# Modeling the satellite communication channel based on stochastic differential equations

Evgenii Glushankov<sup>1</sup>, Anna Lyalina<sup>1</sup> and Evgenii Rylov<sup>2</sup>

<sup>1</sup> Federal State Budget-Financed Educational Institution of Higher Education "The Bonch-Bruevich St. Petersburg State University of Telecommunications", 22/1, Bol'shevikov, St. Petersburg, 193232, Russia

<sup>2</sup> JSC "PKB" RIO", 9G, bldg., 19, Uralskaya st., St. Petersburg, 199155, Russia glushankov57@gmail.com

**Abstract.** The article briefly describes the developed modern satellite communication channel model, which makes it possible to carry out various studies of the quality of the system's functioning. A distinctive feature of the developed model is the presence of a block for modeling random processes based on the SDE. This block can be useful both when studying the effect of fading on the quality of transmission over a channel, as well as assessing the effectiveness of measures to combat them.

Keywords: SDE, satellite channel, Simulink

### 1 Introduction

Today, one of the most relevant and promising directions in the development of technologies for satellite communication systems is the development of high-throughput systems "High Throughput Satellite" (HTS). These systems are able to provide much higher capacity than conventional fixed, broadcast and mobile satellite services (FSS, FSS and MSS) by using multiple "spot beams" to cover the desired service area. This solution has following advantages [1-2]:

- Ability to obtain higher transmit / receive gain: a narrow beam increases power (both transmitted and received), and therefore allows to use smaller user terminals and modulation of higher orders, thus providing a higher data rate per unit orbital spectrum.
- Frequency reuse: covering the desired service area with multiple spot beams allows to use different spacing patterns and the same frequency bands with different polarizations, thus increasing the capacity of the satellite system. An important factor af-

<sup>&</sup>lt;sup>1</sup> Copyright c 2021 for this paper by its authors. Use permitted under Creative Commons License Attribution 4.0 International (CC BY 4.0).

fecting the overall performance of an HTS system is the choice of bandwidth. Satellites can be deployed in several frequency bands, but the Ku and Ka bands are commonly used [3].

- Ka-band uses narrower beams and therefore achieves higher satellite antenna gain, improved link budget, and hence higher throughput for the same antenna size, which is important because antenna size can be limited.
- As the beam size decreases, the noise temperature decreases, so more efficient transmission from the remote station is allowed.
- Better channel budget allows using higher order modulation and coding schemes, thereby spectral efficiency and system throughput are increased.
- Ka-band is more sensitive to atmospheric disturbances. However, this only happens for very limited periods of time and can be mitigated with attenuation mitigation techniques.
- The combination of greater Ka-band spectrum availability and narrower beams allows satellite operators to offer more bandwidth on these frequencies at a more competitive price.

### 2 Materials and methods

### 2.1 Designing a Simulink Model

The developed model is assembled from several parts: DVB-S2X standard channel and blocks of the high-frequency part, including the calculation of losses in the propagation path, taking into account attenuation due to precipitation and fading modeling block. The result is shown in Figure 1 [4-5].

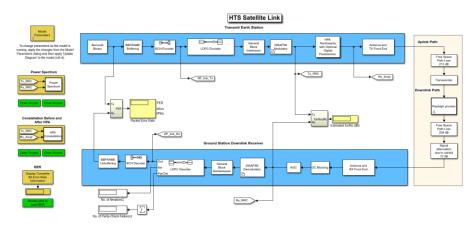


Fig. 1. Model of the satellite communication channel of the HTS system based on the SDE.

Due to the fact that the satellite communication channel is influenced by atmospheric influences, to increase the reliability of the transmission, methods of reducing the in-

fluence of attenuation (Fade mitigation techniques (FMT)) are used. One of these methods is the Adaptive Coding and Modulation (ACM) algorithm. When the channel state changes at the physical transmission layer, the used modulation and coding (ModCods) is dynamically configured. For example, in the case of a clear sky, the use of high coding rates and modulation orders increases spectral efficiency, resulting in a higher system throughput. In poor weather conditions, low coding rates and low modulation orders increase transmission reliability with lower effective spectral efficiency. The research version of this technology is presented in the developed model.

The process describing channel fading is represented in the form of stochastic differential equations.

#### 2.2 Modeling Spatially Coherent Signals

Signal can be written as:

$$s(t) = A(t) \cos[\omega_0 t + \varphi(t)], \qquad (1)$$

where A(t) and  $\varphi(t)$  are random amplitude and phase;  $\omega_0$  is carrier frequency.

