

# Interactive Granular Computing Connecting Abstract and Physical Worlds: An Example

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## Abstract

This short paper is an attempt to clarify the role of Interactive Granular Computing (IGrC) as a computation model which respects that a real cognition about a real physical complex phenomenon and making decisions based on that cannot be formalized only being in the language of mathematics. In this regard, the paper focuses on presenting a real life example of computation where in order to move forward, without stumbling over the obstacles, a blind person needs to explore and learn the surrounding environment through interactions with the environment. The paper simply describes different components and features of IGrC model in the light of the concerned example and explains how this computing model has the potential to handle the grounding problem by bridging a connection between the abstract mathematical modeling and the real physical semantics.

## Keywords

interactions, granular computing, perception, knowledge specification, implementational language, complex granule, informational granule, grounding problem, dynamic transition relation

## 1. Introduction

In a few of our previous papers [8, 9, 19] we already put forward our arguments in favour of a need to develop a model for computing and reasoning which is not purely mathematical and isolated from its real physical semantics, and which has the possibility to learn from the real physical environment through real physical interactions. That such an endeavour is necessary for building an intelligent system, dealing with complex phenomenon, is supported by several opinions of different researchers from different fields of research [1, 2, 3, 4, 6, 7, 10, 11, 12, 13, 16, 18]. Without repeating many such inspiring thoughts of different researchers let us start with citing one from [18].

*[. . .] the often implicit stand one takes with regard to the question of the bridge between physical and symbolic descriptions determines in a fundamental way how one views the problems of cognition. A primary question here is, Exactly what kind of function is carried out by that part of the organism which is variously called the*

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*sensor, the transducer, or in the case of Gibson, the “information pickup”? One’s answer to this question determines the shape of any theory of perception and cognition one subsequently constructs. As I shall argue, unlike the symbolic processes that characterize cognition, the function performed by the transducer cannot be described as a computation, that is, by a purely symbol-manipulation function. Like all primitive operations of the functional architecture, the transducer fundamentally is a physical process; that is, its behavior is explainable (its regularities can be captured) in terms of its intrinsic properties - physical, chemical, biological, and so on. Thus, typically it does not come under study by cognitive psychologists who simply presuppose that an organism is equipped with certain transducer functions. The task of discovering these mechanisms is left for others, for example, “sensory encoding neurophysiologists, biophysicists, and engineers.*

Let us also cite the opinion concerning the need of a new computing model in Cyber-Physical Systems (CPS) [5].

*The inherent cross-disciplinary nature of CPS requires distinct modelling techniques to be employed, thus prompting for a common background formalism that enables communication between all specialities. However, to this date, no such single super-formalism exists to support the multiple dimensions of the design of a CPS. Indeed, to effectively design a CPS, engineers (in the role of modellers) either need to be versed in multiple formalisms, or a **fundamentally new modelling approach has to emerge.***

According to the view of Edmund Husserl, the founder of phenomenology [7], non-standard models of computation such as natural computing, reaction systems, Harel’s algorithmics and Gurevich’s abstract state machines, or neural network computing are closed in the abstract space, in the mathematical manifold.

*Husserl was frustrated by the idea that science and mathematics were increasingly conducted on an abstract plane [treating nature itself as a “mathematical manifold”] that was disconnected from human experience and human understanding, independently of questions of truth and applicability. He felt that the sciences increasingly dealt with idealized entities and internal abstractions a world apart from the concrete phenomena of daily life.*

Till now, in different works (see, e.g., [8, 9, 14, 19, 20]), we tried to introduce what do we mean by *Interactive Granular Computing* (IGrC) and how it is different from other existing theories from the perspective of modeling computations in a complex system. In a very brief description, *Interactive* symbolizes interaction between the abstract world and the real physical world, and *Granular Computing* symbolizes computation over imperfect, partial, granulated information abstracted about the real physical world. Here, once again we take the opportunity to explain briefly the notion of *complex granule* (c-granule), the basic building block of IGrC.

