Design and computer research of a nonlinear stochastic models describing the dynamics of interacting populations

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Abstract

The construction of nonlinear three-dimensional models of interconnected communities number dynamics is considered, taking into account competition in populations of victims. A qualitative research of the systems is carried out, equilibrium states are found, the species number dynamics graphs are constructed. For these models, an estimate of the model parameters is given and local phase portraits are constructed. The transition to the corresponding stochastic models is made. In stochastic cases, the method of constructing self-consistent stochastic models is used. A comparative analysis of deterministic and stochastic models is carried out. Effects typical for three-dimensional models with regard to competition in prey populations are revealed. A software package for the numerical solution of differential equations systems by modified Runge–Kutta methods is used as a software tool for researching the model. The software package allows performing numerical experiments based on the implementation of algorithms for generating trajectories of multidimensional Wiener processes and multipoint distributions and algorithms for solving stochastic differential equations. The formulation of the optimal control problem is proposed. Computer research of the models makes it possible to obtain the results of numerical experiments on the search for trajectories and the estimation of parameters. The results obtained can find application in problems of ecological systems computer modeling, as well as in problems of synthesis, optimal control and analysis of the multidimensional stochastic models stability describing the dynamics of interacting populations.

Keywords

computer modeling, nonlinear model of population dynamics, optimal control, stochastization of one-step processes, symbolic computation libraries, software package

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1. Introduction

An important tool in solving problems of predicting the state of natural systems and managing them is mathematical modeling. To solve these problems, both traditional and new methods and approaches are used. Ecological systems with various interconnections between subsystems and a change in the structure of these interconnections in the process of functioning leads to the mathematical models construction, the analytical research of which is very difficult.

Mathematical models of the interacting communities dynamics taking into account competition and with food chains are considered in [1, 2, 3, 4, 5, 6, 7]. In [2], the stability conditions of the «predator–two preys» system are investigated. In [3], a mathematical model of a system with two competing prey and one predator is analyzed, the influence of predation on the species coexistence is described. A model of two competitors' prey dynamics with the addition of a predator species to change the competition results is studied in [4]. In [5], the deterministic stability of the three-dimensional model «predator–two preys» limit cycles is investigated. In [6, 7], three-dimensional models of population dynamics with competition and with trophic chains are considered.

In the process of stochastic modeling for various dynamical systems a method for constructing self-consistent one-step models [8] is proposed and a software package [9] is developed. Some systems of population dynamics based on the construction of stochastic self-consistent models are considered in [10, 11, 12, 13, 14]. In [12, 13, 14, 15, 16], a number of control problems for the models of population dynamics are considered.

When modeling population systems, various software tools are used that provide ample opportunities for conducting computational experiments. In [17, 18], the models research is carried out using the Python language and symbolic computation libraries.

In this paper we considered several types of three-dimensional models, taking into account the competition among prey species and predation are studied. The stability of these models is investigated, the equilibrium states are calculated. The transition from deterministic models to stochastic ones is performed. A computer research is carried out to study the stability. The estimation of the model parameters is carried out, the phase portraits of the system are constructed, as well as the graphs of the population size dynamics in the deterministic and stochastic cases. The research is carried out using a software package for constructing stochastic dynamic models and searching for appropriate trajectories, as well as the Jupyter application package. The obtained effects are analyzed. The formulations of optimal control problems for the models with trophic chains are proposed.

2. The deterministic models description

We consider a model described by differential equations of the form

$$\begin{aligned} \dot{x}_1 &= x_1(a_1 - \varepsilon x_1 - \delta x_2 - b_1 y), \\ \dot{x}_2 &= x_2(a_2 - \delta x_1 - \varepsilon x_2 - b_2 y), \\ \dot{y} &= y(-c + d_1 x_1 + d_2 x_2 - \gamma y), \end{aligned} \tag{1}$$

where x_1 is the population density of the first competitor, x_2 is the population density of the second competitor, y is the population density of the predator, a_i is the reproduction rate of the competitor's population in the absence of a predator, b_i is the specific rate of consumption of the prey population by the predator population, di is the conversion factor of the prey biomass consumed by the predator in own biomass, $i = 1, 2$, ε is the coefficient of intraspecific competition, δ is the coefficient of interspecific competition, c is the natural mortality of the predator, γ is the intraspecific competition of the predator, $y(0) \geq 0$, $x_i(0) \geq 0$, $i = 1, 2$. According to the ecological sense, the constraints on the coefficients are: $\varepsilon > 0$, $\delta \geq 0$, $\gamma \geq 0$, $a_i > 0$, $b_i > 0$, $d_i > 0$, $c > 0$, $i = 1, 2$.

