Mathematical Models of Pipelines Alternative Stress States

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Abstract

A pipeline mathematical model as moment shell with singularities caused by the domain geometry is built. The parameters that differentiate the various stress states of the pipeline are determined. Two cases of pipeline geometry are considered, which differ in the numerical parameter we have selected. The analysis of the limiting states of the pipeline is performed. The existence of singularities in solutions is established. A numerical analysis of the stress state is carried out for a weakly bent pipeline and a pipeline with a singularity.

Keywords [1]

Pipeline, stress-strain state, singularities, computing experiment

1. Introduction

Currently, many scientific problems associated with pipeline transportation of oil products remain unresolved. Here we analyse two such problems:

1. The problem of pipeline deviation from the design position [1], associated with the problem of studying the pipeline dynamics in a deformable medium. The curvature of the pipe centerline is small.
2. The problem of stress concentration at pipe joints. The curvature of the pipe centerline tends to infinity. The problem of imposing conjugation conditions on the shells junction was studied in [2].

The aim of the work is numerical study of the features of the stress-strain state of the pipeline under these limiting geometries. Research objectives: (1) defining a numerical criterion for detecting limiting cases; (2) building mathematical models; (3) setting up computational experiments.

2. Problem formulation

We investigate a pipeline of length L with a circular cross-section of radius R_0 and a wall thickness h (see [3, p. 908]). The pipe centerline is curved along the curve $\Gamma_0 = \{x_0, y_0 : x_0 = x_0(s), y_0 = y_0(s)\}$, where s is the arc length. The pipe is filled with a stationary fluid flow with a velocity ϑ_{s0}.

We define the curvature parameter

$$\lambda = R_0 \max\left|\kappa_0(s)\right|, \tag{1}$$

where $\kappa_0(s)$ is the initial curvature of the line Γ. Let the basic geometric relation of the theory of elastic cylindrical shells be satisfied: $h/R_0 = 1$.

The parameter (1) allows distinguishing two limiting cases of pipe geometry:

$$\lambda = 1 \text{ – weakly bent pipe;} \tag{2}$$

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$$\lambda \to \infty \text{ – pipe with singularity.} \quad (3)$$

In the case of (2), the pipe is considered as a technical Vlasov shell (see [4]). In the case of (3), the pipe is considered as a moment shell (see [2]).

The pipe geometry at implementation (2) was examined in [3, 5]. Let's introduce curvilinear coordinates: s – defined above; θ and R are the angle and radius of polar coordinates in the section s. We find for the middle surface of the pipe wall following [6]:

$$A = 1 + R_0\kappa(s,t)\sin\theta,\ B = R_0,$$
$$k_1 = \kappa(s,t)\sin\theta \Big/ \left(1 + \kappa(s,t)R_0\sin\theta\right),\ k_2 = 1 \Big/ R_0; \quad (4)$$

where k_1 and k_2 are the main curvatures of a median surface, κ is the axis curvature.

The geometry of a pipeline with a kink in the profile was studied in [7, p. 7553].

Let's consider case (3) on the example of two cylindrical pipes connected at right angles. We denote: R_i – inner radius of the pipe; R_e – outer radius of the pipe; L_1, L_2 – lengths of the first and second pipe sections along the centerline, respectively; $R_0 = 0.5(R_e + R_i)$. The connected pipes are hereinafter referred to as sections (1) and (2). We construct cylindrical coordinate systems on sections (1), (2), and denote these coordinates by s_i (axial), θ_i (angular) and ρ_i (radial), where i is the section number. We fix the numbering of curvilinear coordinates:

$$x_1 = s, \quad x_2 = \theta, \quad x_3 = \rho. \quad (5)$$

2.1. Equations of the pipeline statics and dynamics

The dynamics of a pipeline is governed by the equations of an elastic body [6]:

$$\rho_t a^k = \nabla_i \sigma^{ki}, \quad (6)$$

where ρ_t is the density of the pipe material, a^k are the acceleration components, σ^{ki} are the stress-tensor components, and ∇_i is the covariant derivative.