We represent the random amplitude and phase in the form of one-dimensional stochastic differential equations [6]:

$$\frac{dA(t)}{dt} = f_A[A(t)] + g_A[A(t)]\vartheta_A(t), \qquad (2)$$

$$\frac{\mathrm{d}\varphi(t)}{\mathrm{d}t} = f_{\varphi}[\varphi(t)] + g_{\varphi}[\varphi(t)]\vartheta_{\varphi}(t), \qquad (3)$$

Where  $f_A[A(t)]$ ,  $g_A[A(t)]$ ,  $f_{\phi}[\phi(t)]$ ,  $g_{\phi}[\phi(t)]$  are the unknown coefficients of the SDE;  $\vartheta_A(t)$ ,  $\vartheta_{\phi}(t)$  are white noise of unit intensity. To represent mathematical models in the form of SDE (2) - (3), it is necessary to determine the unknown coefficients. The most common method for determining SDE coefficients is their calculation by the drift and diffusion coefficients from the Fokker - Planck - Kolmogorov equation, based on the given probability distribution densities of Markov random processes. This method is described in [7-8].

The fading amplitudes can be adequately described by the Rice or Rayleigh distributions, depending on the presence or absence of the specular signal component [9]. Fading is Rayleigh if the number of multiple reflective traces is large and there is no dominant line-of-sight propagation path. If a dominant path also exists, then the fading will be Rice spread.

Thus, a channel with Rayleigh fading can be considered as a special case of a Rice fading channel with K = 0. The Rice probability density distribution function is described:

$$p(r_0) = \begin{cases} \frac{r_0}{\sigma_0^2} e^{-\frac{r_0^2 \cdot A^2}{2\sigma_0^2}} I_0\left(\frac{r_0 A}{\sigma_0^2}\right) для r_0 \ge 0, A \ge 0 \end{cases}$$
(4)

Where  $I_0$  is the modified zero-order Bessel function of the first kind. Now, if there is no dominant propagation path, K = 0 and,  $I_0=1$  which gives the worst case - Rayleigh:

$$p(\mathbf{r}_{0}) = \begin{cases} \frac{\mathbf{r}_{0}}{\sigma_{0}^{2}} e^{\frac{\mathbf{r}_{0}^{2}}{2\sigma_{0}^{2}}} \text{для } \mathbf{r}_{0} \ge 0, \\ 0 \end{cases}$$
(5)

The solution of the Fokker-Planck-Kolmogorov equation in this case takes the form [10]:

$$P_{st}(A) = \frac{C_1}{b(A)} \exp\left(2\int_{A}^{A} \frac{a(x)}{b(x)} dx\right),$$
(6)

From here, the drift and diffusion coefficients are determined as follows:

$$b(A) = -\frac{4\sigma^2}{N_0}A + \frac{N_0}{4}, c = \frac{N_0}{2}$$
(7)

Then the coefficients of SDE (2) are determined from:

$$f_{A}(A) = -\frac{4\sigma^{2}}{N_{0}}A + \frac{N_{0}}{4A},$$
 (8)

$$g_A(A) = 1,$$
 (9)

In this case, for the Rayleigh process, the nonlinear stochastic differential equation (2) takes the form:

$$\frac{\mathrm{dA}}{\mathrm{dt}} = -\frac{4\sigma^2}{N_0}A + \frac{N_0}{4A} + \vartheta_A(t), \tag{10}$$

$$dA = -\frac{4\sigma^2}{N_0}Adt + \frac{N_0}{4A}dt + \vartheta_A(t)dt,$$
(11)

$$A(t) = A(t_0) - \int_{t_0}^{t} \frac{4\sigma^2}{N_0} A(\tau) d\tau + \int_{t_0}^{t} \frac{N_0}{4A(\tau)} d\tau + \vartheta_A(t),$$
(12)

The coefficients of drift and diffusion for a more general case can be determined:

$$b(A) = -\alpha\lambda + \frac{\beta}{\lambda}, c = \frac{N_0}{2}, \qquad (13)$$

Then, for the Rice process, the nonlinear stochastic differential equation takes the form:

$$\frac{\mathrm{dA}}{\mathrm{dt}} = -\alpha\lambda + \frac{\beta}{\lambda} + \vartheta_{\mathrm{A}}(\mathbf{t}),\tag{14}$$

$$A(t) = A(t_0) - \int_{t_0}^t \alpha A(\tau) d\tau + \int_{t_0}^t \frac{\beta}{A(\tau)} d\tau + \vartheta_A(t), \qquad (15)$$

Similarly, from the FPK equation for a uniform phase distribution SDE is obtained (3) in the form:

$$\frac{\mathrm{d}\varphi}{\mathrm{d}t} = \vartheta_{\varphi}(t),\tag{16}$$

### 3 Results

The developed model can be successfully applied in various studies regarding the features of the DVB-S2X standard, the study of the ACM algorithm, fading effects and in the study of the HTS system as a whole:

- Study of the signal transmission rate depending on the selected private range
- Study the influence of the selected ModCod on the transmission
- Investigate the noise system immunity in the atmospheric precipitation of varying degrees presence in the selected frequency range.
- Investigate the spectral efficiency depending on the value of the signal-to-noise ratio.

