On Semantics for Testing in Time Petri Nets

Elena Bozhenkova[1], Irina Virbitskaite[1]

[1]A.P. Ershov Institute of Informatics Systems, SB RAS; 6, Lavrentiev av., Novosibirsk, 630090, Russian Federation

Abstract

Dense-Time Petri Nets (TPNs) are now a well-established model to describe and study real time (quantitative) and functional (qualitative) properties of safety-critical, computer-controlled systems. Testing equivalences, used to compare systems' behaviors (processes) and reduce the structure of systems, are defined in terms of tests that processes may or must pass. The intention of the paper is to establish the interrelations between various semantics for must-testing equivalences with extended tests, in the framework of TPNs. This allows for studying in detail the timing behavior in addition to the degree of relative concurrency of processes generated when systems are functioning.

1. Introduction

Dense-Time Petri Nets (TPNs) are suitable for qualitative and quantitative modelling and verifying of safety-critical, computer-controlled, real-time systems. Several semantics (behaviors) are explored in the literature for TPNs, that can be classified according to interleaving – partial order dichotomy. The classical interleaving behavior of the TPN is described by runs — sequences of changes in states by time elapsings and/or transition firings. The semantics allows for analyzing some safety and liveness properties of systems, however concurrency between net transitions is reduced to non-deterministic choice between sequences of transitions firings in any possible order. Step semantics of TPNs generalizes the interleaving approach by allowing several concurrent transitions (forming a step) to fire simultaneously. Partial order semantics of TPNs is most often represented by means of the so-called causal net processes which include events and conditions related by causal dependence (the absence of causality means concurrency) and equipped with timing information. Causal tree semantics summarizes the interleaving and partial order approaches by representing the behavior of the TPN in the form of a tree with nodes corresponding to runs, and edges labeled by actions with their times and causal predecessors.

Testing equivalences [1] are explicitly based on a framework of extracting information about the systems' behaviors (processes) by testing them. Two processes are considered equivalent if there is no test that can distinguish them. In the realm of untimed models, interleaving testing was thoroughly investigated and well-understood in the setting of models of transition systems (see [2, 3] among many others). Interleaving, step and partial order testing equivalences for elementary net systems (safe Petri nets without loops) were studied in [4]. There, the authors

29th International Workshop on Concurrency, Specification and Programming (CS&P'21)
bozhenko@iis.nsk.su (E. Bozhenkova); virb@iis.nsk.su (I. Virbitskaite)
https://pdb.iis.nsk.su/en/person/220 (E. Bozhenkova); https://persons.iis.nsk.su/en/virbitskaite (I. Virbitskaite)
0000-0002-9291-7451 (E. Bozhenkova); 0000-0002-4475-3480 (I. Virbitskaite)

indicated the location of the testings among other behavioral equivalences from linear time – branching time spectrum, providing their hierarchies for the model under consideration with and without invisible actions. Partial order and non-deterministic semantics for different classes of Petri nets are often represented by means of various models of event structures. In the framework of the models, testing equivalences within interleaving – partial order dichotomy were developed and compared in the papers [5, 6, 7]. Moreover, in [7] special attention was paid to the relationships between partial order and causal tree semantics in the context of testing equivalences. To the best of authors' knowledge, causal tree semantics in the framework of Petri nets has not been studied yet.

As safety-critical applications often require verification of real time characteristics, testing equivalences are expanded in concurrent models with time. In the papers [8, 9, 10, 11], alternative characterizations of timed testing are provided for timed generalizations of interleaving models. In [12], the testing relations along with their alternative characterization and discretization were proposed in the framework of Petri nets with time intervals associated to arcs from places to transitions. At the same time, to the best of our knowledge, there are only few mentions of a fusion of timing and partial order semantics, in testing scenario. In this regard, the work [13] is a welcome exception, where time-sensitive testing is investigated within linear time – branching time spectrum, in the setting of event structure models with time characteristics. Also, our origin is the paper [14] the main result of which is the coincidence of poset and causal tree testing equivalences, with the tests as direct extensions of the experiments, in the setting of TPNs. In this paper, we expand the results of [14] to step, poset and causal tree semantics with extended tests[1], and demonstrate the discriminating/matching power of the semantics in the framework of testing equivalences on contact-free TPNs. The results obtained can be useful in formal verification of systems since partial order semantics allows for reducing the number of systems' states to be analyzed.

2. Syntax and Different Semantics of TPNs

In this section, some terminology concerning the model of Petri nets with timing constraints (time intervals on the firings of transitions) and its concurrent semantics in terms of interleaving/step firing sequences, causal net processes and causal trees are defined.

2.1. Syntax and Interleaving/Step Semantics of TPNs

We start with recalling the definitions of the structure and interleaving/step behavior of time Petri nets [15, 16]. We use Act as an alphabet of actions and $Act^{\mathbb{N}}$ as a set of multisets over Act. Let the domain $\mathbb{T}$ of time values be the set of rational numbers. We denote by $[\tau_1, \tau_2]$ the closed interval between two time values $\tau_1, \tau_2 \in \mathbb{T}$. Infinity is allowed at the upper bounds of intervals. Let $Interv$ be the set of all such intervals.

Definition 1. • A (labeled over Act) time Petri net *is a pair* $\mathcal{TN} = (\mathcal{N}, D)$, *where* $\mathcal{N} = (P, T, F, M_0, L)$ *is a (labeled over Act) underlying Petri net (with a finite set P of places, a*

[1] Testing equivalence with extended tests checks, after the executions of the experiments, the tests that are continuations of the experiments with steps/posets of actions, not with single actions.

finite set T of transitions such that $P \cap T = \emptyset$ and $P \cup T \neq \emptyset$, a flow relation $F \subseteq (P \times T) \cup (T \times P)$, an initial marking $\emptyset \neq M_0 \subseteq P$, a labeling function $L : T \to Act$) and $D : T \to Interv$ is a static timing function *associating with each transition a time interval. For a transition $t \in T$, the boundaries of the interval $D(t) \in Interv$ are called the earliest firing time Eft and latest firing time Lft of t. For $x \in P \cup T$, let ${}^\bullet x = \{y \mid (y, x) \in F\}$ and $x^\bullet = \{y \mid (x, y) \in F\}$ be the* preset *and* postset *of x, respectively. For $X \subseteq P \cup T$, define ${}^\bullet X = \bigcup_{x \in X} {}^\bullet x$ and $X^\bullet = \bigcup_{x \in X} x^\bullet$.*