In the stationary case, in the absence of external distributed loads, the equilibrium equations are as follows:

$$\nabla_i \sigma^{ki} = 0. \quad (7)$$

We use the equations (6) to describe the case (2), that is, the dynamics of the slow motion of a curved pipeline. We use the equations (7) to describe a pipeline with singular profile, that is, for the case (3).

2.1.1. Dynamics of a bent pipeline

In the case (2), the movement of the inner flow is considered as quasi stationary. The Darcy's law of a friction [8] was chosen as the law of hydraulic resistance. Ibid also given the equations of stationary motion of an incompressible fluid taking into account the Darcy friction force $\mathbf{\Phi}\left(\vartheta_{s0}\right)$. We denoted: ρ_f – fluid density, μ_f – fluid viscosity, p – fluid pressure.

For a bent pipe, the equations system is obtained [3]:

$$\frac{1}{A}\frac{\partial I^{(0)}}{\partial s} - \frac{1-\nu}{B}\frac{\partial \chi_0}{\partial \theta} + (1-\nu)\left(k_1 k_2 u - \frac{k_2}{A}\frac{\partial w}{\partial s}\right) = -\frac{1-\nu^2}{Eh}X,$$
$$\frac{1}{B}\frac{\partial I^{(0)}}{\partial \theta} + \frac{1-\nu}{A}\frac{\partial \chi_0}{\partial s} + (1-\nu)\left(k_1 k_2 v - \frac{k_1}{B}\frac{\partial w}{\partial \theta}\right) = -\frac{1-\nu^2}{Eh}Y,$$
$$-(k_1 + k_2)\cdot I^{(0)} + \frac{1-\nu}{AB}\left[2ABk_1k_2 w + \frac{\partial}{\partial s}(Bk_2 u) + \frac{\partial}{\partial \theta}(Ak_1 v)\right] - \frac{h^2}{12}\nabla^2\nabla^2 w -$$
$$-\frac{h^2}{12}\nabla^2\left[\left(k_1^2 + k_2^2\right)w\right] = -\frac{1-\nu^2}{Eh}Z; \quad (8)$$

$$\nabla^2 = \frac{1}{AB}\left[\frac{\partial}{\partial s}\left(\frac{B}{A}\cdot\frac{\partial\cdot}{\partial s}\right)+\frac{\partial}{\partial\theta}\left(\frac{A}{B}\cdot\frac{\partial\cdot}{\partial\theta}\right)\right].$$

$$I^{(0)} = \frac{1}{A}\left(\frac{\partial u}{\partial s}+\frac{A}{R_0}\frac{\partial v}{\partial\theta}+\nu\kappa\cos\theta\right)+\frac{1}{A}\left(\frac{A}{R_0}+\kappa\sin\theta\right)w-\frac{1}{2A^2}\left[\left(\frac{\partial w}{\partial s}\right)^2+\left(\frac{\partial v}{\partial s}\right)^2\right],$$

$$\chi_0 = \frac{1}{2AB}\left[\frac{\partial}{\partial s}(Bv)-\frac{\partial}{\partial\theta}(Au)\right];$$

$$\frac{1}{h}X = -\rho_t\frac{\partial^2 u}{\partial t^2}+\frac{1}{h}\Phi_t(\vartheta_{s0}),$$

$$\frac{1}{h}Y = -\rho_t\frac{\partial^2 v}{\partial t^2}-\frac{2\mu u^*\cos\theta}{hR_0\left(0.5-\ln\left|\frac{\gamma}{4}\frac{\rho_e u^*}{\mu}R_0\right|\right)},$$

$$\frac{1}{h}Z = -\rho_t\frac{\partial^2 w}{\partial t^2}+\frac{1}{h}(p-p_e).$$

We denoted: u, v, w – displacements of the median pipe surface along the coordinates s, θ, R; X, Y, Z – the components of the forces density acting on the shell along the coordinates s, θ, R; p_e – external pressure. A, B are defined by formulas (4). The system of equations (8) is supplemented by the boundary conditions of rigid fixing and by the homogeneous initial conditions.