A c-granule is composed of three parts, known as *soft\_suit*, *link\_suit* and *hard\_suit*. These three parts correspond to three sets of physical objects, together which determines the scope of

that particular c-granule<sup>1</sup>. The *soft\_suit* represents the objects from the physical reality which are directly accessible or about which already some information is obtained. The *hard\_suit* corresponds to those objects which are in the scope of the c-granule but not yet accessed or are not in the direct reach at that point of time of the c-granule. The *link\_suit* represents a chain of objects that creates a communication channel between the *soft\_suit* and the *hard\_suit*. A c-granule, when associated with an information layer with it, is known as informational c-granule, in short ic-granule. The information layer of an ic-granule contains different forms of information, such as specification of the already perceived properties of the objects from its *soft\_suit*, specifications of the windows describing where and how some specific part from the scope of the ic-granule can be accessed or reached etc. Based on the purposes and types of the information specifications of an ic-granule there can be different types of ic-granules, such as ic-granule representing perception of the objects from the scope (perception based ic-granule), ic-granule representing domain knowledge (knowledge based ic-granule), ic-granule representing plan of actions (planner ic-granule), ic-granule representing plan into an implementational level language (implementational ic-granule) etc. Here to be emphasized that the ic-granule, responsible for implementation of a plan of actions, serves the task of connecting between the abstract and the real worlds.

A computation process over a c-granule is described by a network of ic-granules lying within the scope of the concerned c-granule. The informational layer of all these ic-granules constitutes the domain knowledge of the *control* of the c-granule, may be also called control of the computation process; this informational content is endowed with a reasoning mechanism. The information content and the reasoning mechanism together designs the control mechanism of a computation process. More specifically, the whole information layer is clustered based on the information relevant to different sub-scopes of the whole scope of the c-granule. For example, in the ic-granules representing the domain knowledge, there can be different sub-clusters in the informational layer corresponding to different aspects of the domain knowledge. That is, an ic-granule may contain several other ic-granules inside its scope. On the other hand the reasoning mechanism of the control of a c-granule is responsible for aggregating, deleting, or generating information from the existing clustered of information layers. Thus new information layers are generated over time based on (i) initial (partial) perception and domain knowledge of the concerned fragment of the environment (ii) initiation of interactions through already accessible objects to access the information about the not directly reachable objects (iii) perception of the physical world after interactions and (iv) verification of the perceived properties of the newly obtained configuration with the expected specifications of the target environment.

Having this much of preliminary relevant details about a computation process over a c-granule, in this paper our target is to present a real life example through which different aspects of computation over a c-granule can be visualised.

In this regard, Section 2 presents an example of computing along with an explanation of how such a real life computation can be modeled in the framework of IGrC. Section 3 presents the concluding remarks explaining how different components and features of IGrC incorporate a

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<sup>1</sup>The scope of a c-granule at a given moment  $t$  of the local time of the c-granule is the part of the physical space corresponding to the formal specifications of all spatio-temporal windows active at  $t$ .

possibility of building a mathematical model which is not a simplified static image of its real physical semantics; rather it is grounded in the real physical world.

## 2. Example of a computation over c-granule

The example described in [15] goes well with the idea of how a computation process, based on perception and interactions, should look like according to IGrC model.

*perceiving is a way of acting. [...] Think of a blind person tap-tapping his or her way around a cluttered space, perceiving that space by touch, not all at once, but through time, by skillful probing and movement. This is or ought to be, our paradigm of what perceiving is.*

Let us consider the above cited example as an example of a computation over a c-granule, where the c-granule has in its scope a blind person<sup>2</sup> and its surrounding. More precisely, the person and the top part of the stick are directly accessible part of this environment, and hence belongs to the soft\_suit. The part of the stick, which is distant from the direct touch of the person, belongs to the link\_suit as a partial information about the end of the stick can be derived based on the part belonging to the soft\_suit, and it creates a link to the not directly accessible objects, such as holes or stones lying in the surrounding environment, that is to the hard\_suit. The goal of the computation is to have a successful forward movement of the blind person by deriving information about the unseen objects based on the already available knowledge and the perceived information about the directly accessible objects. The whole computation process, leading to the goal, is conducted by the control of the mentioned c-granule. The behaviour of the control is based on transformations of collections (finite families) of the actual ic-granules, or more exactly the actual networks of ic-granules, into the new ones.

