$  \hspace{1cm} (7)

The value $E_{i,j,n}(D)$ in accordance with the Directive has several representation variants depending on the economic content of the portfolio components:
Variant 2.1: If we are considering portfolio components with a planned position $Q_{i,j}P$ position (current balances of cash and securities, taking into account future inflows and withdrawals on concluded transactions) in the $i$th foreign currency denominated in the $j$th foreign currency or the planned position in the $i$th security denominated in the $j$th currency, then

$$E_{i,j,n}(D) = P_{i,j} \times Q_{ij}P \times W_{i,n} \times D.$$  

(8)

Variant 2.2: For futures contracts, the value $Q_{ij}F$ value is considered—the difference between the number of futures contracts that provide for the payment of a variation margin (positive or negative difference between the purchase price of a futures contract and its current value) in the client's portfolio when the price of a futures contract increases, and the number of futures contracts that provide for the payment of a variation margin in the client's portfolio when the price of a futures contract decreases; (i-index of a group of such contracts). In this case

$$E_{i,j,n}(D) = V(P_{i,j},D) \times Q_{ij}F \times W_{i,n}.$$  

(9)

Variant 2.3: For the $i$th foreign currency in the client's portfolio in the amount of $Q_{i}C$ let's formulate the other components involved in the goal function:

$$E_{i,j,n}(D) = Rate_i \times (Q_{ij}C + H_i) \times D \times W_{i,n}.$$  

(10)

Let’s formulate the other components involved in the goal function:

$V(P_{i,j}; D)$ - variation margin for a futures contract of the $i$th type when the current settlement price $P_{i,j}$ of this futures contract changes and the value of $D$. The variation margin is calculated according to the specification of the I-th type of futures contract;

$$H_i = \left( \sum_{n=1}^{N} \sum_{g=1}^{N_g} P_{g,j} \times Q_{g} \times W_{i,n} \right) - R_i$$  

(11)

- the amount of the $i$th foreign currency that is a source of currency risk due to the fact that the price of gtx securities and the exchange rate of gtx foreign currencies are expressed in the $i$th foreign currency;

D - parameter that accepts values $(-D_{i,j}^+, (-D_{n,i}^+), (D_{i,j}^-), (D_{n,i}^-), (d_{i,j,n})$. The minimum values of this parameter are set by the clearing organization (a financial intermediary organization that provides netting services between participants in organized trades). Typically, the D parameter is calculated based on the VaR for the corresponding financial instrument;

$D_{i,j}^+$ - the initial risk rate for reducing the price (exchange rate) or the price of the $i$th property or the price of the $i$th type of futures contract (in fractions of a unit), which is calculated based on changes in the indicated prices in the $j$th currency;

$D_{i,j}^-$ - the initial risk rate for an increase in the price (exchange rate) of the $i$th property or the price of the $i$th type of futures contract (in fractions of a unit), which is calculated based on changes in the specified prices in the $j$th currency;

$D_{n,j}^+$ - the initial risk rate for reducing the value of the base indicator of the $n$-th set with dependent prices $C_n$ (in fractions of a unit), calculated based on changes in the specified values expressed in the $j$th currency;
the initial risk rate for an increase the value of the base indicator of the n-th set with dependent prices \( C_n \) (in fractions of a unit), calculated based on changes in the specified values expressed in the jth currency;

\( d_{i,j,n} \)-the risk rate of change relative to the basic indicator the ith property prices or a futures contract of the ith type from the n-th set with dependent prices expressed in the jth currency;

\( U_{i,j,n} \) - an indicator that characterizes the relationship between changes in the prices of the ith property or futures contract of the ith type and the values of the base indicator of the n-th set with dependent prices expressed in units of the jth currency;

\( U_{i,j,n} = 1 \) if there is a direct relationship between the specified price changes (values);

\( U_{i,j,n} = 1 \) if there is an inverse relationship between the specified price changes (values);

\( W_{i,n} \) - the weight of the ith property or volume of the ith type futures contracts from the n-th set with dependent prices. In this case, the weight of the ith property and the volume of the ith type of futures contracts that are not included by the broker in any of the sets with dependent prices are calculated using the formula:

\[
W_{i,n} = 1 - \sum_{i=1}^{N_f} W_{i,n}
\]  

In addition, according to the Directive concept, the proportion of the base indicator in each n-th set with dependent prices is set equal to one, i.e. for the base indicator \( W_{i,n} = 1 \).

\( N_f \) -the amount of the ith property or the price of the futures contract of the i-th type (in fraction of the unit);

\( C_n \) - n-th set with dependent prices;

\( N_G \) - number of g-th securities, exchange rates of g-th foreign currencies;

\( N \) - number of sets with dependent prices.

3.2 The restriction

Risk coverage standards are the restrictions for the optimization task minimizing the margin collateral. Two risk coverage standards are identified by the Directive: the risk coverage standard when executing client orders NRISK1 and the risk coverage standard when changing the value of the client portfolio NRISK2.

NRISK1=S-Mx; NRISK2=S-Mx.  