2.1.2. Static of a pipeline with a singular profile

In the case of (3), we use the equations (7). The problem statement within the framework of the moment shells theory is investigated in [7]. The mathematical model equations are derived in the coordinates (5):

$$\begin{gathered}
\frac{\partial^2 u_i}{\partial s_i^2}+\frac{1-\nu}{2R_0^2}\frac{\partial^2 u_i}{\partial\theta_i^2}+\frac{1+\nu}{2R_0}\frac{\partial^2 v_i}{\partial s_i\partial\theta_i}+\frac{\nu}{R_0}\frac{\partial w_i}{\partial s_i}-\frac{h^2}{12R_0}\frac{\partial^3 w_i}{\partial s_i^3}+\frac{(1-\nu)h^2}{24R_0^3}\frac{\partial^3 w_i}{\partial s_i\partial\theta_i^2}+\frac{1-\nu^2}{Eh}X_i = 0,\\
\frac{1+\nu}{2R_0}\frac{\partial^2 u_i}{\partial s_i\partial\theta_i}+\frac{1}{R_0^2}\frac{\partial^2 v_i}{\partial\theta_i^2}+\frac{1-\nu}{2}\frac{\partial^2 v_i}{\partial s_i^2}+\frac{1}{R_0^2}\frac{\partial w_i}{\partial\theta_i}-\frac{3-\nu}{2}\frac{h^2}{12R_0^2}\frac{\partial^3 w_i}{\partial\theta_i\partial s_i^2}+\frac{1-\nu^2}{Eh}Y_i = 0,\\
-\nu\frac{\partial u_i}{\partial s_i}-\frac{1}{R_0}\frac{\partial v_i}{\partial\theta_i}-\frac{w_i}{R_0}+\frac{h^2}{12}\left(\frac{\partial^3 u_i}{\partial s_i^3}-\frac{2}{R_0^3}\frac{\partial^2 w_i}{\partial\theta_i^2}+\frac{3-\nu}{2R_0}\frac{\partial^3 v_i}{\partial\theta_i\partial s_i^2}-\right.\\
\left.-\frac{1-\nu}{2R_0^2}\frac{\partial^3 u_i}{\partial s_i\partial\theta_i^2}\right)-\frac{h^2}{12}R_0\left(\frac{\partial^4 w_i}{\partial s_i^4}+\frac{2}{R_0^2}\frac{\partial^4 w_i}{\partial s_i^2\partial\theta_i^2}+\frac{1}{R_0^4}\frac{\partial^4 w_i}{\partial\theta_i^4}\right)+R_0\frac{1-\nu^2}{Eh}Z_i = 0.
\end{gathered} \tag{9}$$

Here X_i, Y_i, Z_i are the external force components acting on the shell; u_i, v_i, w_i are components of the middle surface displacement vector of the i-th section.

On the pipes outer ends, we impose the boundary conditions of rigid fastening. We imposed conjugation conditions on the shells connection line. We obtained formulas for the conjugation conditions in [7].

Geometric conjugation conditions:

$$\begin{gathered}
v_2-u_2\cos\theta = v_1+u_1\cos\theta,\quad u_2+v_2\cos\theta+w_2\sin\theta = u_1-v_1\cos\theta-w_1\sin\theta,\\
-u_2\sin\theta-v_2\sin\theta\cos\theta+w_2\left(1+\cos^2\theta\right) =\\
= u_1\sin\theta-v_1\sin\theta\cos\theta+w_1\left(1+\cos^2\theta\right),\\
\frac{\partial w_1}{\partial\theta} = \frac{\partial w_2}{\partial\theta}.
\end{gathered} \tag{10}$$

Conjugation conditions for force factors:

$$M^{(1)} = M^{(2)}; \qquad S^{(1)} = -S^{(2)};$$
$$-Q^{(2)}\sqrt{1+3\cos^2\theta} + N^{(2)}\sin\theta = N^{(1)}\sin\theta + Q^{(1)}\sqrt{1+3\cos^2\theta};$$
$$-Q^{(2)}\sin\theta - N^{(2)}\sqrt{1+3\cos^2\theta} = N^{(1)}\sqrt{1+3\cos^2\theta} - Q^{(1)}\sin\theta. \tag{11}$$

We denoted: $M^{(i)}$ – bending moments, $S^{(i)}$ – shear forces, $Q^{(i)}$ – cutting efforts, $N^{(i)}$ – normal efforts, (i) stands for section number.

