METAGRAPHS WITH TIME-LOGICAL RESTRICTIONS*

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1. Introduction

In recent years, complex graphs with non-standard, granular, hierarchical primitives have been increasingly used in modeling multi-agent systems and network messages [20]. This trend is connected, in particular, with the need for a visual description of the relationships between multiple (and not just pairs) of vertices. One of the earliest examples of complex graphs are hypergraphs, which promise images of a graph in the case when each edge can connect not two, but a favorite number of vertices [2]. Such an edge with a plurality of endpoints is called a hyper-edge.

In this paper, we will consider metagrams, the vertices of which can serve as both single objects (or agents) and a plurality of objects. The concept of a metagraph was introduced by A. Baza and R. Blenning in 1992 [10, 11] as a graph structure in which each vertex is a multiplicity consisting of one or a large number of elements, and edges characterize directed relations between pluralities.

Base-Blanning metagraphs are extensions of both directed graphs (one or more elements in the upper metagraph are allowed) and hypergraphs (directions in edges are introduced). In addition, a hyperreb in a hypergraph is associated only with vertices, whereas a metavershina in a metagraph can include vertices, other metavershina, and edges.

The basics of the theory and a number of attached metagraphs are presented in the book [14], in which the metagraph is generally given in the form of an ordered pair MG = (X, E), where X is the generating multiplication, and E is the multiplication of edges defined on the generating multiplication.

In [1], a hierarchical generating multiplicity is explicitly taken, containing both vertices v and metavershins mv, i.e. $X=V \cup MV$. In [6], various types of odd generating sets are considered and connections between fuzzy graphs and metagraphs are established using the representation theorem. Finally, a model of a recursive annotated metagraph with generating properties is developed in [5].

Bas-Blanning's works of the 1990s are devoted to the use of metagraphs in enterprise modeling [11], decision support [13], creating relationships between metagraphs and Petri nets [12]. Their theoretical results, presented in the books [14], are mainly related to statements about their expressed in terms of paths and metapaths in metagraphs, as well as with various transformations of metagraphs. Attachment areas cover four topics: (1) data relationship models; (2) decision support models; (3) models for Rule - Based Systems; (4) Models for the Workflow Systems system. The publications [9, 10, 15-21] consider a number of applications in economics (stock markets), business (presentation and analysis of business processes), medicine, heterogeneous information networks (solving problems of making recommendations), etc.

2. BASIC DEFINITIONS

We will first give visual representations of the connection between ordinary graphs and metagraphs. So on the trot.1C depicted a metagraph that was taken from the oriented graph shown on the trot.1B. on the trot.1d is given the second image of this metagraph. This metagraph is oriented – its long one is assigned to ordered pairs of metavershins - (finite) multiplication of vertices. If we take unordered pairs of vertices, we get an undirected metagraph. For example, from the pictured to the lynx.1a of an undirected graph, an undirected metagraph is obtained if the ordered pairs of meta-vertices are replaced by unordered ones.

Definition 1. Formally, a (oriented) metagraph is a four

$$MG = (V, H, E, \rho), (1)$$

where $V = \{v_1, v_2,..., v_n\}$ is a multiplication of vertices, $E=\{E_1, E_2,..., e_m\}$ is the multiplication of debt, $H \subseteq 2^V$, the elements of H are called multiplication metaarsenite metagraph ρ is a function $E \rightarrow H \times H$, representing a pair of $\rho(e_j) \in H \times H$ Metaverse her long each $e \in E$.

The beginning and end of the long EJ metagraph MG are the ego metapersons e_j^- and e_j^+ + such that $\rho(e_j) = (e_j^-, e_j^+)$.