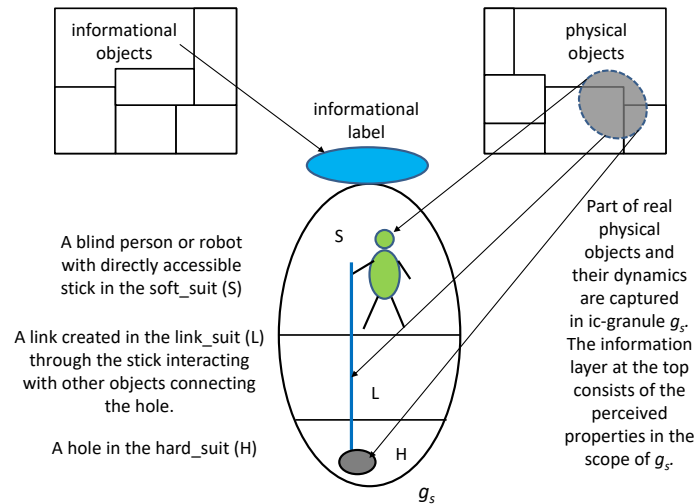
All the information layers corresponding to different of ic-granules involved in a computation is clustered in the control of the computation; using this information the reasoning mechanism of the control makes the computation process to happen and dynamically move from one layer to another layer. Below a step-by-step process of the computation is described in the context of this example.

### Layer:0

1. We assume  $g_s$  to be an ic-granule having in its scope a blind person or robot with a stick and the objects lying in the surrounding. At  $t_0$ , the beginning of the control's cycle,  $g_s$  is labelled with the perceived information of the directly accessible objects from its soft\_suit. Here the directly accessible objects can be the blind person himself and the part of the stick directly in contact with the person (see Figure 1). To be remembered that here  $g_s$  represents a perception based ic-granule. The informational layer of  $g_s$  contains, in particular information concerning perception of the current perceived situation; in more complex situations it may also contain link to the domain knowledge related to general perception of the environment, formal specifications of the transformations of

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<sup>2</sup>One can even consider a robot instead of a human being.

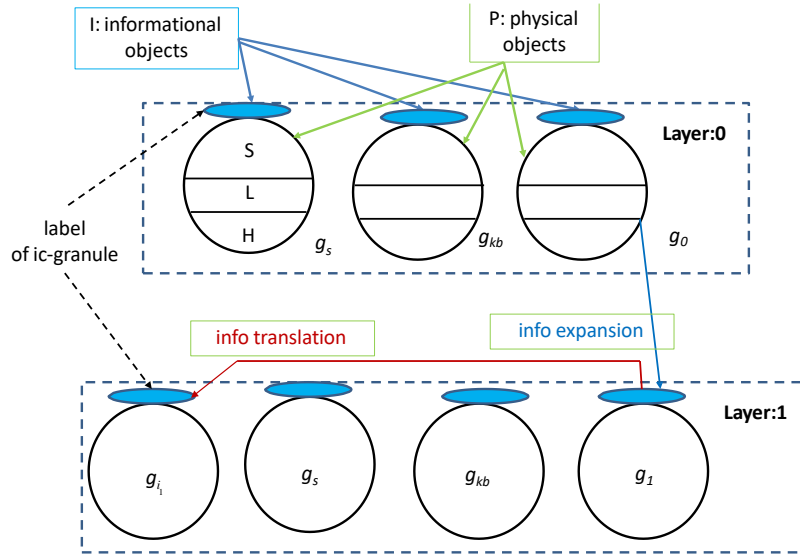


**Figure 1:** ic-granule operating on a particular scope of the physical world.

the ic-granules within its scope, database with rules for selection of transformation of ic-granules for realisation, etc. In our illustrative example we discuss only a very simplified version of  $g_s$ .