Model (1) is a modification of the model described in [1] with the following notation: $\varepsilon_{11} = \varepsilon_{22} = \varepsilon$, $\varepsilon_{12} = \varepsilon_{21} = \delta$.

We introduce the following notation: $\rho = b_1 d_2 + b_2 d_1$, $\phi = b_1 d_1 + b_2 d_2$, $D = \gamma(\varepsilon^2 - \delta^2) - \delta\rho + \varepsilon\phi$,

$$x_1^* = \frac{(a_2 b_1 - a_1 b_2) d_2 + (b_2 \delta - b_1 \varepsilon) c + (a_2 \delta - a_1 \varepsilon)\gamma}{\gamma(\delta^2 - \varepsilon^2) + \delta\rho - \varepsilon\phi},$$
$$x_2^* = \frac{(a_1 b_2 - a_2 b_1) d_1 + (b_1 \delta - b_2 \varepsilon) c + (a_1 \delta - a_2 \varepsilon)\gamma}{\gamma(\delta^2 - \varepsilon^2) + \delta\rho - \varepsilon\phi},$$
$$y^* = \frac{c(\varepsilon^2 - \delta^2) + \delta(a_1 d_2 + a_2 d_1) - \varepsilon(a_1 d_1 + a_2 d_2)}{\gamma(\delta^2 - \varepsilon^2) + \delta\rho - \varepsilon\phi}.$$

Equilibrium states of the (1) in general form are found. The indicated equilibrium states are as follows:

$$E_0(0,0,0), \quad E_1\left(0, 0, \frac{-c}{\gamma}\right), \quad E_2\left(0, \frac{a_2}{\varepsilon}, 0\right), \quad E_3\left(\frac{a_1}{\varepsilon}, 0, 0\right),$$
$$E_4\left(0, \frac{a_2\gamma + b_2 c}{\gamma\varepsilon + b_2 d_2}, \frac{a_2 d_2 - c\varepsilon}{\gamma\varepsilon + b_2 d_2}\right), \quad E_5\left(\frac{a_1\gamma + b_1 c}{\gamma\varepsilon + b_1 d_1}, 0, \frac{a_1 d_1 - c\varepsilon}{\gamma\varepsilon + b_1 d_1}\right),$$
$$E_6\left(\frac{a_2\delta - a_1\varepsilon}{\delta^2 - \varepsilon^2}, \frac{a_1\delta - a_2\varepsilon}{\delta^2 - \varepsilon^2}, 0\right), E_7\left(x_1^*, x_2^*, y^*\right).$$

The state of equilibrium E_7 is an internal state of equilibrium for which the condition of positivity is satisfied. Permanent coexistence of populations in the model (1) is established under the following conditions:

1) $\varepsilon > 0$, $\delta \geq 0$,$\gamma \geq 0$, $a_i > 0$, $b_i > 0$, $d_i > 0$, $c > 0$, $i = 1, 2$;
2) there is a unique internal equilibrium state E_7 such that $D \neq 0$ and $x_1^* > 0$, $x_2^* > 0$, $y^* > 0$;
3) $D > 0$;
4) at least one of the following inequalities holds $a_1\varepsilon > a_2\delta$ or $a_2\varepsilon > a_1\delta$.

Next, we consider the model:

$$\begin{aligned} \dot{x}_1 &= x_1(a - \varepsilon_{11}x_1 - \varepsilon_{12}x_2 - by), \\ \dot{x}_2 &= x_2(a - \varepsilon_{21}x_1 - \varepsilon_{22}x_2 - by), \\ \dot{y} &= y(c + dx_1 + dx_2 - \gamma y), \end{aligned} \tag{2}$$

where ε_{ij} for $i = j = 1$ and $i = j = 2$ are the coefficients of intraspecific competition, ε_{ij} for i not equal to j are the coefficients of interspecies competition. According to the ecological sense, the constraints on the coefficients are: $\varepsilon_{ij} \geq 0$, $i \neq j$, $\varepsilon_{ij} > 0$, $i = j$, $i, j = 1, 2$, $\gamma \geq 0$, $a > 0$, $b > 0$, $d > 0$, $c > 0$. The meaning of other parameters included in the system (2) is similar to the model (1).

Model (2) is a modification of the model described in [1] with the following values: $b_1 = b_2 = b$, $a_1 = a_2 = a$, $d_1 = d_2 = d$.