The results of simulation studies are displayed in Figure 2 and Figure 3 As a consequence, several conclusions can be set:

- For a large coverage area, it is more expedient to use the Ku band, since the Kaband uses narrower beams, which means that a large satellite resource will be required.
- To reduce the atmospheric disturbances influence on transmission the Ka-band is required more power headroom than the Ku-band. Therefore, in regions with a large abundance of precipitation, it is more profitable to work in the Ku-band.
- By increasing the modulation order, the transmission rate increase can be obtained with the same bandwidth. It is actively used in modern systems. In addition, it can be seen that the Ka-band allows the transmission rate to be multiplied for high modulation orders.
- Mode adaptation techniques do indeed cope with rain attenuation fading.
- There is a direct relationship between the value of the signal-to-noise ratio and spectral efficiency. Therefore, it is necessary to find a trade-off between spectral efficiency and bit error rate. On the one hand, a spectral efficiency higher than necessary leads to an increase in throughput, and therefore to an unnecessary error rate

for the system. On the other hand, the use of ModCod with too low spectral efficiency leads to an unprofitable use of the system power.

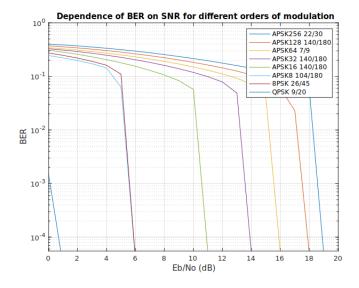
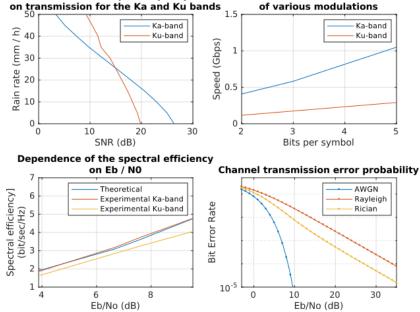


Fig. 2. Dependence of BER on SNR for different orders of modulation.



Assessment of the impact of precipitation Estimation of the transmission rate of various modulations

Fig. 3. Simulation results.

### 4 Discussion

The developed SDE-based communication channel model can be extended with minor additions to simulate other types of fading channels (for example, a correlated Rician fading channel). In addition, this model can be considered as the basis for signal shaping, the characteristics of which have deteriorated due to the effect of fading. Thus, by applying some kind of diversity to provide the receiver with a set of uncorrelated signal copies, and using the powerful error correction code, the effectiveness of the anti-fading measures can be assessed. This can be useful in checking the functionality of a channel in various conditions.

## 5 Conclusion

The developed model has a number of features:

- Uses modern technologies: AKM, DVB-S2X, HTS system parameters
- Allows to set almost any channel parameters: distance to the ground, noise temperatures, rainfall strength, frequencies, antenna diameters, their gain, etc.
- Calculates SNR, packet transmission errors, BER.
- Getes a visual representation of the channel status through the constellation and power spectral density graph.
- Allows to conduct research on the influence on transmission of the effects of Rice and Rayleigh fading arising from multipath propagation of radio waves, as well as changes in the absorbing properties of the medium.

The model can find its application in the development of modern satellite systems.

### References

- Nguyen, Tien M., ed.: Satellite Systems: Design, Modeling, Simulation and Analysis. BoD– Books on Demand (2021).
- 2 Chatzinotas, Symeon, Bjorn Ottersten, and Riccardo De Gaudenzi: Cooperative and cognitive satellite systems. Academic Press (2015).
- 3 Richard Swinford, Bertrand Grau. High Throughput Satellites: Delivering future capacity needs, Arthur D. Little (2015).
- 4 ETSI Standard EN 302 307 V1.1.1: Digital Video Broadcasting (DVB); Second generation framing structure, channel coding and modulation systems for Broadcasting, Interactive Services, News Gathering and other broadband satellite applications (DVB-S2), European Telecommunications Standards Institute, Valbonne, France, (2005).
- 5 ETSI Standard EN 302 307-2 V1.1.1: Digital Video Broadcasting (DVB); Second generation framing structure, channel coding and modulation systems for Broadcasting, Interactive Services, News Gathering and other broadband satellite applications; Part 2: DVB-S2 Extensions (DVB-S2X) European Telecommunications Standards Institute, Valbonne, France (2014).

- 6 Lyalina, A.: Modeling a Satellite Communication Channel Based on Stochastic Differential Equations. Training of professional personnel in the magistracy for the digital economy (PCM-2020). Regional scientific and methodological conference of undergraduates and their leaders; Collection of the best conference reports. SPb.: SPbSUT, 98-102 (2021).
- 7 Glushankov, E.I., Kontorovich, V.Ya.: Mathematical modeling of signals of different spatial coherence in radio communication systems. In the book: Adaptive radio engineering systems with antenna arrays. Publishing house of the Leningrad University, 432-466 (1991).
- 8 Glushankov, E.I., Kontorovich, V.Ya., Savishchenko, N.V.: Digital modeling of vector non-Gaussian random processes describing the parameters of signals and noise in continuous channels. Izvestiya vuzov. Radioelectronics, 38, 3, 69-74 (1995).
- 9 Resnick, S.I.: Adventures in Stochastic Processes. Birkh auser, Boston (1992).
- 10 Tikhonov, V.I., Kharisov V.N.: Statistical analysis and synthesis of radio engineering devices and systems. Radio i Svyaz', Moscow (1991).