The mathematical model consists of: (1) system of equations (9) with fixed constrains as boundary conditions; (2) geometric conditions on the conjugation line (10); (3) interface conditions for force factors (11).

3. Methods of numerical experiments

3.1. Pipe bending problem

Let's introduce dimensionless variables into the equations (8): $\zeta = s/\ell$, $r = R/R_0$, $\theta = \theta$, $\tau = \omega t$. Here ℓ, ω are the characteristic length and frequency of processes in the pipeline. Displacements of the pipe middle surface: $u' = u/R_0$, $v' = v/R_0$, $w' = w/R_0$; fluid pressure: $p' = p/p_a$.

We obtain the dimensionless form of the equation (8). Then we represent their solutions in the form:

$$u'(\zeta,\theta,\tau) = u_0(\zeta) + \lambda u_1(\zeta,\tau)\sin\theta + O(\lambda^2);$$
$$v'(\zeta,\theta,\tau) = \lambda v_1(\zeta,\tau)\cos\theta + O(\lambda^2);$$
$$w'(\zeta,\theta,\tau) = w_0(\zeta) + \lambda w_1(\zeta,\tau)\sin\theta + O(\lambda^2);$$
$$p'(\zeta,\theta,r) = p^{(0)}(\zeta,r) + \lambda p^{(1)}(\zeta,r)\sin\theta + \lambda p^{(2)}(\zeta,r)\cos\theta + O(\lambda^2). \tag{12}$$

Within the scope of this paper, the dynamics of the fluid is considered known and its study is available in [5, 3]. The zero-order solutions (12) are also supposed to be known.

The equations of the first approximation (see [3]):

$$\alpha^2\frac{\partial^2 u_1}{\partial\zeta^2} - \frac{1-\nu}{2}u_1 - \frac{1+\nu}{2}\alpha\frac{\partial v_1}{\partial\zeta} + \nu\alpha\frac{\partial w_1}{\partial\zeta} +$$
$$+ f\left[\frac{1-\nu}{2}u_0 - 2\alpha^2\frac{\partial^2 u_0}{\partial\zeta^2} + \alpha(1-\nu)\frac{\partial w_0}{\partial\zeta}\right] -$$
$$-\alpha^3\left(\frac{\partial w_1}{\partial\zeta}\frac{\partial^2 w_0}{\partial\zeta^2} + \frac{\partial w_0}{\partial\zeta}\frac{\partial^2 w_1}{\partial\zeta^2}\right) + 3\alpha^3 f\cdot\frac{\partial w_0}{\partial\zeta}\frac{\partial^2 w_0}{\partial\zeta^2} = \frac{\rho_t R_0^2\omega^2}{E^*}\frac{\partial^2 u_1}{\partial\tau^2}; \tag{13}$$

$$\frac{1-\nu}{2}\alpha^2\frac{\partial^2 v_1}{\partial\zeta^2} - v_1 - \frac{1}{E^* h^*}\frac{2u_1^*\mu}{R_0\left(0.5 - \ln|\frac{\gamma\rho_e\lambda u_1^*}{4\mu}R_0|\right)} + \frac{1+\nu}{2}\alpha\frac{\partial u_1}{\partial\zeta} +$$
$$+ w_1 + f\left(w_0 - \frac{3-\nu}{2}\alpha\frac{\partial u_0}{\partial\zeta}\right) - \alpha^2\frac{\partial w_0}{\partial\zeta}\frac{\partial w_1}{\partial\zeta} = \frac{\rho_t R_0^2\omega^2}{E^*}\frac{\partial^2 v_1}{\partial\tau^2}; \tag{14}$$