Next, we introduce the following notation:

$$\rho = (\varepsilon_{11} - \varepsilon_{12} - \varepsilon_{21} + \varepsilon_{22}), \quad \phi = (\varepsilon_{11}\varepsilon_{22} - \varepsilon_{12}\varepsilon_{21}), \quad D = bd\rho + \gamma\phi,$$

$$\hat{x}_1^* = \frac{(a\gamma + bc)(\varepsilon_{22} - \varepsilon_{12})}{D}, \quad \hat{x}_2^* = \frac{(a\gamma + bc)(\varepsilon_{11} - \varepsilon_{21})}{D}, \quad \hat{y}^* = \frac{ad\rho - c\phi}{D}.$$

Equilibrium states of the model (2) in general form are found. The indicated equilibrium states are as follows:

$$E_0(0,0,0), \quad E_1\left(0, 0, \frac{-c}{\gamma}\right), \quad E_2\left(0, \frac{a}{\varepsilon_{22}}, 0\right), \quad E_3\left(\frac{a}{\varepsilon_{11}}, 0, 0\right),$$

$$E_4\left(0, \frac{a\gamma + bc}{\gamma\varepsilon_{22} + bd}, \frac{ad - c\varepsilon}{\gamma\varepsilon_{22} + bd}\right), \quad E_5\left(\frac{a(\varepsilon_{22} - \varepsilon 12)}{\phi}, \frac{a(\varepsilon_{11} - \varepsilon 21)}{\phi}, 0\right),$$

$$E_6\left(\frac{a\gamma + bc}{\gamma\varepsilon_{11} + bd}, 0, \frac{ad - c\varepsilon_{11}}{\gamma\varepsilon_{11} + bd},\right), \quad E_7\left(\hat{x}_1^*, \hat{x}_2^*, \hat{y}^*\right).$$

The state of equilibrium E_7 is an internal state of equilibrium for which the condition of positivity is satisfied. Permanent coexistence of populations in the model (1) is established under the following conditions:

1) $\varepsilon_{12} \geq 0$, $\varepsilon_{21} \geq 0$, $\varepsilon_{11} > 0$, $\varepsilon_{22} > 0$, $\gamma \geq 0$, $a > 0$, $b > 0$, $d > 0$, $c > 0$;
2) there is a unique internal equilibrium state E_7 such that $D \neq 0$ and $\hat{x}_1^* > 0$, $\hat{x}_2^* > 0$, $\hat{y}^* > 0$;
3) $D > 0$;
4) at least one of the following inequalities holds $\varepsilon_{22} > \varepsilon_{12}$ or $\varepsilon_{11} > \varepsilon_{21}$.

In [1], a theoretical research of the generalized model «predator–two preys» stability is carried out. However, the numerical research of this model in order to identify the conditions of oscillating modes causes a number of difficulties. We carry out a computer research of the models (1) and (2), which are special cases of the model [1]. Subsequently, the transition to the stochastic case is made based on the method of self-consistent stochastic models.

3. Transition to stochastic models

We carry out the transition to stochastization of the models (1) and (2) using the method of constructing self-consistent stochastic models. This method is based on a combinatorial methodology.

We write down the scheme of interaction between elements for the «predator–two preys» system in general form

$$
\begin{gathered}
X_i \xrightarrow{a_i} 2X_i, \quad i = 1, 2, \\
X_i + X_j \xrightarrow{\varepsilon_{ij}} X_j, \quad i, j = 1, 2, \\
X_i + Y \xrightarrow{d_i} 2Y, \quad X_i + Y \xrightarrow{b_i} Y, \quad i = 1, 2, \\
Y + Y \xrightarrow{\gamma} Y, \\
Y \xrightarrow{c} 0.
\end{gathered} \tag{3}
$$

In the interaction scheme (3), the first line corresponds to the natural reproduction of prey in the absence of other factors, the second line symbolizes intraspecific (at $i = j$) interspecific (at $i \neq j$) competition, and the third describes the predator-prey relationship. The fourth line is responsible for intraspecific competition among predators, and the fifth describes their natural mortality.

The method of constructing self-consistent stochastic models assumes, in the course of mathematical transformations, a transition from the interaction scheme to obtaining the coefficients of the Fokker–Planck equation. This transition is carried out using the upgraded software package described in [9]. This software package is implemented in the Python programming language using the NumPy and SyPy libraries.

The software package consists of the following modules: `IS_to_SDE.py` and stochastic.py. The `IS_to_SDE.py` module is designed to obtain the coefficients of the Fokker–Planck equation from the interaction scheme. The stochastic.py module is a module for obtaining solutions for the stochastic model.

The algorithm of the software package is shown in Diagram 1.

The `IS_to_SDE.py` module takes as input the matrices M and N of the system states before and after interaction, vectors `K_plus` and `K_minus` interaction coefficients) and a vector X (the system state vector). As a result, we obtain a symbolic representation of the Fokker–Planck equation coefficients. Using the SymPy library allows you to get the code of these coefficients in TeX, which makes them easier to read.

The `IS_to_SDE.py` module consists of several functions. Hereafter there is a description of the main ones.