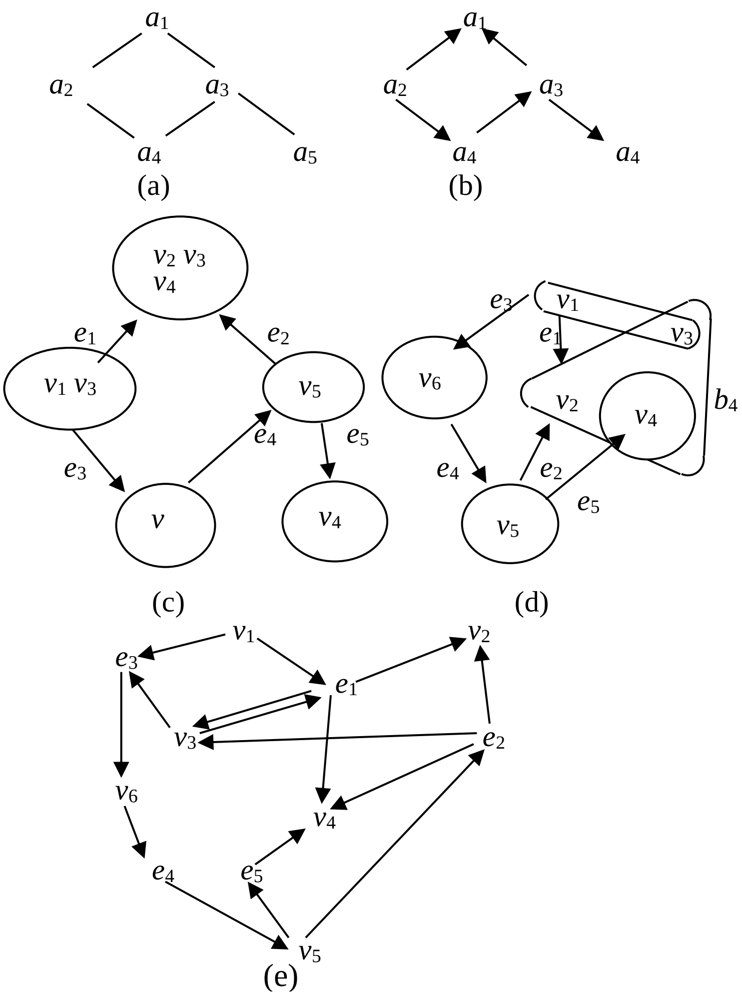

Figure 1. (a) Undirected graph; (b) Oriented graph; (c) metagraph; (d) The image of the Basu and Blanning metagraph; (e) The bipartite graph G (MG) for the MG metagraph

Example 1. Let's say

$V = \{v_1, v_2, v_3, v_4, v_5, v_6\}$, $E = \{e_1, e_2, e_3, e_4, v_5\}$,

$H = \{\{v_2, v_3, v_4\}, \{v_1, v_3\}, \{v_4\}, \{v_5\}, \{v_4\}\}$,

$\rho(e_1) = (\{v_1, v_3\},\{v_2, v_3, v_4\})$, $\rho(e_2) = (\{v_5\}, \{v_2, v_3, v_4\})$,

$\rho(e_3) = (\{v_1, v_3\}, \{v_6\})$, $\rho(e_4) = (\{v_6\}, \{v_5\})$, $\rho(e_5) = (\{v_5\}, \{v_5\})\}$.

This metagraph (V, H,E, p) is shown in Figure 1 (c) and (d). For its arcs, the beginnings and ends are $e_1^- = \{v_1, v_3\}$, $e_1^+ =\{v_2, v_3, v_4\}$, $e_2^- = \{v_5\}$, $e_2^+ =\{v_2, v_3, v_4$, etc.

