Mathematical Modelling of Forming Processes in the Conditions of Uniaxial Compaction of Powder Wax-Like Materials

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Abstract
Mechanical treatment of castings is primarily used in manufacturing of critical parts with high dimensional and geometric accuracy. Investment casting can be used to reduce the specific content of metal and production time. Casting defects are the result of thermophysical impact at various stages of technological process and usually reduce casting accuracy. Forming investment patterns by compacting wax-like powder helps to rectify shrinkage cavities which increases pattern accuracy by 1–2 tolerance grades. In a number of cases the geometry of such pressings is distorted as the result of pressing overcompaction due to material's elastic response. The search for an adequate mathematical model of wax-like powder material compaction process determines the relevance of using finite element method to predict stress-strain state of pressings. The paper presents a comparative analysis of design values and experiment results.

Keywords 1
Mathematical modelling, wax-like powder materials, stress-strain state, compaction, plastic deformation, density, dimensional and geometric accuracy, elastic response.

1. Introduction

Worldwide, parts of structures with complex surface geometry and low roughness are mainly obtained by mechanical treatment of castings or solid metal. This approach to precise steel product formation determines that options to reduce production costs when conforming to the required technological and operational parameters are required. One of the most sought-after technological method of precise casting formation is investment casting. This method is used in various areas and constantly improved.

The specified method makes it possible to obtain castings up to 500 mm with wall thickness up to 1 mm, surface roughness up to Ra = 1.25 μm corresponding to 11–16 tolerance grades [5]. The traditional investment casting sequence includes forming investment patterns, their assembly into clusters, lamination and drying of fire-retardant ceramic shell, dewaxing, heat treatment of the shell and pouring liquid metal into it. At first three stages, the combination of heat and physical impact causes geometry distortion in form of shrinkage cavities on investment pattern surface (shrinkage cavity size can reach up to 8–14 % of the entire product) and crack formation in the shells at the heat treatment stage. The rectification of specified defects done by forming porous investment patterns by compacting wax-like pattern compound powders without external heat sources [6]. Operational strength of pressings is ensured by their structure consisting of local material particles melting and

VI International Conference Information Technologies and High-Performance Computing (ITHPC-2021),
September 14–16, 2021, Khabarovsk, Russia
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pores in such pressings solve the problem of crack formation during dewaxing. It was experimentally confirmed that porous pattern accuracy is 1–2 tolerance grades higher than that of the traditional ones.

Alongside the obvious advantages of the specified method, the disadvantage is the possibility of overcompaction of different pressing areas in narrow parts, mutually perpendicular elements, etc. The process of plastic material powder compaction is performed at three main stages [7]: structural powder body deformation; local contacting particles deformation; plastic deformation of the whole compacted substance. As the result of a series of preliminary experiments, it was established that mutual overlapping of pressing stages which determines the simultaneity of all processes accompanying compaction [8]. Overcompaction leads to elastic material response and further resultsin geometry distortion. It was experimentally established that elastic response during pressing formation using wax-like material PS 50/50 (50% paraffin + 50% stearin) with 6–10% porosity is 0.7...1.2% in a direction coaxial to the pressing direction, and 0.4...0.5% in the transverse directionю Material load relief affects the final size of the pressing after product extraction from the press matrix. This is expressed by linear pressing size exceeding the size of the corresponding press matrix forming elements and depends both on the properties and the volume of compacted material as well as on the volume and degree of air compression in the pressing. The values of compacted material elastic response in various pressing areas also depend on its configuration due to the presence of varying density areas.

In relation to this it becomes obvious that the main factor in forming pressings with homogeneous density distribution is material particles movement at the first and second stages of compaction. Equal-value specific indicators of compacted material density depend on the form and fraction of components, pressing speed, time the material is under load [9, 10]. The latter should ensure the equality of density values in the plasticised wax-like material due to the material flow.

In cases when multi-component materials (consisting of 20% ammonium nitrate granules and 80% paraffin powder of equal fraction) are used for pressing formation, uniform mixture component distribution at the end of the second compaction stage ensures the localisation of wax-like component deformation points around more dense elements. This significantly affects pressing formation with the same elastic response values in all its parts.

Preliminary studies established the minimal value difference for compacted material relaxation in longitudinal and transverse directions relative to the pressing axis of powder body consisting of wax-like spherical elements of equal fraction [11, 12].