The `S_plus` function for obtaining the forward interaction coefficients has the following description:

```
def S_plus(X, K_plus, M):
""" interaction coefficient [s^{-}_{1}(x),...,s^{-}_{s}(x)]"""
        res = []
        for i in range(len(K_plus)):
                Prod_s = [Prod_(x, int(n)) for (x, n) in zip(X, M[i, :])]
                res.append(K_plus[i]* sp.prod(Prod_s))
        return res
```

The derivation function `drift_vector` for obtaining the drift vector A in the Fokker–Planck equation is described as follows:

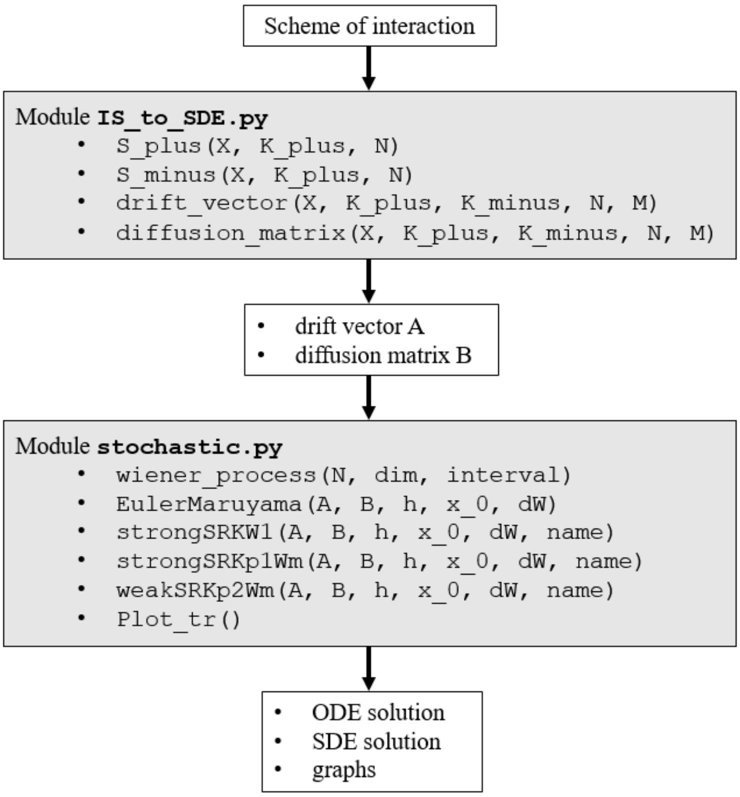

Figure 1: Algorithm of the software package.

```
def drift_vector(X, K_plus, K_minus, N, M):
""" drift vector"""
        res = sp.zeros(rows=len(X), cols=1)
        R = M.T - N.T
        for i in range(len(K_plus)):
                res += R[:, i] * S(X, K_plus, K_minus, N, M)[i]
        return res
```

In addition, we use the following description of the `diffusion_matrix` function to obtain the diffusion matrix B in the Fokker–Planck equation:

```
def diffusion_matrix(X, K_plus, K_minus, N, M):
""" diffusion matrix"""
        res = sp.zeros(rows=len(X), cols=len(X))
        R = M.T - N.T
        R = sp.Matrix(R)
        for i in range(len(K_plus)):
```

```
            res += R[:, i] * R[:, i].T * S(X, K_plus, K_minus, N, M)[i]
    return res
```

In fig. 2. the result of the functions for obtaining the Fokker–Planck equation coefficients for the model (1) is presented.

```
[101] A = drift_vector(X, K_plus, K_minus, N, M)
      B = diffusion_matrix(X, K_plus, K_minus, N, M)
```

```
[102] A
```

$$\begin{bmatrix} a_1x_1 - b_1x_1y - \delta x_1x_2 - \varepsilon x_1^2 \\ a_2x_2 - b_2x_2y - \delta x_1x_2 - \varepsilon x_2^2 \\ -cy + d_1x_1y + d_2x_2y - y^2\gamma \end{bmatrix}$$

```
[103] B
```

$$\begin{bmatrix} a_1x_1 + b_1x_1y + \delta x_1x_2 + \varepsilon x_1^2 & \delta x_1x_2 & 0 \\ \delta x_1x_2 & a_2x_2 + b_2x_2y + \delta x_1x_2 + \varepsilon x_2^2 & 0 \\ 0 & 0 & cy + d_1x_1y + d_2x_2y + y^2\gamma \end{bmatrix}$$

Figure 2: The Fokker-Planck equation coefficients for the model (1).