$$w_1 + \frac{h^{*2}}{12}\left(\alpha^4\frac{\partial^4 w_1}{\partial\zeta^4} - \alpha^2\frac{\partial^2 w_1}{\partial\zeta^2}\right) + \nu\alpha\frac{\partial u_1}{\partial\zeta} - v_1 +$$
$$+ f\left[2\nu w_0 + (1-\nu)\alpha\frac{\partial u_0}{\partial\zeta}\right] - \alpha^2\frac{\partial w_0}{\partial\zeta}\frac{\partial w_1}{\partial\zeta} + \frac{\alpha^2}{2}f\left(\frac{\partial w_0}{\partial\zeta}\right)^2 =$$
$$= \frac{1}{E^* h^*}\left[\rho_f\vartheta_{s0}^2 f - \frac{2u_1^*\mu}{R_0\left(0.5 - \ln|\frac{\gamma\rho_e\lambda u_1^*}{4\mu}R_0|\right)}\right] - \frac{\rho_t R_0^2\omega^2}{E^*}\frac{\partial^2 w_1}{\partial\tau^2}. \tag{15}$$

Thus, the three-dimensional problem (8) is reduced to a one-dimensional formulation. A difference scheme for numerical solution of equations (13)–(15) was constructed in [3].

3.2. Numerical method for a pipeline with a singular profile

In the problem with a kinked pipe, there are points of singularity of the stress field, as indicated in [2]. The problem of calculating such a stress field in a mathematical sense is close to the problem of calculating stresses in the L-shaped domain, see [9]. Therefore, to solve the problem under the condition (3), it is necessary to develop a new computational algorithm. We plan to create this algorithm based on the approach developed in [9, 10, 11, 12].

Numerical experiments on calculating the stress-strain state of a pipeline with a break in the profile were performed in the FreeCAD software package (see [13]) to illustrate the existence of a singularity and estimate the limiting stress values.

We created a pipeline modeling algorithm in the FreeCAD software package. It provides for the creation of a solid 3D model, mesh generation by the finite element method, data entry, solver setup, and visualization of calculation results. The CalculiX finite element method package and the NetGen meshing package are used.

4. Numerical results

4.1. The pipeline bending problem

Physical and geometric parameters of the test problem: $h = 0.005\ \mathrm{m}$, $\rho_e = 1700\ \mathrm{kg/m^3}$, $\mu = 10000\ \mathrm{N\cdot s/m^2}$, $\rho_t = 7200\ \mathrm{kg/m^3}$, $E = 2.07\cdot 10^{11}\ \mathrm{N/m^2}$, $\nu = 0.24$, $R_0 = 0.3\ \mathrm{m}$, $\mu_f = 0.667\ \mathrm{N\cdot s/m^2}$, $\rho_f = 850\ \mathrm{kg/m^3}$, $L = 12000\ \mathrm{m}$, $\vartheta_{s0} = 1\ \mathrm{m/s}$. Calculation time $T_{end} = 691200\ \mathrm{s}$. The centerline of the pipeline is described by a fractional rational function:

$$y = 40(1 - 0.001x)\big/\left(1 + 10^{-6}x^2\right), -6000 \le x \le 6000.$$

In numerical experiments, the following are found: displacements of the centerline, angular deformations of the walls, coordinates of the centerline $x(t,s)$, $y(t,s)$, longitudinal displacements of the first approximation u_1.