2. Let the description of the general goal of the computation be attached as the information layer of a planner ic-granule  $g_0$ . So, here  $g_0$  corresponds to those particular cells of a human brain where the goal description is set. So, for  $g_0$  the soft\_suit, link\_suit and the hard\_suit can be like different layers of those brain cells where the reachability to more deeper layer in the hard\_suit happens through the directly reachable layer in the soft\_suit and reactions of the brain cells propagating from the soft\_suit to the hard\_suit.
3. The role of the knowledge base is represented by another ic-granule  $g_{kb}$ . Here,  $g_{kb}$  can be considered as the brain parts related to the memory locations. The information layer of  $g_{kb}$  is labelled with the addresses of different relevant properties of different fragments of the c-granule. The soft\_suit of  $g_{kb}$  consists of the objects which form the outer box of the memory location whose address is attached to the information layer; in order to access the detailed information about some fragments some more inner boxes, lying in the hard\_suit, are to be opened. Such ic-granules representing knowledge base may be called information granules. Their physical parts create local memories for storing and transmitting information. One may also look on  $g_{kb}$  as on a compound ic-granule representing a network of ic-granules determining the structure of  $g_{kb}$ .
4. Now based on the information gathered from the informational layers of  $g_s$ ,  $g_0$  and  $g_{kb}$ , the control with its reasoning mechanism aims to better understand the perceived situation necessary for decomposition; this leads to construction of a more detailed plan of actions. This detailed plan is represented in informational layer of the ic-granule  $g_1$  at the next time point  $t_1$ . For a visual representation the readers are referred to Figure 2.

In our simplified illustrative example, we mention only one mechanism for enriching the



**Figure 2:** Computation over ic-granules passing from layer-0 to layer-1.

(i)  $g_s$ : Current perception of some objects of P at time  $t_0$ , indicating the scope. S contains directly perceivable objects, L contains objects creating communication channel to the objects in H where some actions are to be performed.

(ii)  $g_{kb}$ : Relevant information about general laws related to objects in  $g_s$  and specifications of where this information is stored. Here S contains directly accessible part of the storage memory and L contains the objects linking to the directory at H.

(iii)  $g_0$ : Relevant information regarding a goal that to be implemented in the H part of  $g_s$ . This specification of goal is stored in some object in the H of  $g_0$ , and is accessible by some object lying in the S of  $g_0$ .

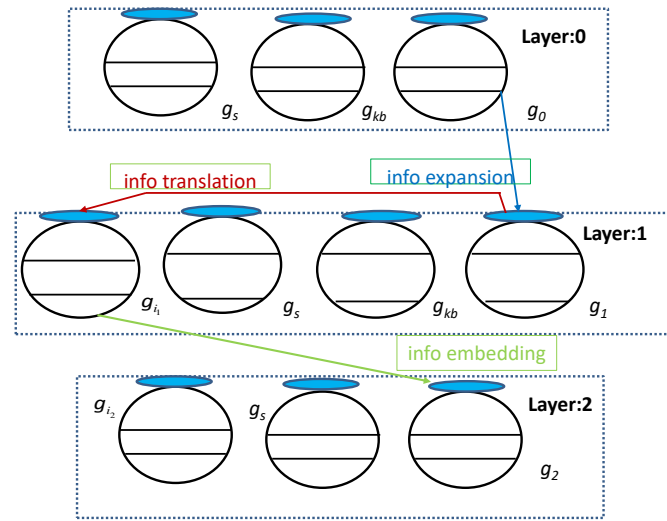
(iv) info expansion denotes decomposition of the plan available at  $g_0$  to a more detailed plan.

informational layer of  $g_s$ . In a more realistic example, one should consider other mechanisms, e.g., measuring different parameters by sensors (e.g., stick in our example), recording the perceived results of measurements in the corresponding informational layers of ic-granules etc. In a more general case, a sequence of steps of reasoning is realised, which are based on transformations of ic-granules, leading to better understanding of the currently perceived situation. It should be also noted that in general decomposition problems are challenging and they are related, in particular to the idea of information granulations and computing with words [23, 24, 25, 26].