Figure 3 shows the derivation and the result of the functions for obtaining the Fokker–Planck equation coefficients for the model (2).

```
[143] A = drift_vector(X, K_plus, K_minus, N, M)
      B = diffusion_matrix(X, K_plus, K_minus, N, M)
```

```
[144] A
```

$$\begin{bmatrix} ax_1 - bx_1y - \varepsilon_{11}x_1^2 - \varepsilon_{12}x_1x_2 \\ ax_2 - bx_2y - \varepsilon_{21}x_1x_2 - \varepsilon_{22}x_2^2 \\ -cy + dx_1y + dx_2y - y^2\gamma \end{bmatrix}$$

```
[145] B
```

$$\begin{bmatrix} ax_1 + bx_1y + \varepsilon_{11}x_1^2 + \varepsilon_{12}x_1x_2 & 0 & 0 \\ 0 & ax_2 + bx_2y + \varepsilon_{21}x_1x_2 + \varepsilon_{22}x_2^2 & 0 \\ 0 & 0 & cy + dx_1y + dx_2y + y^2\gamma \end{bmatrix}$$

Figure 3: The Fokker-Planck equation coefficients for the model (2).

Further, the obtained coefficients are transferred to the `stochastic.py` module for the numerical solution of the generated stochastic differential equations and drawing the graphs of these solutions.

The `stochastic.py` module can be used to study and numerically solve systems of ordinary differential equations and their corresponding stochastic differential equations based on the Runge–Kutta method and its modifications. A detailed description of this module is performed in [8, 9].

4. Results of computer experiments

For the models (1) and (2), we carry out a series of computational experiments using the above-described software package designed to study and numerically solve systems of ordinary differential equations and the corresponding stochastic differential equations. Computational experiments are carried out for both the deterministic case and the stochastic case.

For the model (1), calculations are carried out at $\varepsilon = \delta$. We consider the following sets of parameters: $(x_1, x_2, y) = (2, 1.6, 1.2)$, $(a_1, a_2, c, \varepsilon, b_1, b_2, d_1, d_2, \gamma) = (0.2, 0.4, 0.8, 0, 0.2, 0.4, 0.3, 0.6, 1)$. With this set of parameters, the approximate equilibrium states are obtained.

Further, for the model (1), we consider the following sets of parameters: $(x_1, x_2, y) = (0.5, 0.4, 0.3)$, $(a_1, a_2, c, \varepsilon, b_1, b_2, d_1, d_2, \gamma) = (0.2, 0.4, 0.8, 0.2, 0.2, 0.4, 1.3, 2.4, 0.1)$. With this set of parameters, we obtained the approximate equilibrium states.

Figures 4 and 5 show the dynamics of population density for given sets of initial conditions. The dashed line indicates the fluctuations of species number of the deterministic model (1), the solid line indicates the stochastic one. Taking into account the results presented in Fig. 4, we note that the trajectories have an oscillating character with conservation of amplitudes. For the corresponding stochastic model, damping of oscillations takes place with an approximation to the stationary mode. Taking into account the results presented in Fig. 5, we note the proximity of the trajectories of the deterministic and stochastic models. In the case under consideration, stochastization does not affect the of the system behavior, which is characterized by damping of oscillations.

For the model (2), the following sets of parameters are considered: $(x_1, x_2, y) = (2, 1, 6)$, $(a, c, \varepsilon_{11}, \varepsilon_{12}, \varepsilon_{21}, \varepsilon_{22}, b, d, \gamma) = (8, 2.5/3, 8, 4, 4.125, 1, 1, 1, 0)$. With this set of parameters, the approximate equilibrium states are obtained.

Fig. 6 shows the population density dynamics for two sets of initial conditions indicated above. For stochastic and deterministic cases, the trajectories are located close to each other. As in the deterministic case, the mean values graphs of various realizations of the stochastic differential equation solutions reach a stationary mode.

Computational experiments show that the character of the systems (1) and (2) stability is significantly influenced by the coefficients of intraspecific and interspecific competition. Oscillations are formed if $\varepsilon = \delta = 0$ and if $a_1 = ka_2$,$d_1 = ld_2$ at $l = k$. At $\varepsilon = \delta \neq 0$, the oscillations have a damping character. At $a_1 = ka_2$, $d_1 = ld_2$ at $l \neq k$ one of the prey populations dies out. Next, we proceed to the consideration of controlled models and formulate the optimal control problem.