- *A marking M of $\mathcal{TN}$ is any subset of P. A transition $t \in T$ is* enabled at a marking M *if ${}^\bullet t \subseteq M$. Let $En(M)$ be the set of transitions enabled at M. A non-empty subset $\emptyset \neq U \subseteq T$ is a* step enabled at a marking M, *if $(\forall t \in U \diamond t \in En(M))$ and $(\forall t \neq t' \in U\colon ({}^\bullet t \cup t^\bullet) \cap ({}^\bullet t' \cup t'^\bullet) = \emptyset)$.*

  A state *of $\mathcal{TN}$ is a pair (M, I), where M is a marking and $I : En(M) \longrightarrow \mathbb{T}$ is a* dynamic timing function. *The* initial state *of $\mathcal{TN}$ is a pair $S_0 = (M_0, I_0)$, where M_0 is the initial marking and $I_0(t) = 0$, for all $t \in En(M_0)$.*

  A step U enabled at a marking M can fire from a state $S = (M, I)$ after a delay time $\theta \in \mathbb{T}$ *if $(Eft(t) \leq I(t) + \theta)$, for all $t \in U$, and $(I(t') + \theta \leq Lft(t'))$, for all $t' \in En(M)$.*

  The firing *of a step U that can fire from a state $S = (M, I)$ after a delay time θ* leads *to the new state $S' = (M', I')$ (denoted $S \xrightarrow{(U,\theta)} S'$) given by:*

  (a) $M \xrightarrow{U} M'$,

  $$(b)\ \ \forall t' \in T \diamond I'(t') = \begin{cases} I(t') + \theta, & \text{if } t' \in En(M \setminus {}^\bullet t), \\ 0, & \text{if } t' \in En(M') \setminus En(M \setminus {}^\bullet t), \\ \text{undefined}, & \text{otherwise.} \end{cases}$$

  Then, we write $S \xrightarrow{(A,\theta)} S'$, if $A = L(U) = \Sigma_{t \in U} L(t) \in Act^{\mathbb{N}}$, i.e. A is a multiset over the set $\{a \in Act \mid a = L(t)$ and $t \in U\}$. We use the notation $S \xrightarrow{\sigma} S'$ iff $\sigma = (U_1, \theta_1) \ldots (U_k, \theta_k)$ and $S = S^0 \xrightarrow{(U_1,\theta_1)} S^1 \ldots S^{k-1} \xrightarrow{(U_k,\theta_k)} S^k = S'$ ($k \geq 0$). In this case, σ is a step firing sequence of $\mathcal{TN}$ from S (to S'), *and S' is a* reachable state of $\mathcal{TN}$ from S. *Whenever $|\, U_i \,| = 1$ for all $1 \leq i \leq k$, we call σ an* interleaving firing sequence of $\mathcal{TN}$. *Let $\mathcal{FS}_{s(i)}(\mathcal{TN})$ be the set of all step (interleaving) firing sequences of $\mathcal{TN}$ from S_0, and $RS(\mathcal{TN})$ be the set of all reachable states of $\mathcal{TN}$ from S_0. For $\sigma = (U_1, \theta_1) \ldots (U_k, \theta_k) \in \mathcal{FS}_{s(i)}(\mathcal{TN})$, $L(\sigma) = (A_1, \theta_1) \ldots (A_k, \theta_k)$ iff $A_i = L(U_i)$ for all $1 \leq i \leq k$.*

  We call $\mathcal{TN}$ T-restricted *iff ${}^\bullet t \neq \emptyset \neq t^\bullet$, for all transitions $t \in T$;* contact-free *iff whenever a step U can fire from the state $S = (M, I)$ after some delay time θ, then $(M \setminus {}^\bullet U) \cap U^\bullet = \emptyset$, for all $S \in RS(\mathcal{TN})$. In what follows, we shall consider only T-restricted and contact-free time Petri nets.*

Example 1. *A (labeled over $Act = \{a, b, c\}$) time Petri net $\widetilde{\mathcal{TN}}$ is shown in Figure 1. Here, the places are represented by circles, and transitions — by bars; the names are depicted near the net elements, the flow relation is drawn by the arcs, the initial marking is represented as the set of the places with tokens (bold points), and the values of the labeling and timing functions are*

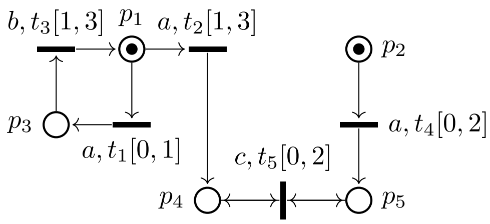

Figure 1: The TPN $\widetilde{\mathcal{TN}}$.

printed next to the transitions. It is not difficult to check that t_1, t_2 and t_4 are transitions enabled at the initial marking $M_0 = \{p_1, p_2\}$, and, moreover, $\{t_1, t_4\}$ and $\{t_2, t_4\}$ are steps enabled at M_0. The steps can fire from the initial state $S_0 = (M_0, I_0)$ after time delay $\theta_1 = 1$, where $I_0(t) = \begin{cases} 0, & \text{if } t \in \{t_1, t_2, t_4\}, \\ undefined, & \text{otherwise.} \end{cases}$ The sequence $\sigma = (\{t_1, t_4\}, 0.5)\ (t_3, 1)\ (t_2, 2)$ $(t_5, 2)$ is a step firing sequence of $\widetilde{\mathcal{TN}}$ from S_0. Also, we have $L(\sigma) = (2'a, 0.5)\ (b, 1)\ (a, 2)$ $(c, 2)$. Furthermore, it is easy to see that $\widetilde{\mathcal{TN}}$ is really T-restricted and contact-free.

2.2. Causal Net Process Semantics of TPNs

In this subsection, the concept of causality-based net processes is presented in the context of TPNs.

We start with the definition of time causal nets, whose events and conditions are related by causal dependence and concurrency (absence of causality), and whose timing function associates with the events their occurrence times.