### Layer:1

1. From the perspective of the example, at  $t_0$  the information attached to  $g_0$  encodes the general goal of the blind person that primarily gets registered in his brain, i.e., in the soft\_suit of  $g_0$ . At time  $t_0$  the hard\_suit of  $g_0$ , such as more deep analytical brain cells, remains still unaccessed. Based on the collected information of  $g_s$  and  $g_{kb}$ , at time  $t_1$  the person ponders more analytically; this in a sense activates interaction with the previously unaccessed part of  $g_0$ . This gradually gives access to the hard\_suit of  $g_0$ , and thus at time

- $t_1$  the hard\_suit of  $g_0$  becomes the soft\_suit of  $g_1$ , labelled with a more detailed plan for the person.
- Now in order to implement the abstract description of the plan available at  $g_1$  through real physical actions, the plan needs to be transformed from the abstract level to an implementational level language. From the perspective of our example, this can be a translation of the plan from the person's analytical brain cells to a language of actuators, like hands, legs, and the stick of the person. So, a new ic-granule is manifested at this layer. We call it as  $g_{i_1}$ , an implementational ic-granule. To be noted that  $g_{i_1}$  does not concern about the actual actuators; rather it is like another hard-drive in the brain of the person where the action plan can be stored in the language of actuators. The information layer of  $g_{i_1}$  also contains the specification of the conditions for initiating the implementation plan through a real actuator. Figure 3 presents the computation process described in layer-1.



**Figure 3:** Computation over ic-granules passing from layer-1 to layer-2.

- $g_1$ : at time  $t_1$  specification of the plan of  $g_0$  is expanded. Detailed specification is generated based on  $g_s$ ,  $g_{kb}$  and  $g_0$  of Layer:0. In particular the H part of  $g_0$  can be now the S of  $g_1$  which is gradually reached through the L part of  $g_0$ .
- $g_{i_1}$ : Specification of how the abstract plan of  $g_1$  can be implemented in  $g_s$ , that is the specification of the plan in a lower level language which can be implemented via physical objects. This lower level language is also a built-in language of a hardware lying at the H part of  $g_{i_1}$ .
- info translation denotes translation of the plan from the level of abstract description to the level of implementational language.
- $g_2$ : Specification in lower level language is embedded to a physical object lying in the S part of  $g_2$ , which prepares the ground to run the plan via a physical object in H part of  $g_2$ .
- info embedding represents embedding the plan of actions on a real physical object.

## Layer:2

- The specification of the plan of implementation of  $g_{i_1}$  is now realized through a physical object at time  $t_2$ . Let this object belong to the scope of the ic-granule  $g_2$ . In case of the



example, it can be the stick of the blind person on which the abstract implementation plan is embedded, and  $g_2$  represents the ic-granule containing the stick in its scope. The physical interaction of the stick with other objects in  $g_2$  is encoded in the information layer of  $g_2$ . If this information matches to a significant level to the condition for initiating implementation plan stored at  $g_{i_1}$  then an action compilation signal is passed to the next implementation granule, may be named as  $g_{i_2}$ .

2. With the action compilation specification of  $g_{i_2}$  the objects lying in its link\_suit and hard\_suit propagate actions to realize a desired configuration in the hard\_suit of  $g_s$ . In the context of our example,  $g_{i_2}$  represents the ic-granule which specifies how to move the stick forward until it touches a stone on its way. This chain of objects between the stick and a stone creates a communication channel.
3. Through this communication channel the computation process enters into the hard\_suit of  $g_s$ , which was inaccessible at time  $t_0$ . The initiation of the action compilation via  $g_{i_2}$  creates a link to the hard\_suit of  $g_s$ . This new interaction gives access to the hard\_suit of  $g_s$  which was previously inaccessible.
4. A new cycle starts by perceiving properties of the newly accessible part of  $g_s$ .