The metagraph MG can also be represented by an oriented bipartite graph, denoted by G (MG). The vertices of the graph G (MG) = (U, F) are the vertices and arcs of the metagraph, i.e. $U = V \cup E$ (where V is one fraction of the metagraph and E is another fraction), and the arcs of the graph G (MG) are determined by the following condition: $(v_i,e_j) \in F$ e_j^- и $(e_j,v_i) \in F$ $e_j^+ \in F$ e_j^-

In typical graph applications, their vertices often denote actions, works, events, in a word, objects that change over time. In general, the concept of temporal graphs is interpreted quite widely: from time graphs to oriented acyclic graphs and Petri nets. Temporal graphs and their applications to the modeling of complex systems are described, for example, in [3, 17].

In this paper, we will consider metagraphs with which temporal intervals are associated. For such objects, the concept of a temporal metagraph (T-metagraph) is introduced.

Previously, we recall that the elementary relations between temporal intervals were first considered by James Allen (J. F. Allen) in the fundamental work [2], where he introduced the logic of intervals, which found numerous applications in solving computer science problems (and especially artificial intelligence problems). Table 1 shows the 7 main relations of J. Allen between intervals.

In Allen's logic, intervals contain only qualitative information. To represent the quantitative dependencies between intervals, metric information is added to the atomic sentences of Allen's logic. Deduction algorithms for extensions of Allen's interval logic are considered in [4].

We will describe the temporal interval by a pair of natural numbers (a,b) with the condition a <b. Let T be the set of all temporal intervals.

Definition 2. The T-metagraph is defined by the five

$$TMG = (V, H, E, p, \tau), (2)$$

where (V, H, E, p) is a metagraph by definition 1 and τ: V→T is a partial function that assigns temporal intervals to the vertices of the metagraph. The vertices vi∈V for which the values of τ (vi) are not defined are considered variables that take values in the set T of all intervals.

Table 1.

Elementary relations between intervals

Allen's attitude	***Title***	Illustration
X ***b*** *Y*	Before	|====*X*====| |===*Y*===| $X^+ < Y^-$
X ***m*** *Y*	Meets	|====*X*====|===*Y*===| $X^+ = Y^-$
X ***s*** *Y*	Starts	|====*X*====| |======*Y*=====| $X^- = Y^-$, $X^+ < Y^+$
X ***f*** *Y*	Finishes	|====*X*====| |=====*Y*=====| $X^- < Y^-$, $X^+ = Y^+$
X ***o*** *Y*	Overlaps	|===*X*===| |===*Y*===| $Y^- < X^-$, $X^- < Y^+$, $Y^+ < X^+$
X d Y	During	|==*X*==| |=====*Y*=====| $Y^- < X^-$, $X^+ < Y^+$
A e B	Equals	|====*A*====| |====*B*====| $X^- = Y^-$, $X^+ = Y^+$

Example 2. Let MG be the metagraph from Example 1. Let's put Положим $\tau(v_2) = (1, 4)$, $\tau(v_4) = (2, 7)$ and $\tau(v_6) = (5, 8)$. Then, adding the function τ to the metagraph, we get a T-metagraph. The vertices v_1, v_3, and v_5 in this T-metagraph are variables.

Next, we will use the following notation: |X| is the length of the temporal interval X, and **b** and **o** are names for elementary temporal relations: X **b** Y means that the interval X lies before the interval Y, and X **o** Y means that the interval X overlaps the interval Y.

The intervals τ (v_2), τ (v_4) and τ (v_6) have corresponding lengths and are in the following temporal relations:

- $|\tau(v_2)| = |(1, 4)| = 3$, $|\tau(v_4)| = |(2, 7)| = 5$, $|\tau(v_6)| = |(5, 8)| = 3$;
- • τ(v_2) b τ(v6), since the interval (1, 4) is located before (to the left) of the interval (5, 8));
- τ(v_2) on τ(v_4), since the interval (1, 4) covers the interval (2, 7).

By definition 2 T-metagraph TMG contains vertices that are variable (as a function of τ is not everywhere defined). Then TMG will actually be a schema from which specific T-metagraphs are obtained by assigning specific values to all its variables.