Definition 2. *A* (labeled over Act) time causal net *is a finite, acyclic net $TN = (B, E, G, l, \tau)$ with a set B of conditions; a set E of events; a flow relation $G \subseteq (B \times E) \cup (E \times B)$ such that $\mid {}^\bullet b \mid \leq 1 \geq \mid b^\bullet \mid$, for all $b \in B$, and ${}^\bullet B = E = B^\bullet$; a labeling function $l : E \to Act$, and a timing function $\tau : E \to \mathbb{T}$ such that $e\ G^+\ e' \Rightarrow \tau(e) \leq \tau(e')$.*

For a time causal net $TN = (B, E, G, l, \tau)$ and $e, e' \in E$, define:

- $TN^\bullet = \{b \in B \mid b^\bullet = \emptyset\}$;
- $\prec = G^+$, $\preceq = G^*$ (causality);
- $Predec(e) = \{e' \in E \mid e' \prec e\}$ (causal predecessors of e), $Earlier(e) = \{e' \in E \mid \tau(e') < \tau(e)\}$ (time predecessors of e), and $Cut(e) = (Earlier(e)^\bullet \cup {}^\bullet TN) \setminus {}^\bullet Earlier(e)$;
- E' is a *downward-closed subset of* E iff $E' \subset E$ and $Predec(e') \subseteq E'$, for all $e' \in E'$; a *timely sound subset of* E iff $E' \subset E$ and $Earlier(e') \subseteq E'$, for all $e' \in E'$;
- $e \smile e' \iff \neg((e \prec e') \vee (e' \prec e))$ (concurrency); $\emptyset \neq V \subseteq E$ is a *step* iff $e \smile e'$ and $\tau(e) = \tau(e')$ for all $e \neq e' \in V$. Let $\tau(V) = \tau(e)$ for some $e \in V$ (time of V);

- a sequence $\rho = V_1 \ldots V_k$ ($k \geq 0$) of steps of TN is an *s-linearization of* TN iff $\bigcup_{1 \leq i \leq k} V_i = E$ and $\bigcap_{1 \leq i \leq k > 1} V_i = \emptyset$ (i.e. every event of TN appears in the sequence exactly once), and for all $1 \leq i < j \leq k$ it holds: $\bigcup_{1 \leq l \leq i} V_l$ is a downward-closed and timely sound subset of $\bigcup_{1 \leq m \leq j} V_m$ (i.e. both causal and time order are preserved: for all $1 \leq i < j \leq k$, $\neg((e' \prec e) \vee (\tau(e') < \tau(e)))$, for all $e \in V_i$, $e' \in V_j$). Whenever $\mid V_i \mid = 1$ for all $1 \leq i \leq k$, ρ is an i-linearization;

- $\eta(TN) = (E_{TN}, \preceq_{TN} \cap (E_{TN} \times E_{TN}), l_{TN}, \tau_{TN})$ is a time poset[2]. For time posets $\eta = (E, \preceq, l, \tau)$ and $\eta' = (E', \preceq', l', \tau')$, η is a *pos-extension of* η' iff E' is a downward-closed and timely sound subset of E, $\preceq' = \preceq \cap E' \times E'$, $l' = l \mid_{E'}$, and $\tau' = \tau \mid_{E'}$.

We are now ready to define the concept of causal net based processes of TPNs, proposed in [15].

Definition 3. *Given a time Petri net* $\mathcal{TN} = ((P,\ T,\ F,\ M_0,\ L),\ D)$ *and a time causal net* $TN = (B, E, G, l, \tau)$,

- *a mapping* $\varphi : B \cup E \to P \cup T$ *is a* homomorphism *from* TN *to* $\mathcal{TN}$ *iff the following hold:*
  - $\varphi(B) \subseteq P$, $\varphi(E) \subseteq T$;
  - *the restriction of* φ *to* ${}^\bullet e$ *is a bijection between* ${}^\bullet e$ *and* ${}^\bullet \varphi(e)$, *and the restriction of* φ *to* $e^\bullet$ *is a bijection between* $e^\bullet$ *and* $\varphi(e)^\bullet$, *for all* $e \in E$;
  - *the restriction of* φ *to* ${}^\bullet TN$ *is a bijection between* ${}^\bullet TN$ *and* M_0;
  - $l(e) = L(\varphi(e))$, *for all* $e \in E$.
- *a pair* $\pi = (TN, \varphi)$ *is a* time process of a time Petri net $\mathcal{TN}$ *iff* TN *is a time causal net and* φ *is a homomorphism from* TN *to* $\mathcal{TN}$;
- *a time process* $\pi = (TN, \varphi)$ *of* $\mathcal{TN}$ *is* correct *iff for all* $e \in E$ *it holds:*

  (a) $\tau(e) \geq \boldsymbol{TOE}_\pi({}^\bullet e, \varphi(e)) + Eft(\varphi(e))$,

  (b) $\forall t \in En(\varphi(Cut(e))) \diamond \tau(e) \leq \boldsymbol{TOE}_\pi(Cut(e), t) + Lft(t)$.

  Here, for a subset $B' \subseteq B_{TN}$ *and a transition* $t \in En(\varphi(B'))$, *the time of enabling (TOE) of* t, *i.e. the latest global time moment when tokens appear in all input places of* t, *is defined as follows:* $\boldsymbol{TOE}_\pi(B', t) = \max\left(\{\tau_{TN}({}^\bullet b) \mid b \in B'_{[t]} \setminus {}^\bullet TN\} \cup \{0\}\right)$, *where* $B'_{[t]} = \{b \in B' \mid \varphi_{TN}(b) \in {}^\bullet t\}$.
  Let $\mathcal{CP}(\mathcal{TN})$ *denote the set of correct time processes of* $\mathcal{TN}$.

We now present for the time Petri net the relationships between its correct time processes and its interleaving/step firing sequences from the initial state.

Proposition 1. *[15, 16] Let* $\mathcal{TN}$ *be a time Petri net. Then,*

[2] A *(labeled over Act) time poset (partially ordered set)* is a tuple $\eta = (X, \preceq, \lambda, \tau)$ consisting of a finite set X of elements; a reflexive, antisymmetric and transitive relation $\preceq$; a labeling function $l : X \to Act$; and a timing function $\tau : X \to \mathbb{T}$ such that $x \preceq x' \Rightarrow \tau(x) \leq \tau(x')$.