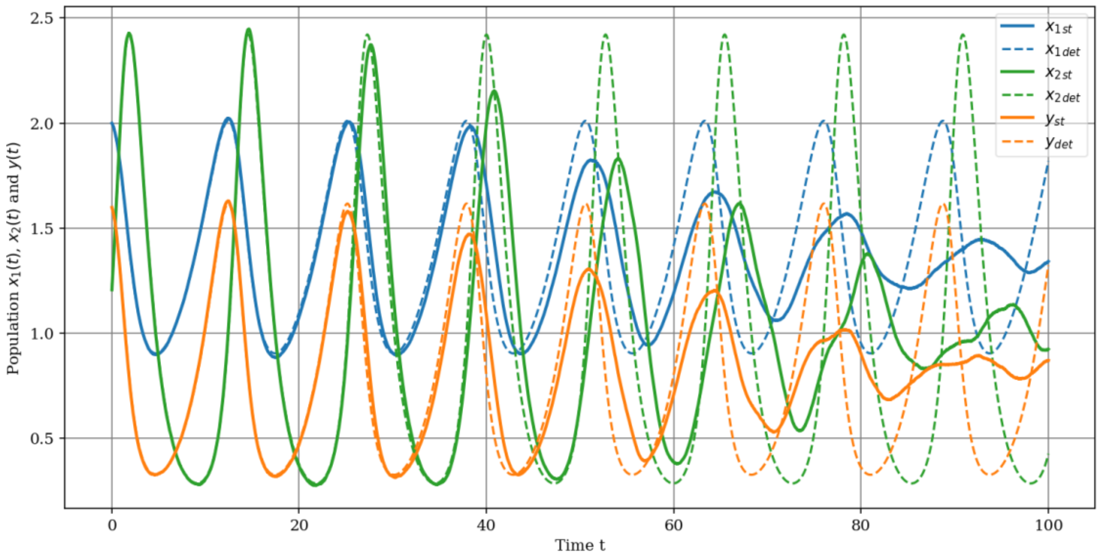

Figure 4: The populations dynamics x_1, x_2, y of the model (1) under initial conditions: $(x_1, x_2, y) = (2, 1.6, 1.2)$, $(a_1, a_2, c, \varepsilon, b_1, b_2, d_1, d_2, \gamma) = (0.2, 0.4, 0.8, 0, 0.2, 0.4, 0.3, 0.6, 1)$.

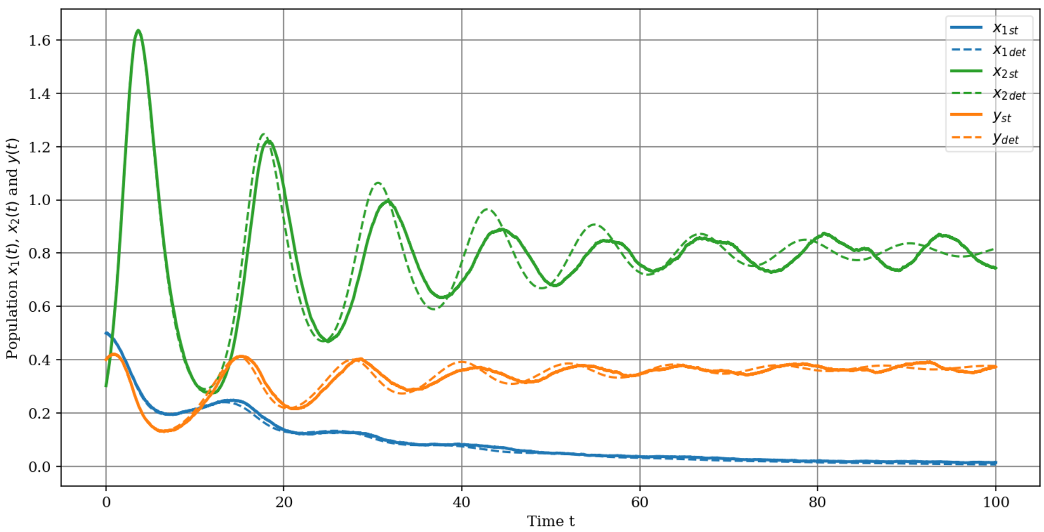

Figure 5: The populations dynamics x_1, x_2, y of the model (1) under initial conditions: $(x_1, x_2, y) = (0.5, 0.4, 0.3)$, $(a_1, a_2, c, \varepsilon, b_1, b_2, d_1, d_2, \gamma) = (0.2, 0.4, 0.8, 0.2, 0.2, 0.4, 1.3, 2.4, 0.1)$.

5. The problem of optimal control

Let us formulate an optimal control problem for a three-dimensional model of the interconnected communities number dynamics, taking into account competition in populations of preys. For a three-dimensional model (1), the controlled model is given by a system of differential equations