An application modeled using a T-metagraph has semantics that can be defined using restrictions on the values of temporal variables. To formally express these restrictions, it is natural to use interval logics. Suppose, for example, that in a multi-agent system, one agent can perform actions a and b, and another agent can perform action c. Let these actions be represented in a T-metagraph by vertices-variables with which the temporal intervals A, B and C are associated. Let's also assume that the following statement is true for the simulated application: "If action a is performed no earlier than action b, then there is a moment when actions a and c occur simultaneously." Then, in interval logic with Allen and propositional connectives, this knowledge can be expressed by the formula ~ A b B → A o C.

In this paper, we will define 2 interval logics IL-1 and IL-2 and show (by examples) how knowledge in the form of constraints for a T-metagraph can be represented in these logics. We will also define an output method based on analytical tables in these logics.

3. INTERVAL LOGICS

The interval logics IL-1 and IL-2 will be used to specify temporal-logical constraints in T-metagraphs.

3.1. IL-1 Logic

The propositions (formulas) of the IL-1 and IL-2 logics are constructed on the basis of a signature, which is a finite set of propositional variables and interval variables – names for temporal intervals. Using IL-1 (∑) and IL-2 (∑), we also denote the sets of all sentences of these logics written in the signature ∑.

Table 1 above shows the Allen relations b, m, s, f, o, d and e with an indication of their meaning. Let

$$Ω = \{b, b^*, m, m^*, s, s^*, f, f^*, o, o^*, d, d^*, e\},$$

where * denotes the inversion of the binary relation, i.e. X θ*Y □ Y θ X.

Atomic sentences from IL-1 (∑) are expressions of the form X θ Y, |X| ≥ r and X| ≤ r, where θ∈Ω, *X, Y* ∈ ∑and r are a natural number or an arithmetic term with variables taking natural values. Arbitrary sentences (formulas) of the IL-1 (∑) logic are constructed inductively according to the following rules:

- propositional variables belong to IL-1 (∑);
- atomic sentences belong to IL-1 (∑);
- suggestions ~φ, (φ ∧ ψ), (φ ∨ ψ) and (φ →ψ) belongs to IL-1 (σ), if φ and ψ belong to the IL-1 (σ).

These three rules make up the syntax of the IL-1 (∑) logic. Its semantics are set using interpretations. The interpretation of the signature is an arbitrary function"•": ∑ → {0,1} ∪ T is such that "p "∈ {0,1} for each propositional variable p ∈ ∑ and "X " "T for each name of the interval X ∈ T. The interpretation of atomic sentences is obtained by extending the function " • "to arbitrary sentences according to the rules:

- •" X θ Y "= 1 □ "X" θ" Y";
- • "| X | ≥ r "= 1 □ | "X "| ≥ r □ b– a ≥ r if" X " = (a, b);
- • "| X | ≤ r "= 1 □ | "X "| ≤ r □ b– a ≤ r if" X " = (a, b);
- • "| X | = r "= 1 □ "| X | ≥ r "= 1 and "| X | ≤ r " = 1.

Finally, continuing the function " • " according to the rules

"~ φ"= 1 □ "φ"= 0, "(φ ∧ ψ)" = 1 □ " φ" = 1 and "ψ" = 1,

"(φ ∨ ψ)" = 1 □ " φ" = 1 or "ψ" = 1,

"(φ → ψ)" = 1 □ " φ" = 1 or "ψ" = 1,

we get an interpretation of arbitrary formulas from IL-1 (∑).

Like any logic, IL-1 induces the ' | = ' relation of the logical sequence, which is defined in the usual way. Let E be an arbitrary set of formulas from IL-1 and ψ be any formula from IL-1. Then ψ logically follows from E if and only if there is no interpretation in which all the formulas from E are true, but the formula ψ is false:

E $\models \psi$ □ there is no interpretation of " * " such,

that "ψ" = 0 and " φ" = 1 for every formula φ E. E.