(i) for any $\pi = (TN, \varphi) \in \mathcal{CP}(\mathcal{TN})$ with an $s(i)$-linearization $\rho = V_1 \dots V_k$ of TN, there is a unique step (interleaving) firing sequence $FS_\pi(\rho) = (\varphi(V_1), \tau(V_1) - 0) \dots (\varphi(V_k), \tau(V_k) - \tau(V_{k-1})) \in \mathcal{FS}_{s(i)}(\mathcal{TN})$;

(ii) for any step (interleaving) firing sequence $\sigma \in \mathcal{FS}_{s(i)}(\mathcal{TN})$, there is a unique (up to an isomorphism[3]) correct time process $\pi_\sigma = (TN, \varphi) \in \mathcal{CP}(\mathcal{TN})$ with a unique $s(i)$-linearization ρ_σ of TN such that $FS_{\pi_\sigma}(\rho_\sigma) = \sigma$.

For correct time processes $\pi = (TN, \varphi)$, $\pi' = (TN', \varphi') \in \mathcal{CP}(\mathcal{TN})$, we say that π is a pos-extension of π' in $\mathcal{TN}$ iff $\eta(TN)$ is pos-extension of $\eta(TN')$, $B' \subset B$, $G' = G \cap (B' \times E' \cup E' \times B')$, and $\varphi' = \varphi\mid_{B' \cup E'}$.

We expand the above results to pos-extensions of correct time processes of TPNs.

Lemma 1. *Given a time Petri net $\mathcal{TN}$, $\sigma \in \mathcal{FS}_{s(i)}(\mathcal{TN})$, and $\pi \in \mathcal{CP}(\mathcal{TN})$ such that $\sigma = FS_\pi(\rho)$, where ρ is an $s(i)$-linearization of TN_π,*

(i) if $\widetilde{\pi}$ is a pos-extension of π in $\mathcal{TN}$, then there is $\sigma\sigma' \in \mathcal{FS}_{s(i)}(\mathcal{TN})$ such that $\sigma\sigma' = FS_{\widetilde{\pi}}(\rho\rho')$, where $\rho\rho'$ is an $s(i)$-linearization of $TN_{\widetilde{\pi}}$;

(ii) if $\sigma\sigma' \in \mathcal{FS}_{s(i)}(\mathcal{TN})$, there is $\widetilde{\pi} \in \mathcal{CP}(\mathcal{TN})$ such that $\widetilde{\pi}$ is a pos-extension of π in $\mathcal{TN}$ and $\sigma\sigma' = FS_{\widetilde{\pi}}(\rho\rho')$, where $\rho\rho'$ is an $s(i)$-linearization of $TN_{\widetilde{\pi}}$.

2.3. Causal Tree Semantics of TPNs

Causal trees [17] are synchronisation trees which carry in their labels additional information about causes of actions thus providing us with an interleaving description of concurrent processes, which faithfully expresses causality. Time causal trees are generalizations of causal trees by adding timing. In the time causal tree of the TPN, the nodes are the interleaving firing sequences of the TPN, and an edge exists between two nodes if the second one is a direct extension of the first one. The causes in the edge labels are calculated based on the causality relations in the correct time processes of the TPN corresponding to the nodes (the interleaving firing sequences).

Definition 4. *The* time causal tree *of the TPN $\mathcal{TN}$, $TCT(\mathcal{TN})$, is a tree $(\mathcal{FS}_i(\mathcal{TN}), Ed, \phi)$, where $\mathcal{FS}_i(\mathcal{TN})$ is the set of nodes with the root ϵ; $Ed = \{(\sigma, \sigma(t,\theta)) \mid \sigma, \sigma(t,\theta) \in \mathcal{FS}(\mathcal{TN})\}$ is the set of edges; ϕ is the labeling function such that $\phi(\epsilon) = \epsilon$ and $\phi(\sigma, \sigma(t,\theta)) = (L_{\mathcal{TN}}(t), \theta, K)$, with $\sigma = FS_{\pi_\sigma}(\rho_\sigma = e_1 \dots e_n)$, $\sigma(t,\theta) = FS_{\pi_{\sigma(t,\theta)}}(\rho_{\sigma(t,\theta)} = e_1 \dots e_n e)$, $K = \{n-l+1 \mid e_l \prec_{TN_{\pi_{\sigma(t,\theta)}}} e\}$. Let $path(\sigma)$ be the path starting from the root and finishing in the node σ of $TCT(\mathcal{TN})$[4].*

[3]Time processes $\pi = (TN, \varphi)$ and $\pi' = (TN', \varphi') \in \mathcal{CP}(\mathcal{TN})$ are *isomorphic* (denoted $\pi \simeq \pi'$) iff there exists a bijective mapping $\beta : B \cup E \to B' \cup E'$ such that (i) $\beta(B) = B'$ and $\beta(E) = E'$; (ii) $x\, G\, y \iff \beta(x)\, G'\, \beta(y)$, for all $x, y \in B \cup E$; (iii) $l(e) = l'(\beta(e))$, for all $e \in E$; (iv) $\tau(e) = \tau'(\beta(e))$, for all $e \in E$; (v) $\varphi(x) = \varphi'(\beta(x))$, for all $x \in B \cup E$.

[4]We assume $path(\epsilon) = \epsilon$. Notice that in $TCT(\mathcal{TN})$, for any node $\sigma \in \mathcal{FS}_i(\mathcal{TN})$, there is a path starting from the root and finishing in σ.

Example 2. *Consider the time Petri net $\widetilde{\mathcal{TN}}$ (see Figure 1) and its interleaving firing sequence $\sigma = (t_1, 0.5)\ (t_4, 0)\ (t_3, 1)\ (t_2, 2)\ (t_5, 2) \in \mathcal{FS}_i(\widetilde{\mathcal{TN}})$. It is easy to get that $\phi(path(\sigma)) = (a, 0.5, \emptyset)\ (a, 0.5, \emptyset)\ (b, 1, \{2\})\ (a, 2, \{1\})\ (c, 2, \{1, 3\})$.*

We finally establish some relationships between correct time processes and labeled paths in the time causal trees of two time Petri nets.