Here to be noted, that in the example we only have mentioned about the decomposition of the plan of actions from the initial stage  $g_0$  to a stage  $g_1$ , from where it gets translated to the implementational level language. But in practice decomposition of the action plan, say  $\alpha : g_0 \Rightarrow \beta : g_1$  available in the information layer of  $g_0$  specifying the target ic-granule  $g_1$  with property  $\beta$  can have several layers of decomposition in between. For a visual representation the readers are referred to the Figure 4, which will be discussed in Section 3. One should also consider that some actions are lunched in the process of perception while the other ones are related to the main decisions.

The above described idea of computation over a c-granule is in the line of the idea that Luc Steels has characterised in [21]; complex dynamical systems (complex systems, for short) are considered as systems consisting of a set of interacting elements

*[. . .] where the behavior of the total is an indirect, non-hierarchical consequence of the behavior of the different parts. Complex systems differ in that sense from strictly hierarchical systems [...] where the total behavior is a hierarchical composition of the behavior of the parts. In complex systems, global coherence is reached despite purely local non-linear interactions. There is no central control source. Typically the system is open.*

### 3. Beyond pure mathematical modeling: a concluding remark

From the above exemplification of the process of computation over a c-granule, moving from a configuration of ic-granules to another, it is quite clear that the process deals with a set of hunks of real physical matter associated with their information layers; the information layers indicate where, when and how they can be touched, or their properties can be achieved or verified. So, this already clarifies how in the model of IGrC by a c-granule both abstract world, that is the information layer, and the real physical semantics, that is the three-layered hunk of objects,



together are referred to. One more point, that needs to be clarified, is how the model designs its real physical implementation by lifting the static description of a process to the level of actions.

The implementational ic-granules create a specific interface between the abstract and the physical world. In particular, at some level of the implementational phase the control of the c-granule launches actions linking the abstract world associated to the informational layer of the control of the c-granule with the real physical world. These actions are not from abstract mathematical space and the model of IGrC keep those action functions free from mathematical formalizations. Their syntactic descriptions can be formalised in the informational layers of the control of the c-granule. However, their implementation should be realised in the real physical world and the model only can mathematically formalize their performance quality by perceiving the changes in the world after initiation of the actions. Of course, the expected properties of the real physical environment, after the implementation of the actions, also can be formalized within the description of the action specification.

Thus the IGrC model keeps the possibility of mismatch between expected and real physical outcome open as we never can a priori formalize all possible outcomes of an action, which is supposed to be initiated in the physical world based on an abstract description of the action specification. We can only expect a desired outcome, specified in the information layer. So, after each implementational phase of a computation over a c-granule the control starts a new cycle again by perceiving the properties of the objects lying in its scope. Then those observed properties are verified with the expected properties. Hence, accordingly there is a need to modify the induced models based on their comparisons with the recorded data.

In Figure 4 we illustrate the idea of decomposition of the description of the action plan in order to transform an ic-granule with a given property, say  $\alpha$ , into another one with the property  $\beta$ . After several levels of decomposition it reaches a level at which the relevant actions are launched by the control so that the expected realisation of the whole chain of actions can lead to a situation satisfying, to a satisfactory degree, the property  $\beta$ . Though the conditions of the chain of actions and the expected properties after the actions are specified by  $\alpha$ 's and  $\beta$ 's, the actual actions  $ac_1, \dots, ac_k$  are not in the realm of mathematical formulation. The model of IGrC keeps this juncture between abstract and physical bridging free from mathematical formulation. Instead, IGrC proposes to formalize the results of actions by perceiving the properties of outcome configurations and verifying them with the expected property  $\beta$ .

Let us outline a scheme of the behaviour of the control of a c-granule.