Here S- the client portfolio value, Mx – the minimum margin.

The minimum margin Mx is defined by Directive as the part of the initial margin M0. This part proportion is fixed at 0.5: Mx =0.5 M0. This is due to the fact that when
futures contracts were included in the portfolio, the calculated margin formulas of the previous regulation could lead to a paradoxical situation when \( \text{Mx} = \text{M0} \). To exclude this case, the direct dependence of one variable on another was introduced. In this case, the values of \( \text{NRISK1} \) and \( \text{NRISK2} \) are interdependent: \( \text{NRISK2} = 0.5(\text{S} + \text{NRISK1}) \).

The regulator has established the broker’s actions in violation of both of these standards. When \( \text{NRISK1} < 0 \), the broker no longer provides the client with borrowed funds. When \( \text{NRISK2} < 0 \), the broker forces the closing of client positions. The value of the client’s portfolio, in this case, will be:

\[
\sum_{i=1}^{I} Q_i P_{i,j} \text{Rate}_j
\]

Here \( Q_i \) - the value of the planned position for the \( i \)th security or \( i \)th currency (hereinafter - the \( i \)th property);

\( I \) - the number of the planned position values in calculating the value of the specified client portfolio;

\( P_{i,j} \) - the price of the \( i \)th security expressed in the \( j \)th currency or the exchange rate of the \( i \)th currency in relation to the \( j \)th currency. The price of one \( i \)th security \( P_{i,j} \) is determined based on information about the price of the traded \( i \)th security under the last concluded agreement (hereinafter - the price of the last transaction), except for the cases listed in the Directive.

The value of the planned position for the \( i \)th property \( Q_i \) is calculated by the following formula:

\[
Q_i = A_i - L_i
\]

According to the Directive \( A_i \) and \( L_i \) are defined in two variants:

**Variant 1:** If the \( i \)th property is cash

\[
A_i = Q_{i,0}^A + \sum_{n=1}^{N_Q} Q_{i,n}^A + \sum_{m=1}^{N_F} F_{i,m}^A
\]

Here \( Q_{i,0}^A \) - funds in the \( i \)th currency in the client’s portfolio;

\( Q_{i,n}^A \) - funds in the \( i \)th currency that are the subject of the \( n \)-th obligation in the client’s portfolio;

\( F_{i,m}^A \) - the positive value of revaluation of the \( m \)th futures contract (one lot of the \( m \)th futures contract). It is expressed in the currency and calculated as the variation margin provided for by this futures contract and (or) in the clearing rules. If the value of the revaluation of the \( m \)th futures contract expresses the amount of the variation
margin that is conditionally payable from the funds included in the client's portfolio, then \( F_{i,m}^A = 0; \)

\( N_Q \) - the number of nth obligations that the broker accepts as the part of the client's portfolio;

\( N_M \) - the number of lots of the mth futures contract.

\[
L_i = \sum_{k=1}^{N_K} Q_{i,k}^l + Q_{i,broker}^l + Q_i^l + \sum_{m=1}^{N_M} F_{i,m}^l
\]  
(17)

Here

\( Q_{i,broker}^l \) - remuneration and (or) reimbursement of the broker's expenses in the ith currency under the brokerage service agreement, if this is provided for in this agreement;

\( Q_i^l \) - funds in the ith currency received in the client’s portfolio from a third party, with the exception of some persons specified by the Directive;

\( Q_{i,k}^l \) - funds in the ith currency, which are the subject of the kth obligation, the execution of which should be carried out at the expense of the property that is part of the client's portfolio;

\( F_{i,m}^l \) - positive revaluation of the mth futures contract (one lot of the mth futures contract) in the ith currency. It is calculated as the variation margin provided for by this futures contract and (or) in the clearing rules. If the revaluation of the m-th futures contract corresponds to the variation margin that is conditionally payable to the client’s portfolio, then \( F_{i,m}^l = 0; \)

\( N_K \) - the number of kth obligations that must be fulfilled at the expense of the property that is part of the client's portfolio.

Variant 2: If the ith property is a security:

\[
A_i = Q_{i,0}^S + \sum_{n=1}^{N_Q} Q_{i,n}^S
\]  
(18)

Here

\( Q_{i,0}^S \) - balance of ith securities in the client’s portfolio;

\( Q_{i,n}^S \) - the number of ith securities that are the subject of the n-th obligation, the execution of which is accepted by the broker as part of the client’s portfolio.

\[
L_i = Q_i^l + \sum_{k=1}^{N_K} Q_{i,k}^A
\]  
(19)
Here $Q_{ik}$ is the number of $i$th securities that are the subject of the $k$th obligation, the execution of which should be carried out at the expense of the property included in the portfolio of the client.

Thus, the constraint for the minimization problem (1) formulated in accordance with the Directive is the inequality:

$$NRISK1 = S - M_o \geq 0$$ (20)

Thus, the complex multivariate Directive of financial regulator [14] is formalized and reduced to the minimization problem (1) with restriction (20). This allows you to select the components of the financial portfolio that provide a minimum of margin collateral when meeting the requirements of the regulator.