Proposition 2. *Let $\mathcal{TN}, \mathcal{TN}'$ be time Petri nets. Then,*

(i) for any $\pi \in \mathcal{CP}(\mathcal{TN})$ and $\pi' \in \mathcal{CP}(\mathcal{TN}')$ with an isomorphism $f : \eta(TN_\pi) \to \eta(TN_{\pi'})$, $\phi(path(FS_\pi(\rho))) = \phi'(path(FS_{\pi'}(f(\rho))))$, for any i-linearization ρ of TN_π;

(ii) for any $\sigma \in \mathcal{FS}_i(\mathcal{TN})$ and $\sigma' \in \mathcal{FS}_i(\mathcal{TN}')$ such that $\phi(path(\sigma)) = \phi'(path(\sigma'))$, there is an isomorphism $f : \eta(TN_{\pi_\sigma}) \to \eta(TN_{\pi_{\sigma'}})$ such that $f(\rho_\sigma) = \rho_{\sigma'}$.

3. Testing Equivalences

Interleaving testing equivalence deals with the experiments on the TPN — sequences of actions with their times — and the behaviors which are tested for after the experiments — sets of actions with their times. So, it checks whether actions with times, given as a test, can be executed after a sequence of actions with times, specified as an experiment. Here, both the experiments and tests represent interleaving semantics.

Definition 5. *Given time Petri nets $\mathcal{TN}$ and $\mathcal{TN}'$,*

- *for a sequence $w \in (Act \times \mathbb{T})^*$ and a set $W \subseteq (Act \times \mathbb{T})$, $\mathcal{TN}$ **after** w $\mathbf{MUST}^{int}_{int}$ W iff for all firing sequences $\sigma \in \mathcal{FS}_i(\mathcal{TN})$ such that $L(\sigma) = w$, there exists an element $(a, \theta) \in W$ and a firing sequence $\sigma(t, \theta) \in \mathcal{FS}_i(\mathcal{TN})$ such that $L(\sigma(t, \theta)) = w(a, \theta)$;*
- *$\mathcal{TN}$ and $\mathcal{TN}'$ are* interleaving testing equivalent *(denoted $\mathcal{TN} \sim^{int}_{int} \mathcal{TN}'$) iff for all sequences $w \in (Act \times \mathbb{T})^*$ and for all sets $W \subseteq (Act \times \mathbb{T})$, it holds:*

$$\mathcal{TN}\ \textbf{after}\ w\ \mathbf{MUST}^{int}_{int}\ W \iff \mathcal{TN}'\ \textbf{after}\ w\ \mathbf{MUST}^{int}_{int}\ W.$$

In step testing, the experiments on the TPN are sequences of multisets over sets of actions with their times and the tests checked after the experiments are sets of multisets over sets of actions with their times. So, it checks whether multisets over sets of actions with times, given as a test, can be executed after a sequence of multisets over sets of actions with times, specified as an experiment. Thereby, both the experiments and tests respect step semantics.

Definition 6. *Given time Petri nets $\mathcal{TN}$ and $\mathcal{TN}'$,*

- *for a sequence $w \in (Act^{\mathbb{N}} \times \mathbb{T})^*$ and a set $W \subseteq (Act^{\mathbb{N}} \times \mathbb{T})$, $\mathcal{TN}$ **after** w $\mathbf{MUST}^{step}_{step}$ W iff for all firing sequences $\sigma \in \mathcal{FS}_s(\mathcal{TN})$ such that $L(\sigma) = w$, there exists an element $(A, \theta) \in W$ and a firing sequence $\sigma(U, \theta) \in \mathcal{FS}_s(\mathcal{TN})$ such that $L(\sigma(U, \theta)) = w(A, \theta)$;*

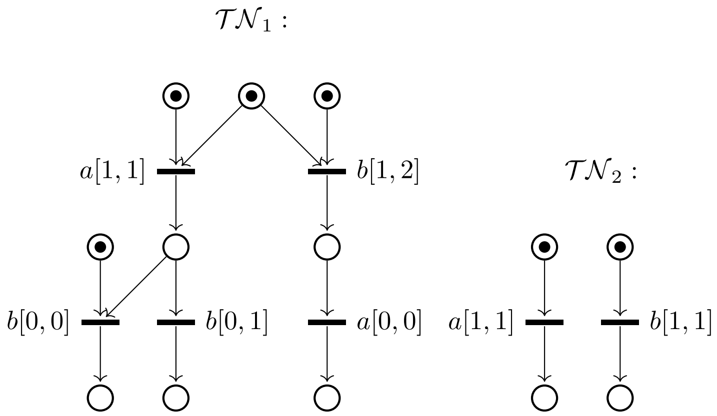

Figure 2: The $\sim_{int}^{int}$–equivalent but neither $\sim_{step}^{step}$– nor $\sim_{pos}^{pos}$–equivalent TPNs $\mathcal{TN}_1$ and $\mathcal{TN}_2$.

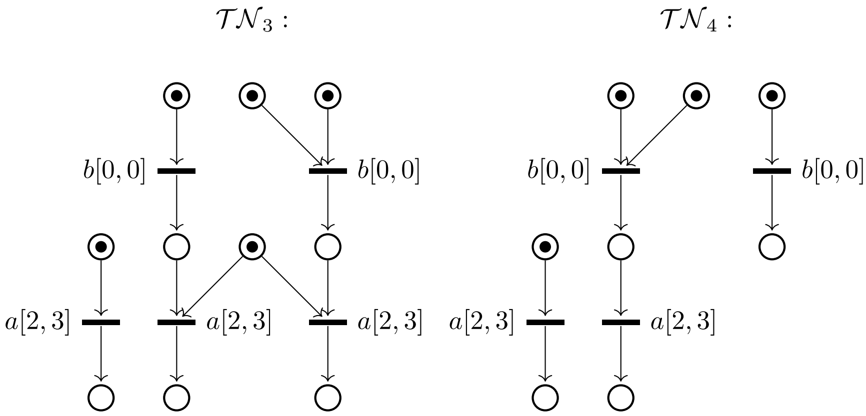

Figure 3: The $\sim_{step}^{step}$–equivalent but not $\sim_{pos}^{pos}$–equivalent TPNs $\mathcal{TN}_3$ and $\mathcal{TN}_4$.