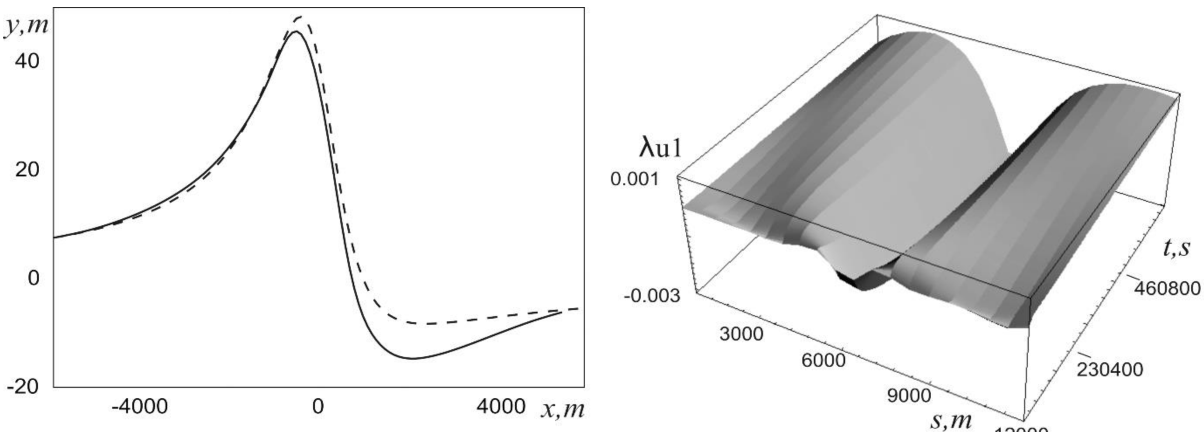
From (12) it follows that the physical meaning of $\lambda u_1 R_0$ is warping of pipe cross sections. Warping of the cross-sections of a cylindrical tube were observed in the experiments of V.S. Vlasov [4].

The coordinates of the centerline at the beginning and end of the calculation are shown in Figure 1 (a). This illustrates the consistency of the numerical solution with the mechanics laws: the profile displacement is directed towards distributed load from the fluid flow.

Figure 1 (b) shows the graph of λu_1, repeating the cross-sections warping. Warping has a maximum of about $0.003R_0$. In another numerical experiment for a cubic parabola profile, the warping reached $0.02R_0$.

4.2. The pipeline with singularity

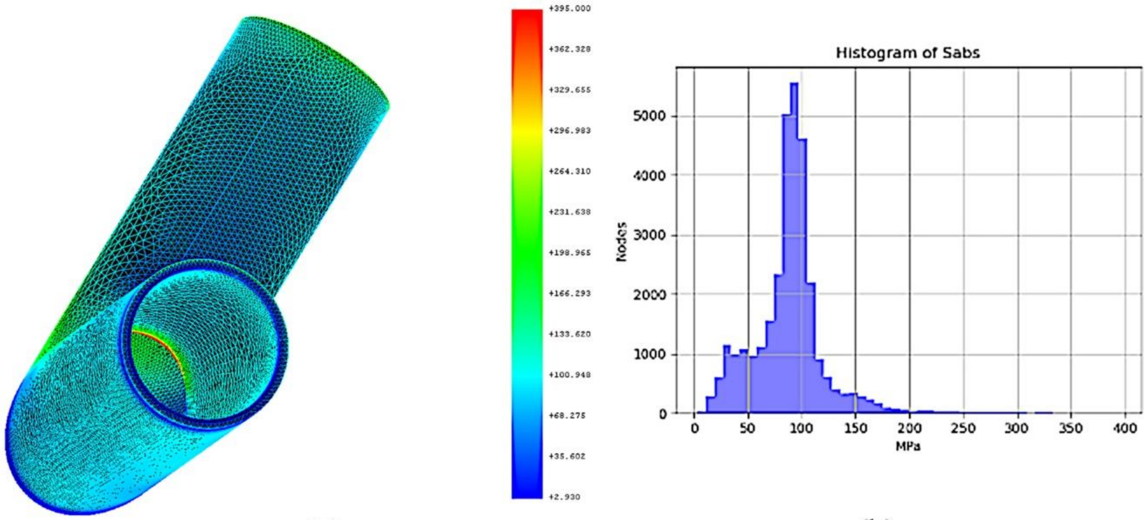
Physical and geometric parameters of the test problem: $h = 5$ mm, $R_0 = 47.5$ mm, $L_1 = 250$ mm, $L_2 = 250$ mm. Material: S335JO steel, $\rho_t = 7800\,\mathrm{kg/m^3}$, $E = 210$ GPa, $\nu = 0.3$, $\sigma_t = 343$ MPa. Rigid fixing conditions are imposed on the outer ends. Pressure $p = 10$ MPa is applied to the inner pipes surface.



(a) (b)

Figure 1: (a) – profile coordinates at the beginning (dashed line) and at the end of calculation; (b) – longitudinal displacement in the first approximation.