4 Example

We will consider the distribution of the weight for finding the minimum margin collateral in the client's high-risk portfolio, consisting of the share $A$ with the current price $P_A=250$ rubles, the bonds $B$ with the current price $P_B=230$ rubles, and the national currency (rubles) in the amount of 27,950 rubles; Rate=1. For share $A$, the planned position is $Q_1=1000$, for bond $B$, the planned positions are $Q_2=-2000$, $Q_3=-1000$ (a negative value corresponds to a short position, for bond $B$, two short positions are included in the portfolio). The broker uses the sets with dependent prices. Thus, for each security with different weights in the portfolio, the ratio $E_{i,j,n}(D)$ (see 8) represents the following values:

- $E_{A,1,1}(D)=25000D$, $W_{A,1}=1$ (share $A$ is the base indicator);
- $E_{B,1,1}(D)=-460000W_{B,1}D$;
- $E_{B,1,2}(D)=-230000W_{B,2}D$.

Since the broker uses sets with dependent prices, here are the parameters corresponding to this option:

- for security $A$, which is the basic indicator of this set: $\tilde{D}_A^+ = 4\%$; $\tilde{D}_A^- = 5\%$; $d_A = 0\%$; $U_A = 1$;
- for security $B$: $\tilde{D}_B^+ = 3\%$; $\tilde{D}_B^- = 4\%$; $d_B = 6\%$; $U_B = 1$.

Then $R^+ = 7700$; $R^- = 0$; $R^* = 3450$.

Thus, the necessary margin provision for maintaining the client's position, which is the minimum of the goal function (1) is $M_0 = 19200$ with the restriction $NRISK1=0$.

The distribution of weights in this case: $W_{A,1}$ (share $A$ is the base indicator), for bond $B$ $W_{B,1}=0.125$, $W_{B,2}=0.875$ respectively.

If the broker does not use weight distribution, then the client's margin provision is not minimized. Then in this example, without the distribution of weights, the necessary margin collateral to maintain the client's position will be: $M_o=20350$.

So by distributing the weights when calculating the necessary margin collateral to maintain a client position, the minimum level of margin collateral for the portfolio of heterogeneous financial instruments is achieved when performing the restrictions.
5 Conclusion

Creating the model for optimizing the margin collateral of a portfolio of heterogeneous financial instruments is a multivariate problem. This approach is a complement and extension of the classical model for optimizing a portfolio of homogeneous financial instruments. This also expands the concept of financial risk. This paper presents the problem of minimizing margin collateral for the client's investment portfolio, which includes stocks, bonds, futures contracts, and foreign currencies.

Note that the new Directive [14] does not yet include non-linear financial instruments, such as options contracts or options. The peculiarity of non-linear financial instruments, including options, is a nonlinear dependence of the current value on changes in the value of the underlying asset, i.e. the subject of a fixed-term transaction (securities, goods, currency, etc.). As a rule, the formation of the price of options depends on many factors: the time until its maturity, the internal cost, the type of option (American, European), volatility, and other. Open positions of clients (the amount of obligations for transactions that were not settled on the date in question) from the options currently brokers take into account separately from the positions of the stock section, such as shares and bonds. This is due to the fact that it is very difficult to determine the margin provision for all financial instruments jointly. Besides, client risk management for options differs significantly from risk management for linear instruments, such as futures contracts. This is due to the complexity of the revaluation of options and insufficient liquidity. The development of the proposed approach to determining the optimal amount of margin provision for the client portfolio will be the construction of a nonlinear model that includes, among other things, options contracts. In addition, the proposed approach can be adapted to assess the minimum margin provision of the broker's client's portfolio under other regulatory rules adopted in other countries.

References

3. LNCS Commission Implementing Regulation (EU) No 827/2012 of 29 June 2012 laying down implementing technical standards with regard to the means for public disclosure of net position in shares, the format of the information to be provided to the European Securi-
ties and Markets Authority in relation to net short positions, the types of agreements, arrangements and measures to adequately ensure that shares or sovereign debt instruments are available for settlement and the dates and period for the determination of the principal venue for a share according to Regulation (EU) No 236/2012 of the European Parliament and of the Council on short selling and certain aspects of credit default swaps Text with EEA relevance, https://eur-lex.europa.eu/legal-content/EN/TXT/?uri=CELEX%3A32012R0827, last accessed 2020/02/10.


14. LNCS Bank of Russia Directive dated 08.10.2018 N 4928-U On requirements for brokering when a broker makes certain transactions with securities and concludes contracts that are derivative financial instruments, liquidity criteria for securities provided as security for a client’s obligations to a broker when making such transactions by the broker and con-
cluding such agreements, as well as on the mandatory standards of the broker making such transactions and concluding such agreements (Registered in the Ministry of Justice of Russia 04.03.2019 N 53942), http://www.consultant.ru/document/cons_doc_LAW_319691/, last accessed 2020/02/10.