- *$\mathcal{TN}$ and $\mathcal{TN}'$ are* step testing equivalent *(denoted $\mathcal{TN} \sim_{step}^{step} \mathcal{TN}'$) iff for all sequences $w \in (Act^{\mathbb{N}} \times \mathbb{T})^*$ and for all sets $W \subseteq (Act^{\mathbb{N}} \times \mathbb{T})$, it holds:*

$$\mathcal{TN} \textbf{ after } w \textbf{ MUST}_{step}^{step} W \iff \mathcal{TN}' \textbf{ after } w \textbf{ MUST}_{step}^{step} W.$$

The idea of partial order testing is that the experiments on the TPN are time posets and the tests, that are examined after the experiments, are sets of pos-extensions of the experiments. This contrasts with partial order based testing investigated in the paper in [14], where the tests contain sets of time posets extending the experiments by single actions with their times. From now on, we denote pos-extensions of a time poset TP by $\mathbf{TP}_{TP}$.

Definition 7. *Given time Petri nets $\mathcal{TN}$ and $\mathcal{TN}'$,*

- *for a time poset TP and a set $\mathbf{TP} \subseteq \mathbf{TP}_{TP}$, $\mathcal{TN}$* **after** *TP* **MUST**$_{pos}^{pos}$ **TP** *iff for all time processes $\pi = (TN, \varphi) \in \mathcal{CP}(\mathcal{TN})$ and for all isomorphisms $f : \eta(TN) \longrightarrow TP$,*

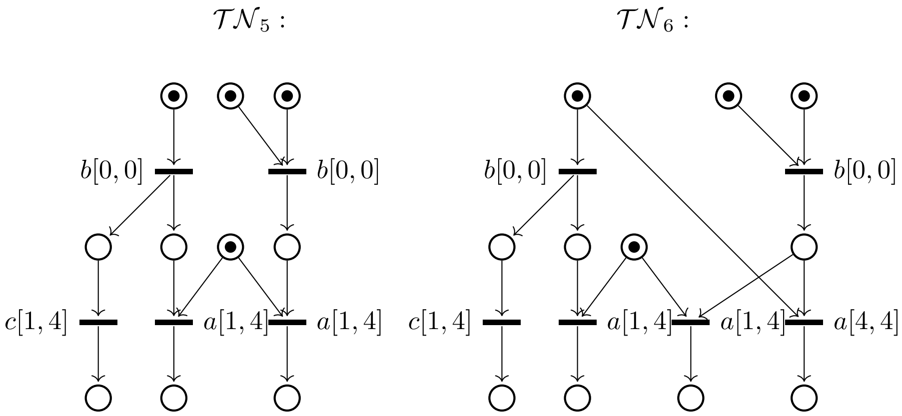

Figure 4: The $\sim_\star^\star$–equivalent TPNs $\mathcal{TN}_5$ and $\mathcal{TN}_6$, for $\star \in \{int, step, pos\}$.

there exists a time poset $TP' \in \mathbf{TP}$, a time process $\pi' = (TN', \varphi') \in \mathcal{CP}(\mathcal{TN})$, and an isomorphism $f' : \eta(TN') \longrightarrow TP'$, such that π' is a pos-extension of π and $f \subset f'$;

- *$\mathcal{TN}$ and $\mathcal{TN}'$ are* poset testing equivalent *(denoted $\mathcal{TN} \sim_{pos}^{pos} \mathcal{TN}'$) iff for all time posets TP and for all sets $\mathbf{TP} \subseteq \mathbf{TP}_{TP}$, it holds:*

$$\mathcal{TN} \textbf{ after } TP \textbf{ MUST}_{pos}^{pos}\mathbf{TP} \iff \mathcal{TN}' \textbf{ after } TP \textbf{ MUST}_{pos}^{pos}\mathbf{TP}.$$

Theorem 1. *Given time Petri nets $\mathcal{TN}$ and $\mathcal{TN}'$,*

$$\mathcal{TN} \sim_{pos}^{pos} \mathcal{TN}' \implies \mathcal{TN} \sim_{step}^{step} TN \implies \mathcal{TN} \sim_{int}^{int} \mathcal{TN}'.$$

The implications in the theorem above do not hold in the opposite directions, as demonstrated in the example below.

Example 3. *The time Petri nets $\mathcal{TN}_1$ and $\mathcal{TN}_2$, shown in Figure 2, are $\sim_{int}^{int}$–equivalent but they are neither $\sim_{step}^{step}$– nor $\sim_{pos}^{pos}$–equivalent. First, check that $\mathcal{TN}_1$ and $\mathcal{TN}_2$ are not $\sim_{step}^{step}$–equivalent. It is easy to see that $\mathcal{TN}_1$ **after** $w = (1'a + 1'b, 1)$ $\mathbf{MUST}_{step}^{step}$ $W = \emptyset$, because in $\mathcal{FS}_s(\mathcal{TN}_1)$ there is no firing sequence σ such that $L(\sigma) = w$. However, this is not the case in $\mathcal{TN}_2$, since in $\mathcal{FS}_s(\mathcal{TN}_2)$ there exists a firing sequence σ such that $L(\sigma) = w$ and it is impossible to find any element (A, θ) of W so as to locate in $\mathcal{FS}_s(\mathcal{TN}_2)$ a firing sequence $\sigma(U, \theta)$ such that $L(\sigma(U, \theta)) = w(A, \theta)$. Hence, it hods that $\neg(\mathcal{TN}_2$ **after** $w = (1'a + 1'b, 1)$ $\mathbf{MUST}_{step}^{step}$ $W = \emptyset)$. Second, verify that $\mathcal{TN}_1$ and $\mathcal{TN}_2$ are not $\sim_{pos}^{pos}$–equivalent. Define a poset $TP = (\{x_1, x_2\}, \preceq, \lambda, \tau)$ (with $\preceq = \{(x_1, x_1), (x_2, x_2)\}$, $\lambda(x_1) = a$, $\lambda(x_2) = b$, $\tau(x_1) = \tau(x_2) = 1$). For any time process $\pi_1 = (TN_1, \varphi_1) \in \mathcal{CP}(\mathcal{TN}_1)$, there is no isomorphism $f_1 : \eta(TN_1) \longrightarrow TP$. So, it is true that $\mathcal{TN}_1$ **after** TP $\mathbf{MUST}_{pos}^{pos}$ $\mathbf{TP} = \emptyset$. However, this is not the case in $\mathcal{TN}_2$ because there is a time process $\pi_2 = (TN_2, \varphi_2) \in \mathcal{CP}(\mathcal{TN}_2)$, with E_{TN_2} containing two concurrent events labeled by a and b, both with time value 1, and an isomorphism $f_2 : \eta(TN_2) \longrightarrow TP$, and we cannot find any pos-extension of TP in $\mathbf{TP}$. Hence, it hods that $\neg(\mathcal{TN}_2$ **after** TP $\mathbf{MUST}_{pos}^{pos}$ $\mathbf{TP} = \emptyset)$.*