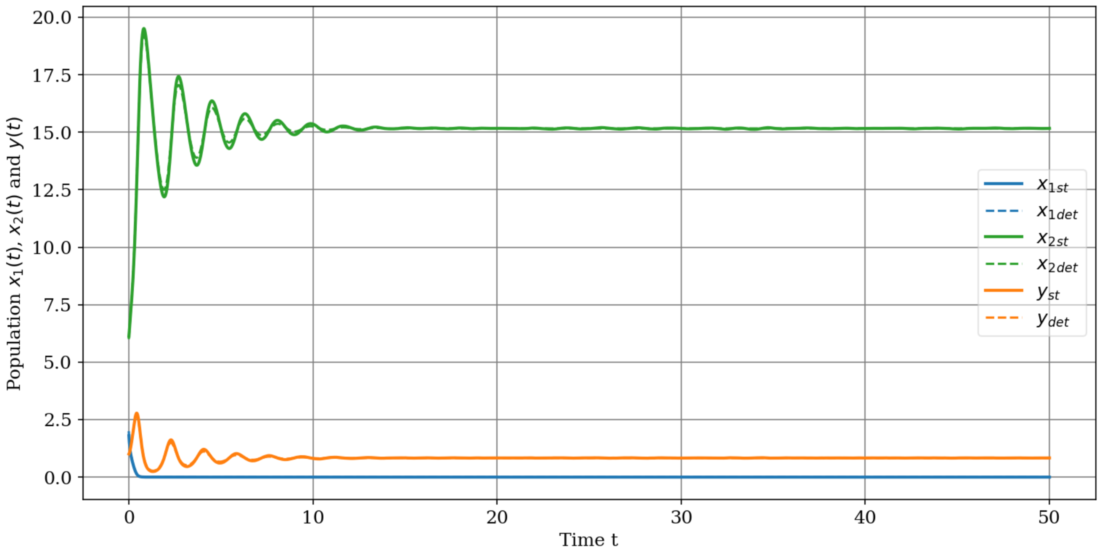

Figure 6: The populations dynamics x_1, x_2, y of the model (2) under initial conditions: $(x_1, x_2, y) = (2, 1, 6)$, $(a, c, \varepsilon_{11}, \varepsilon_{12}, \varepsilon_{21}, \varepsilon_{22}, b, d, \gamma) = (8, 2.5/3, 8, 4, 4.125, 1, 1, 1, 0)$.

$$
\begin{aligned}
\dot{x}_1 &= x_1(a_1 - \varepsilon x_1 - \varepsilon x_2 - b_1 y) - u_1 x_1,\\
\dot{x}_2 &= x_2(a_2 - \varepsilon x_1 - \varepsilon x_2 - b_2 y) - u_2 x_2,\\
\dot{y} &= y(-c + d_1 x_1 + d_2 x_2 - \gamma y) - u_3 y,
\end{aligned}
\tag{4}
$$

where $u_i = u_i(t)$ are control functions. The input parameters is explained in section 2.

Constraints for model (4) are specified in the form

$$
x_1(0) = x_{10}, x_2(0) = x_{20}, x_3(0) = x_{30}, x_1(T) = x_{11}, x_2(T) = x_{21}, x_3(T) = x_{31}, t \in [0, T], \tag{5}
$$

$$
0 \le u_1 \le u_{11}, 0 \le u_2 \le u_{21}, 0 \le u_3 \le u_{31}, t \in [0, T]. \tag{6}
$$

With regard to problem (4)–(6), we consider the functional to be minimized in the form

$$
J(u) = \int_0^T \sum_{i=1}^{3} k_i u_i(t) dt. \tag{7}
$$

Control quality criterion (4) corresponds to minimizing losses from regulation of the population, and in this case, the positive coefficients are denoted by k_i in (7).

For the model (4), the optimal control problem can be formulated as follows: find the minimum of functional (7) under conditions (5), (6) taking into account $x_i \ge 0$.

We also generalize model (2) for the controlled case and formulate the corresponding optimal control problem. At the same time, we consider the criterion of control quality, which also consists in minimizing losses from regulation of the population.

It is possible to construct the control laws u_1, u_2, u_3 by different methods. For example, it is possible to use PID controllers [19] or controllers using sliding mode [20].

For a population model with competition and migration flows in [13], the authors use a polynomial control of the form

$$u_i(t) = \|RT\|, \quad R = (r_{i1}, r_{i2}, ..., r_{in})^T, \quad T = (t^0, t^1, ..., t^m), \quad i = 1, 2, 3.$$

In this case, the model parameters are the coefficients $r_{i1}, r_{i2}, ..., r_{in}$ of polynomial functions. Methods of global parametric optimization [21, 22] are usually used to calculate the parameters.

We propose to use control methods based on machine learning and regulators using artificial intelligence. In particular, it is possible to use machine learning in combination with controllers based on fuzzy logic or artificial neural networks [23, 24]. The generalized algorithm for constructing the optimal trajectory of the model (4) based on machine learning is shown in Fig. 7.

Thus, the optimal control problem is to find $u_i = u_i(t)$ those that satisfy, firstly, the phase constraints of problem (5), and secondly, the optimality criterion (7). To solve the problem, it is necessary:

a) to construct the loss function,
b) to build a parametric control model,
c) to use the global parametric optimization algorithm for search the coefficients of the control model with the minimum loss function.