It is difficult to perform compaction calculations for a powder body made of wax-like materials in form of spherical elements. The attempts to calculate stress-strain state of more than 2 uniaxially located spherical elements lead to triangulation errors when plotting the spacial grid of modelling objects. It is obvious that the results of such calculations need to be confirmed experimentally.

Studying compaction of wax-like material powder in the course of comparative analysis of the results obtained by experiments and mathematical calculations is aimed at establishing the operability of stress-strain state of powder bodies affected by uniaxial compaction. The relevance of the study results presented here is based on the necessity to create an adequate form change model for powder mediums consisting of wax-like materials which will allow predicting finite properties of pressings.

2. Purpose and Goals

The purpose hereof is mathematical modelling of pressing formation in the closed cylindrical matrix in the course of unilateral compaction of wax-like powder materials.

The following goals were set herein:

- analysis of models using finite element method to describe the processes of wax-like material compaction based on the evaluation of stress-strain state of pressings;
- composing a logarithmic equation describing unilateral compaction of wax-like material powder body;
- determination of applicability of regressional relationships to formation of porous pressings obtained by wax-like powder material compaction.

3. Methods and Approaches

Modelling form change of wax-like material powder to obtain pressings of certain configuration is difficult due to the lack of information on their properties. Considering such materials like paraffin are not used in mechanical engineering as structural materials, it is not possible to find a standard value, for example, Young's modulus, for them in known sources.

To fulfill the purpose of the study, the specified goals were reached in the following sequence:

- determination of material constants required to virtually obtain the characteristics of pressings stress-strain state;
- determination of maximum obtainable pressing material density in the conditions of uniaxial vertical loading in a rigid matrix;
- determination of equation parameters describing the stress relief of compacted powder body from wax-like material obtained as the result of experimental unilateral vertical compaction of a powder body in a closed matrix.

Finite element method should be used for modelling wax-like material powder properties. This method is used in most applied software as it has an option of adaptive automated generation of finite element grid and allows to take into account elastic and visco-plastic properties of material. In this case the method is used to solve the combined mechanical and temperature problem.

The accuracy of calculations for compaction of wax-like powder body is affected by a number of constants and parameters that have to be determined. The following is to be determined for compacted powder body material: density, deformation resistance curve depending on such parameters as deformation, deformation speed, temperature. The following is to be determined for mold material: deformation resistance curve, density, Young's modulus which is an important physical quantity characterising the property of material to resist compaction during elastic deformation. Steel 45 was chosen for the experiment as material imitating soluble components. Steel 45 properties correspond to GOST 1050-88 (Young's modulus: 210 GPa, density 7810 kg/m³).

Wax-like single-component material, T1 grade paraffin with properties corresponding to GOST 23683-89 was chosen as pattern medium. As T1 properties range is rather wide, the material properties required for calculations shall be determined experimentally. The density of T1 obtained by free pouring (taking into account distributed porosity) comprised 0.86 g/cm³. Melting temperature determined using a differential and thermal analyser Shimadzu DTG-60H comprised 60 ^{0}C. Cylindrical samples were destroyed by compaction at AG-X plus Shimadzu to determine Young's modulus of T1 paraffin. The samples were checked according to the following:

$$\varepsilon_c^* \leq 0{,}4\frac{D^2}{h^2}, \quad (1),$$

where ε_c^* is maximum nominal relative deformation at compaction, D is cylinder size, mm, h is the height of the obtained sample, mm [13].

The samples with a diameter to height ratio ≈ 1:1.5 fully conform to condition 1.

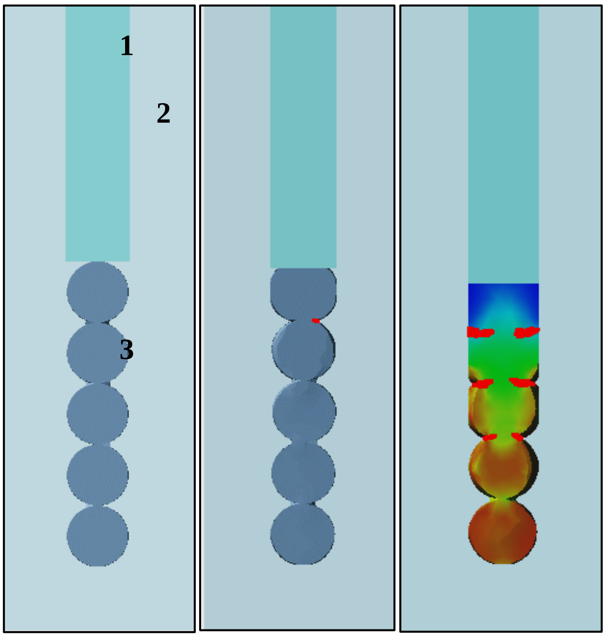
Young's modulus was calculated according to the following formula:

$$E_i = \frac{F_i h}{S\Delta h}, \quad (2),$$

whereF_i is normal strength component, H; S is sample surface area, mm² ($S=\pi D^2/4$); h is sample height, mm; Δh is modulus of sample height change as the result of elastic deformation, mm [14]. Average value E used for further calculations of load dependence from movement during compaction for T1 paraffin powder comprised 81.91 GPa.

Triangulation errors are possible when modelling the compaction of powder bodies consisting of spherical elements using known computational software. To exclude such errors, the calculation shall be performed for single-component powder bodies consisting of horizontally-oriented cylindrical elements with diameter Ø = 10 mm and height h=10 mm made of T1 material.

Mold rigidity is insignificant, and its forming elements are considered elastic. Figure 1,a shows the layout of cylindrical elements in a mold before the compaction.



a b c

Figure 1: Experiment Scheme: a is frontal scheme of wax-like elements layout in the mold (1 is forming mold walls; 2 is wax-like cylindrical elements; 3 is pressing punch); b is surface layer deformation stage; c is powder body deformation.

The experiment imitates mold's punch reaching the point where plastic deformation of surface layer particles (Fig. 1, b) and the whole powder body (Fig. 1, c) occurs with extensive plastic deformations $\varepsilon \geq 20\%$ characteristic of local cylindrical element sintering zones, the deformations become irreversible, the load increases rapidly. Pressing punch test speed was 1 mm/s during experimental deformation of powder body at Shimadzu AGX250. The data obtained during mathematical calculations is further compared to the data of field study.

4. Results and Discussion

Figure 2 shows design and experimental relationship of deformation (%) and compacted medium resistance to the load (kN) which occurs during the punch movement during unilateral compaction of cylindrical elements made of T1 material. The depictions in Figure 2 correspond to the deformation values $\varepsilon \leq 40\%$. At these deformation values, the volumetric density (ρ_{vol}) of wax-like material comprises 1.3–1.4 of pour density (ρ_{pour}). The pour density of the material considered in the experiment is $\rho_{pour} \sim 0.6$ kg/m^3. Experimental deformation up to the values exceeding $\varepsilon > 40\%$ leads to an increased load characterised by increased pressing resistance at relatively small punch movements. The values of the design and experimental loads become aligned.

Figure 2 shows that the above-mentioned compaction stages of powder body which is imitated in the experiment using cylindrical elements occur almost simultaneously. At the same time, it is rather difficult to account for a number of actual pressing conditions in calculations obtained with software based on using finite element methods. These conditions are as follows: friction of powder body wall layers and mold's forming cavity walls; the impact of fraction and form of compacted elements; ambient temperature determining visco-plastic properties of wax-like materials where compaction occurs due to the emergence of the liquid phase of the material as a result of pressing. Therefore, design curves represent a description of compacted material front movement with increased density right under the pressing punch and least density area at the opposite side of the pressing.