The time Petri nets $\mathcal{TN}_3$ and $\mathcal{TN}_4$, shown in Figure 3, are $\sim_{step}^{step}$–equivalent but not $\sim_{pos}^{pos}$–equivalent. Let's make sure of the latter. Define posets $TP = (\{x_1\}, \preceq, \lambda, \tau)$ (with $\preceq = \{(x_1, x_1)\}$, $\lambda(x_1) = b$, $\tau(x_1) = 0$) and $TP' = (\{x_1, x_2, x_3, x_4\}, \preceq', \lambda', \tau')$ (with $\preceq' = \{(x_i, x_i) \mid 1 \leq i \leq 4\} \cup \{(x_2, x_3)\}$, $\lambda'(x_1) = \lambda'(x_2) = b$, $\lambda'(x_3) = \lambda'(x_4) = a$, $\tau'(x_1) = \tau'(x_2) = 0$, and $\tau'(x_3) = \tau'(x_4) = 2.9$. It is easy to see that TP' is a pos-extension of TP. For any time process $\pi_1 = (TN_1, \varphi_1) \in \mathcal{CP}(\mathcal{TN}_3)$, with E_{TN_1} consisting of an event with label b and time value 0, and any isomorphism $f_1 \colon \eta(TN_1) \longrightarrow TP$, we can find a pos-extension $\pi_1' = (TN_1', \varphi_1') \in \mathcal{CP}(\mathcal{TN}_1)$, with $E_{TN_1'}$ consisting of two concurrent events, both with label b and time value 0, and two concurrent events, both with label a and time value 2.9, and, moreover, one of the two events labeled by a is causally preceded by the added event labeled by b, and an isomorphism $f_1' \colon \eta(TN_1') \longrightarrow TP'$ such that $f_1 \subset f_1'$. But this is not the case in $\mathcal{TN}_4$.

The time Petri nets $\mathcal{TN}_5$ and $\mathcal{TN}_6$, depicted in Figure 4, are $\sim_\star^\star$–equivalent, for $\star \in \{int, step, pos\}$. □

At last, the definition of testing equivalence based on the causal trees of TPNs is developed. In doing so the experiments are considered as sequences over the alphabet $(Act \times \mathbb{T} \times 2^{\mathbb{N}})$ (corresponding to labeled paths from the roots in the causal trees) and the tests are specified as sets of non-empty sequences over the same alphabet (corresponding to sets of extensions of the labeled paths in the causal trees). In the paper [14], the tests directly extend the experiments by single elements, not by sequences of elements, from the set $(Act \times \mathbb{T} \times 2^{\mathbb{N}})$.

Definition 8. *Given time Petri nets $\mathcal{TN}$ and $\mathcal{TN}'$ with their time causal trees $TCT(\mathcal{TN})$ and $TCT(\mathcal{TN}')$, respectively,*

- *for a sequence $w \in (Act \times \mathbb{T} \times 2^{\mathbb{N}})^*$ and a set $\mathbf{W} \subseteq (Act \times \mathbb{T} \times 2^{\mathbb{N}})^+$, we say $TCT(\mathcal{TN})$* **after** w $\mathbf{MUST}_{ct}^{ext}$ $\mathbf{W}$ *iff for all paths u in $TCT(\mathcal{TN})$ from its root to a node n such that $\phi(u) = w$, there exists $w' \in \mathbf{W}$ and a path u' starting from the node n, such that $\phi(u') = w'$;*
- *$\mathcal{TN}$ and $\mathcal{TN}'$ are causal tree testing equivalent ($\mathcal{TN} \sim_{ct}^{ext} \mathcal{TN}'$) iff for all sequences $w \in (Act \times \mathbb{T} \times 2^{\mathbb{N}})^*$ and sets $\mathbf{W} \subseteq (Act \times \mathbb{T} \times 2^{\mathbb{N}})^+$, it holds:*

$$TCT(\mathcal{TN}) \textbf{ after } w \, \mathbf{MUST}_{ct}^{ext}\mathbf{W} \iff TCT(\mathcal{TN}') \textbf{ after } w \, \mathbf{MUST}_{ct}^{ext}\mathbf{W}.$$

We finally expand the main result of [14] by establishing the coincidence of poset and causal tree testing equivalences with extended tests, in the setting of TPNs.

Theorem 2. *Given time Petri nets $\mathcal{TN}$ and $\mathcal{TN}'$,*

$$\mathcal{TN} \sim_{pos}^{pos} \mathcal{TN}' \iff \mathcal{TN} \sim_{ct}^{ext} \mathcal{TN}'.$$

4. Concluding Remarks

We have specified and studied several testing equivalences based on concurrent semantics, in the setting of contact-free time Petri nets. In doing so, we dealt with various conceptions of the behavior of the time Petri net: interleaving/step firing sequences, time processes, from causal nets of which partial orders are derived, and time causal tree, constructed from interleaving firing sequences and partial orders. We have demonstrated that interleaving testing equivalence (with the experiments as labeled interleaving firing sequences and with the tests as experiments extensions by single actions with their times) is coarser than step testing (with the experiments as labeled step firing sequences and the tests as sequences extensions by steps of concurrent actions with their times), which, in turn, is coarser than poset testing (with time posets as experiments and their pos-extensions as tests). As the main result, the latter equivalence has been established to coincide with causal tree testing (based on labeled paths and their extensions in time causal trees of TPNs).

As for future work, we plan to investigate the equivalences and semantics under consideration in the framework of Petri nets with weak timing policy [18]. Also, it would be interesting to see whether open intervals in the specification of TPNs influence the results obtained here.