We propose the construction of stochastic controlled models taking into account the «predator–two preys» interaction. To study such models, it is advisable to consider the control laws u_1, u_2, u_3 using the algorithm for constructing the optimal trajectory of the model (4). For population models with model migration, a number of computer experiments are carried out in [12, 14]. In [14], the «predator–prey» model is studied taking into account migration flows. In [12], multidimensional models of competing populations with migration, without trophic chains, are studied. In [13, 14], the indicated approach to the construction of stochastic controlled models is effective. A similar approach can be used for the models (1) and (2).

6. Conclusion

Computer research of two competing individuals interaction of prey with a predator population nonlinear models makes it possible to study the proposed models stability. The obtained results of the models research at different variables sets and initial conditions make it possible to estimate the influence of the predator species on the result of the prey competition. It is established that the presence of trophic chains has a positive character on the result of competition, and this, in turn, contributes to the coexistence of species. By the aid of developed software package, graphs of population dynamics are constructed. For the systems (1) and (2), the transition to the corresponding stochastic differential equations is carried out. The introduction of stochastics makes it possible to take into account the probabilistic nature of the reproduction processes and species death, as well as random fluctuations occurring in the environment in time and leading to random fluctuations of the model parameters. We formulated an optimal control problem,

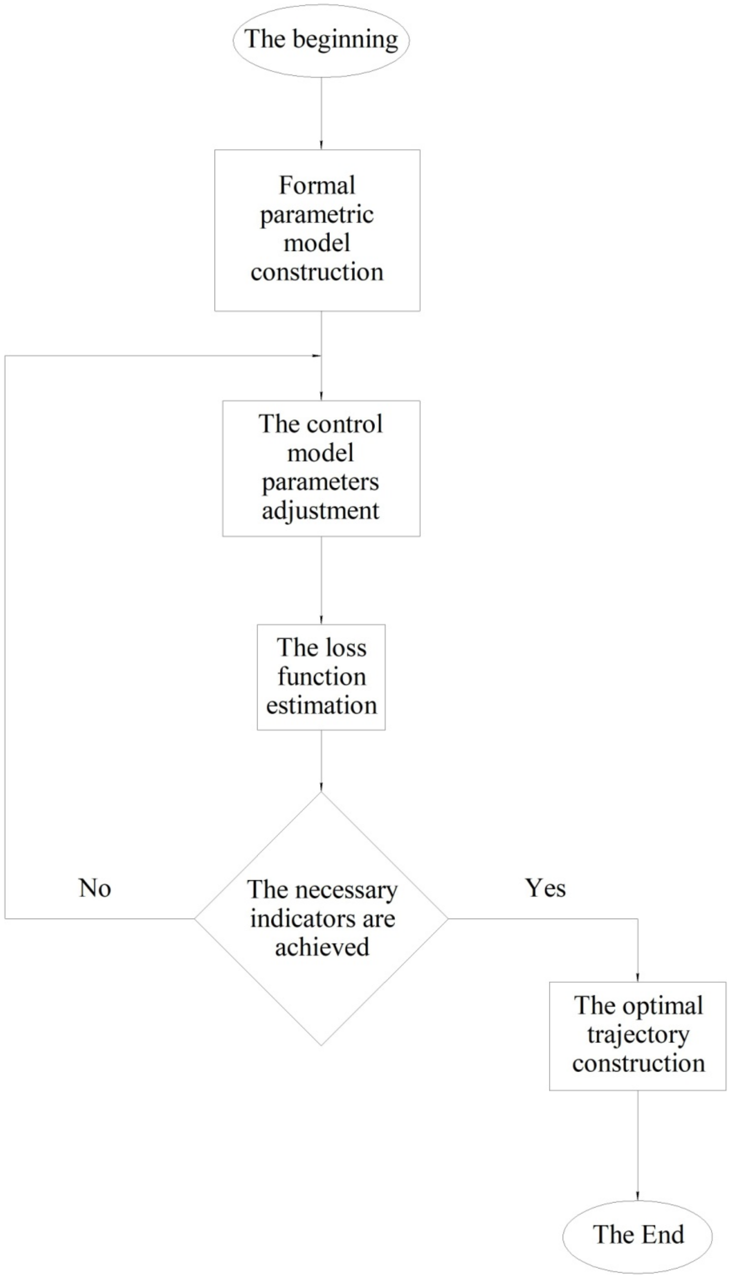

Figure 7: Generalized algorithm for solving the optimal control problem.

proposed a criterion for the quality of control and developed the corresponding generalized algorithm. To solve the problem of optimal control, it is proposed to use numerical optimization methods and intelligent algorithms for symbolic computations. The obtained results can find application in the research of the proposed models, taking into account the requirements of control and optimization.

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