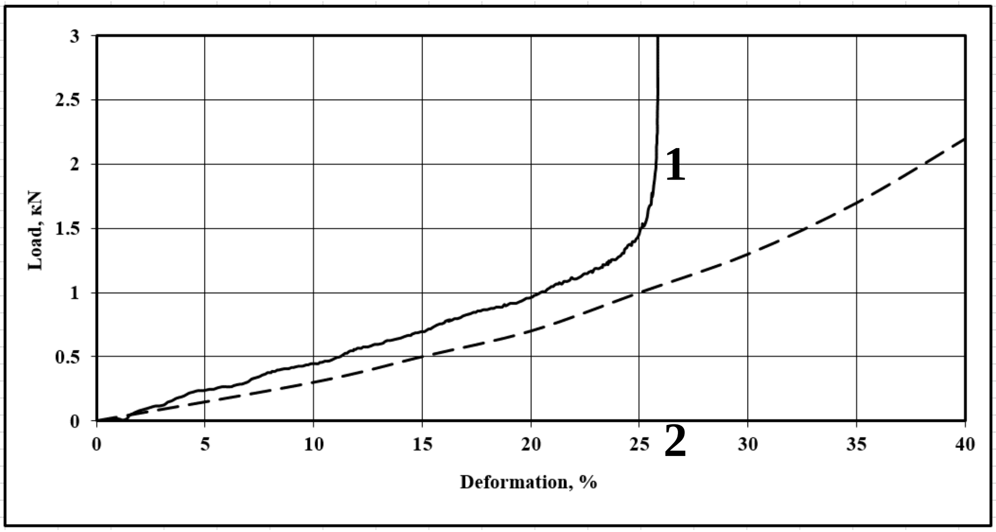


Figure 2: Comparison of Relationship Between Load (kN) and Deformation (%) Obtained as the Result of Calculations (1) and Experimentally (2) at Shimadzu AGX250 unit.

This makes it obvious that using finite element method to model unilateral pressing of powder bodies consisting of wax-like material fractions in order to predict stress-strain state of the pressings is possible to describe the initial stages of body compaction with base area to final pressing height ration max. 1 to 5.

Significant difference between calculation and experimental results found at relatively low pressing punch movement speed 1 mm/s determines the necessity of a universal approach to description of wax-like powder body compaction. Let us use a dimensionless logarithmic equation containing two material constants where one is identically equal to 1 [15] to select an adequate mathematical model, allowing to describe the relationship of powder material compaction and applied pressure.

As the result of a series of experiments, it was established that density values at pressing layers are pretty close in quasistatic mode. In this mode, pressing material stays under set pressure for a few hours until the redistribution of stresses inside compacted body is achieved.

Two-parameter dimensionless pressing equation used to describe wax-like powder body compaction is [16].

$$\rho = b \times \ln P + a, \tag{3}$$

where $\rho = \rho_{cur} / \rho_{theor}$ is current pressing density referred to theoretical (maximum possible) material density, i.e. density in fractions of a unit (experimentally confirmed that T1 material density is 0.912 g/cm^3 at critical pressure 5 MPa); $P = P_{cur} / P_{crit}$ is relative (dimensionless) pressure; P_{cur} is current pressure value; P_{crit} is critical pressing pressure at which theoretical density is achieved. Constant $a = 1$ corresponds to the relative pressing density at $P = 1$. Constant *b* characterises the ability of material to compact and is determined experimentally.

The problem of deriving a logarithmic equations the minimum acceptable pressures where pressing density corresponding to the free pouring in the range of technically acceptable pressures at which pressing density corresponding to the free pouring 0.860 g/cm^3. When the density of T1 is determined by 0.943 and the purpose of 5.7 %, relative density is 0.943 and pressing pressure density is in the range 0.1–0.6 MPa. The range of relative density values at this pressure is 0.70–0.98 which corresponds to the density value range 74–104 % of density occurring during free pouring.

As the constant of dimensionless logarithmic equation is b = 0.0218, the equation of T1 material pressing is as follows:

$$\rho = 0.0218 \times \ln P + 1 \tag{4}$$

To obtain technologically based casting density and required relative pressure comprised 0.01 – 0.04.

5. Conclusion

The study established that properties of pressings formed using wax-like material powders used as investment patterns can be predicted with the help of two-parameter logarithmic equation. Equation coefficients determined experimentally at vertical unilateral compaction in a rigid cylindrical matrix with a wide range of properties are intended for obtaining approximate compaction curves. The obtained curves, in turn, allow determination of pressing pressure which ensures technologically acceptable product density.

The given approach to describing compaction is aimed at realisation of possibility to obtain coefficients of lateral, wall and inter-particle friction as well as physical properties of non-structural wax-like materials, such as elastic modulus and Poisson's ratio. The specified parameters shall further allow to form adequate predictions of compacted material elastic response and take its size into account during work design of molds intended for investment casting.

6. Acknowledgements

The study was performed as part of state assignment of Khabarovsk Federal Research Center.

The studies were carried out using the resources of the Center for Shared Use of Scientific Equipment "Center for Processing and Storage of Scientific Data of the Far Eastern Branch of the Russian Academy of Sciences", funded by the Russian Federation represented by the Ministry of Science and Higher Education of the Russian Federation under project No. 075-15-2021-663.

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