References

[1] R. de Nicola, M. Hennessy, Testing equivalence for processes, in: J. Díaz (Ed.), Automata, Languages and Programming, 10th Colloquium, Barcelona, Spain, July 18-22, 1983, Proceedings, volume 154 of *Lecture Notes in Computer Science*, Springer, 1983, pp. 548–560. URL: https://doi.org/10.1007/BFb0036936. doi:10.1007/BFb0036936.

[2] R. de Nicola, Extensional equivalences for transition systems, Acta Informatica 24 (1987) 211–237. URL: https://doi.org/10.1007/BF00264365. doi:10.1007/BF00264365.

[3] R. Cleaveland, M. Hennessy, Testing equivalence as a bisimulation equivalence, Formal Aspects Comput. 5 (1993) 1–20. URL: https://doi.org/10.1007/BF01211314. doi:10.1007/BF01211314.

[4] L. Pomello, G. Rozenberg, C. Simone, A survey of equivalence notions for net based systems, in: G. Rozenberg (Ed.), Advances in Petri Nets 1992, The DEMON Project, volume 609 of *Lecture Notes in Computer Science*, Springer, 1992, pp. 410–472. URL: https://doi.org/10.1007/3-540-55610-9_180. doi:10.1007/3-540-55610-9_180.

[5] L. Aceto, History preserving, causal and mixed-ordering equivalence over stable event structures, Fundam. Informaticae 17 (1992) 319–331.

[6] L. Aceto, R. de Nicola, A. Fantechi, Testing equivalences for event structures, in: M. V. Zilli (Ed.), Mathematical Models for the Semantics of Parallelism, Advanced School, Rome, Italy, September 24 - October 1, 1986, Proceedings, volume 280 of *Lecture Notes in Computer Science*, Springer, 1986, pp. 1–20. URL: https://doi.org/10.1007/3-540-18419-8_9. doi:10.1007/3-540-18419-8_9.

[7] U. Goltz, H. Wehrheim, Causal testing, in: W. Penczek, A. Szalas (Eds.), Mathematical Foundations of Computer Science 1996, 21st International Symposium, MFCS'96, Cracow, Poland, September 2-6, 1996, Proceedings, volume 1113 of *Lecture Notes in Computer Science*, Springer, 1996, pp. 394–406. URL: https://doi.org/10.1007/3-540-61550-4_165. doi:10.1007/3-540-61550-4_165.

[8] R. Cleaveland, A. E. Zwarico, A theory of testing for real-time, in: Proceedings of the Sixth Annual Symposium on Logic in Computer Science (LICS '91), Amsterdam, The

Netherlands, July 15-18, 1991, IEEE Computer Society, 1991, pp. 110–119. URL: https://doi.org/10.1109/LICS.1991.151635. doi:10.1109/LICS.1991.151635.

[9] M. Hennessy, T. Regan, A process algebra for timed systems, Inf. Comput. 117 (1995) 221–239. URL: https://doi.org/10.1006/inco.1995.1041. doi:10.1006/inco.1995.1041.

[10] F. Corradini, W. Vogler, L. Jenner, Comparing the worst-case efficiency of asynchronous systems with PAFAS, Acta Informatica 38 (2002) 735–792. URL: https://doi.org/10.1007/s00236-002-0094-3. doi:10.1007/s00236-002-0094-3.

[11] L. F. L. Díaz, D. de Frutos-Escrig, Denotational semantics for timed testing, in: M. Bertran, T. Rus (Eds.), Transformation-Based Reactive Systems Development, 4th International AMAST Workshop on Real-Time Systems and Concurrent and Distributed Software, ARTS'97, Palma, Mallorca, Spain, May 21-23, 1997, Proceedings, volume 1231 of *Lecture Notes in Computer Science*, Springer, 1997, pp. 368–382. URL: https://doi.org/10.1007/3-540-63010-4_25. doi:10.1007/3-540-63010-4_25.

[12] E. Bihler, W. Vogler, Timed Petri nets: Efficiency of asynchronous systems, in: M. Bernardo, F. Corradini (Eds.), Formal Methods for the Design of Real-Time Systems, International School on Formal Methods for the Design of Computer, Communication and Software Systems, SFM-RT 2004, Bertinoro, Italy, September 13-18, 2004, Revised Lectures, volume 3185 of *Lecture Notes in Computer Science*, Springer, 2004, pp. 25–58. URL: https://doi.org/10.1007/978-3-540-30080-9_2. doi:10.1007/978-3-540-30080-9_2.

[13] M. V. Andreeva, I. B. Virbitskaite, Observational equivalences for timed stable event structures, Fundam. Informaticae 72 (2006) 1–19. URL: http://content.iospress.com/articles/fundamenta-informaticae/fi72-1-3-02.

[14] E. N. Bozhenkova, I. B. Virbitskaite, Testing equivalences of time Petri nets, Program. Comput. Softw. 46 (2020) 251–260. URL: https://doi.org/10.1134/S0361768820040040. doi:10.1134/S0361768820040040.

[15] T. Aura, J. Lilius, A causal semantics for time Petri nets, Theor. Comput. Sci. 243 (2000) 409–447. URL: https://doi.org/10.1016/S0304-3975(99)00114-0. doi:10.1016/S0304-3975(99)00114-0.

[16] I. B. Virbitskaite, D. Bushin, E. Best, True concurrent equivalences in time Petri nets, Fundam. Informaticae 149 (2016) 401–418. URL: https://doi.org/10.3233/FI-2016-1454. doi:10.3233/FI-2016-1454.

[17] P. Darondeau, P. Degano, Refinement of actions in event structures and causal trees, Theor. Comput. Sci. 118 (1993) 21–48. URL: https://doi.org/10.1016/0304-3975(93)90361-V. doi:10.1016/0304-3975(93)90361-V.

[18] P. Reynier, A. Sangnier, Weak time Petri nets strike back!, in: M. Bravetti, G. Zavattaro (Eds.), CONCUR 2009 - Concurrency Theory, 20th International Conference, CONCUR 2009, Bologna, Italy, September 1-4, 2009. Proceedings, volume 5710 of *Lecture Notes in Computer Science*, Springer, 2009, pp. 557–571. URL: https://doi.org/10.1007/978-3-642-04081-8_37. doi:10.1007/978-3-642-04081-8_37.