Mesh parameters are set to high precision. As a result, the stress distribution was found, see Figure 2. Figure 2 (a) shows the von Mises stresses. Figure 2 (b) shows a histogram of stress distribution by the number of mesh nodes. Limiting stress values: maximum stress 395.5 MPa (reached on the inner side in the reentrant corner of the domain), average stress 89.5 MPa, minimum stress 2.9 MPa.



(a) (b)

Figure 2: Stress field in the pipe: (a) – graphic isolines on the grid, MPa; (b) – histogram.

5. Conclusion

A mathematical model of the pipeline as a moment shell of irregular geometry, with singularities caused by the domain geometry, has been built. The parameters that differentiate the various stress states of the pipeline are determined.

A constructive algorithm for the asymptotic solution of the original three-dimensional boundary value problem for a pipeline is created, and on the basis of the obtained asymptotics, reduced mathematical models are constructed. The parameters of the problem are found for which various models are applicable. A numerical criterion has been determined that makes it possible to distinguish between two limiting states of the pipeline geometry.

Numerical experiments were performed. In the case (2), the results consistency of numerical experiments on the proposed mathematical model with the mechanics laws is shown. The existence of the pipe cross-sections warping is proved.

In the case (3), the pipeline modeling algorithm in the FreeCAD package is created. The stress fields are found by the finite element method. The presence of a singularity in the stress field is proved.

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7. References

[1] I. Towhata, Geotechnical Earthquake Engineering, Springer-Verlag, Berlin, Heidelberg, 2008.

[2] A. L. Gol'denveizer, Theory of Elastic Thin Shells, Published for the American Society of Mechanical Engineers by Pergamon Press, 1961.

[3] V. A. Rukavishnikov, O. P. Tkachenko, Dynamics of a fluid-filled curvilinear pipeline, Applied Mathematics and Mechanics 39 (2018) 905–922. doi:10.1007/s10483-018-2338-9.

[4] V. S. Vlasov, Basic Differential Equations in General Theory of Elastic Shells, Technical Report NACA-TM-1241, National Advisory Committee for Aeronautics, Washington, 1951. URL: https://ntrs.nasa.gov/search.jsp?R=20050028489.

[5] V. A. Rukavishnikov, O. P. Tkachenko, Numerical and asymptotic solution of the equations of propagation of hydroelastic vibrations in a curved pipe, Journal of Applied Mechanics and Technical Physics 41 (2000) 1102–1110. doi:10.1023/A:1026619009228.

[6] L. I. Sedov, Mechanics of Continuous Media, World Scientific Publishing Company, Singapore, 1997.

[7] V. A. Rukavishnikov, O. P. Tkachenko, Mathematical model of the pipeline with angular joint of elements, Mathematical Methods in the Applied Sciences 43 (2020) 7550–7568. doi:10.1002/mma.5751.

[8] L. G. Loitsyanskii, Mechanics of Liquids and Gases, Pergamon Press, Oxford-New York, 1966.

[9] V. A. Rukavishnikov, S. G. Nikolaev, On the R_ν -generalized solution of the Lamé system with corner singularity, Doklady Mathematics 92 (2015) 421–423. doi:10.1134/S1064562415040080.

[10] V. A. Rukavishnikov, On the uniqueness of the R_ν -generalized solution of boundary value problems with noncoordinated degeneration of the initial data, Doklady Mathematics 63 (2001) 68–70.

[11] V. A. Rukavishnikov, E. I. Rukavishnikova, The finite difference method for boundary value problem with singularity, Lecture Notes in Computer Science (including subseries Lecture Notes in Artificial Intelligence and Lecture Notes in Bioinformatics) 9045 (2015) 72–83. doi: 10.1007/978-3-319-20239-6_7.

[12] V. A. Rukavishnikov, A. O. Mosolapov, E. I. Rukavishnikova, Weighted finite element method for elasticity problem with a crack, Computers and Structures 243 (2021) 106400. doi: 10.1016/j.compstruc.2020.106400.

[13] B. Collette, FreeCAD for Inventors: Practical Examples and Clear Descriptions, Purple Squirrel Productions LLC, Gurley, USA, 2018.