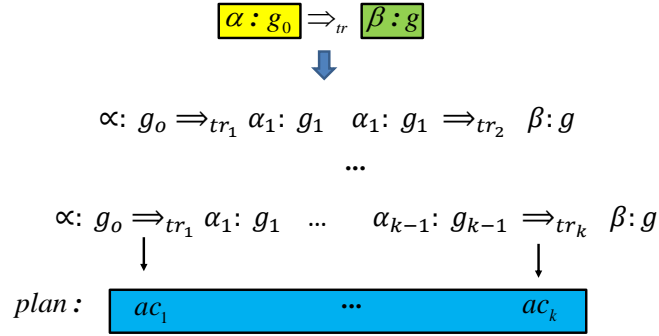
Often the control aims at achieving its target goal, expressed in the information layer of the ic-granule  $g_0$ , using complex vague concepts, *e.g.*, related to safeness of the perceived situation, from a natural language. Below, we assume that for a given specification a set of rules for selection of the relevant transformations of ic-granules was learned by the control or is given a priori. The pre-condition of each rule is a condition. The degree of matching of this condition by the current status of the perceived situation determines selection of the transformation of ic-granules from the post-condition of the rule.<sup>3</sup>

The outline of the general procedure of a computation in IGrC, realised by the control of a c-granule, is based on searching for relevant transformations of ic-granules and their

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<sup>3</sup>Note that other learning paradigms such as lazy learning can be used in inducing models of complex vague concepts different from the discussed here.

Transformation specification  $tr$  from an ic-granule with property  $\alpha$  to an ic-granule with property  $\beta$  available at the planner ic-granule  $g_0$



**Figure 4:** Illustration of a simple case of decomposition of transformation.

implementations; it looks as follows.

perceive accessible parts of the abstract and physical world to understand (up to a satisfactory degree) the current situation for making decision concerning the selection of the formal specification of transformation of ic-granules for implementation; the current status of perception is represented in the informational layer of  $g_s$ ; this is realised by the control in several steps and one of them is listed below

↓

consult domain knowledge and required goal of computation ( $g_{kb}, g_0$ ) to enrich the information about the status of currently perceived situation

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...

(it may be necessary to perform several steps of reasoning before having a satisfactory understanding of the perceived situation; it can be achieved by allowing the control to select the relevant formal specification of transformation of ic-granules for implementation; some of these steps may be related, *e.g.*, to measurements (by sensors) of features of the perceived physical objects in the scope of active ic-granules<sup>4</sup>

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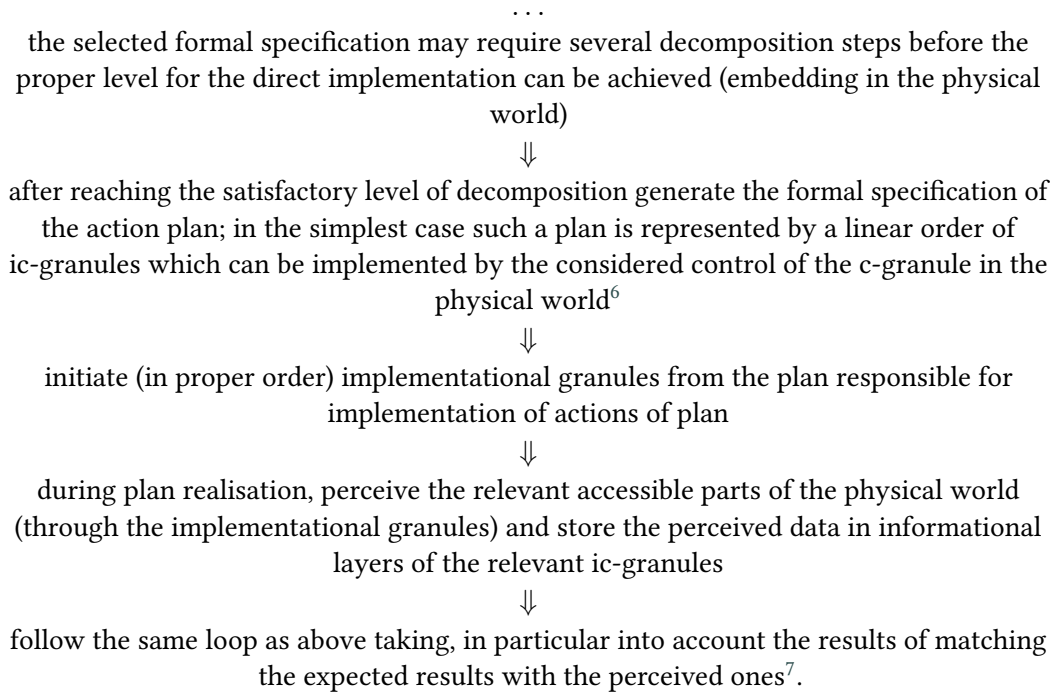
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select (from the proper knowledge base) the relevant formal specification of the transformation of the ic-granules for implementation<sup>5</sup>

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<sup>4</sup>It should be noted that the mentioned steps of reasoning are also realised by transformations of some ic-granules.

<sup>5</sup>This step is especially compound; details will be discussed in an extended version of our paper.



The discussed above issues of decomposition and implementation should be treated as an illustrative example only. In the real-life projects one should take into account many other issues.

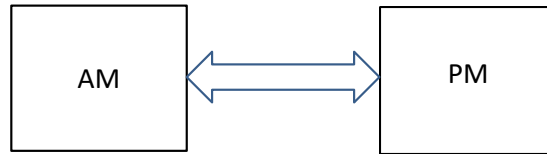
Here to be noted that the transition from one configuration of ic-granules to the other, as described above in the general plan of computation, by creating and accessing different ic-granules and their information layers is not also purely mathematical. Usually, the state transition relation is presented by a given family of sets  $\{X_i\}_{i \in I}$  where a transition relation is represented as a relation  $tr_i \subseteq X_i \times X_i$ . Here, this cannot be purely mathematical as we need to incorporate the components which can specify (i) how elements of  $X_i$  are perceived in the real physical environment, and (ii) how the transition relation  $tr_i$  is implemented in the real physical world. This reasoning mechanism, as described in the Introduction, is conducted by the control of the c-granule over which the computation process is running.

So, in contrary to a static transition relation, in IGrC the control of a c-granule incorporates the possibility of dynamic as well as not purely mathematical formulation of a state transition relation. The structure of a control is designed to have two interacting modules, called the abstract module (AM) and the physical module (PM) (see Figure 5).

Communication between AM and PM is designed by two mechanisms. The first one provides a possibility of encoding given information from AM by a relevant state of a set of physical objects from PM, and the second one provides a possibility of encoding the considered state of

<sup>6</sup>Plans may be generated on different levels of decomposition, *e.g.*, for better understanding the perceived situation helping the control to generate the high quality plans for realisation of the target goals.

<sup>7</sup>In a more compound case of the control of a c-granule, strategies for adaptation of the previously used plan to the new situation are used. Moreover, the decision about the necessity of plan adaptation may be taken often during of the actual plan.



**Figure 5:** Interacting abstract (AM) and physical (PM) modules in control of c-granules.

a set of physical objects from PM by a relevant information represented in the language of AM. These two mechanisms can be implemented by atomic actions.

AM module sends to PM a formal specification of a transformation of a ic-granule as well as formal specifications of some spatio-temporal windows. Some of them are labelled by already perceived information or properties from the scope of the c-granule and others are labelled by formal specifications of the required information from PM. In case when the delivered specification can be directly embedded by PM in the physical world then PM, by creating a network of interacting ic-granules, aims to deliver a network of physical pointers matching information, perceived by AM, with the expected properties, expressed formally in AM. Otherwise, PM sends to AM a message about the necessity of the specification decomposition.

AM may send to PM messages consisting of formal specification(s) of the required transformation(s) of ic-granules together with the expected results provided by AM. The messages sent by AM are encoded by the states of some physical objects in PM. In this way, atomic actions changing states of physical objects to the ones specified by the given information are implemented. However, it is to be noted that the modeling of the behaviour of AM and communication of AM with PM can be based on mathematical modeling; whereas PM is composed out of physical objects which, being from the real world, are outside of the abstract mathematical modeling. However, partial information about properties of these objects and their interactions may be communicated to AM through interaction of PM with AM. Thus, IGrC incorporates both dynamic as well as mathematical modeling grounded in the real physical world.

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