The concept of logical impracticability is closely related to the concept of logical consequence. The set of E formulas of logic is called impossible if there is no interpretation in which all the formulas from E are true. It is easy to see that (if the logic contains negation), then E $\models \varphi$ if and only if the set E $\cup$ $\{\sim\varphi\}$ is impossible.

To recognize the impracticability, we will use an algorithm based on the method of analytical tables [7]. This algorithm builds a tree whose vertices are the indicated formulas and inequalities. The indicated formulas have the following values when interpreting"•":" + φ " = " φ " and:" – φ " = " ~ φ".

Example 3. Even in a T-metagraph three vertices (say, v_1, v_2, and v_3) are the intervals A, b and C, have the following restrictions

$$p \wedge A\,\boldsymbol{s}\,C,\;\; \sim p \rightarrow (B\,\boldsymbol{d}\,C) \wedge (A\,\boldsymbol{b}\,B),\; |A| \geq a,\; |B| \geq b, \qquad (3)$$

where it is assumed that $a \geq 1$, $b \geq 1$. We prove that these constraints imply the constraint $|C / \geq a+b+2$, i.e. O $\models |C| \geq a+b+2$, where O is an "ontology" composed of formulas (3) (here the term "ontology" means a finite set of sentences interpreted as constraints). To do this, applying the output rules from the Table.2 and Table.3, we have built a tree (Figure 2).

$+ (p \wedge (A\,\boldsymbol{s}\,C))$ [1, T2, 3]

$+ (p \rightarrow ((B\,\boldsymbol{d}\,C) \wedge (A\,\boldsymbol{b}\,B))$ [6, T2, 7]

$+ (|A| \geq a)$ [2, T3, 3]

$+ (|B| \geq b)$ [3, T3, 3]

$- (|C| \geq a+b+2)$ [4, T3, 4]

1: $+p$ [7]

1: $+ (A\,\boldsymbol{s}\,C)$ [5, T2, 17]

2: $A^{+} - A^{-} \geq a$

3: $B^{+} - B^{-} \geq b$

4: $C^{-} - C^{+} \geq - a - b - 1$

5: $A^{-} = C^{-}$

5: $C^{+} - A^{+} \geq 1$

Left branch:

6: $-p$ [7]

7: X

Right branch:

6: $+ (B\,\boldsymbol{d}\,C) \wedge (A\,\boldsymbol{b}\,B))$ [8,T2, 3]

8: $+ B\,\boldsymbol{d}\,C$ [9, T2, 19]

8: $+ A\,\boldsymbol{b}\,B$ [10, T2, 9]

9: $B^{-} - C^{-} \geq 1$

9: $C^{+} - B^{+} \geq 1$

10: $B^{-} - A^{+} \geq 1$

Figure 2. The output tree for proving the logical consequence from Example 3

The construction of the tree began with a branch containing the plus-sign formulas of the ontology O, as well as the indicated formula $- (|C| \geq a+b+2)$.

Table 2.
Inference rules for sentences with Allen relations and for inequalities

Number	Antecedent	Consequences
1	$+ \sim \varphi$	$- \varphi$
2	$- \sim \varphi$	$+ \varphi$
3	$+ (\varphi \wedge \psi)$	$+ \varphi,\ +\psi$
4	$- (\varphi \wedge \psi)$	$- \varphi \ \vert - \psi$
5	$+ (\varphi \vee \psi)$	$+ \varphi \ \vert + \psi$
6	$- (\varphi \vee \psi)$	$- \varphi, - \psi$
7	$+ (\varphi \rightarrow \psi)$	$- \varphi \ \vert + \psi$
8	$- (\varphi \rightarrow \psi)$	$+ \varphi,\ - \psi$
9	$+ (X\ \boldsymbol{b}\ Y)$	$Y^{-}–X^{+} \geq 1$
10	$- (X\ \boldsymbol{b}\ Y)$	$X^{+}–Y^{-} \geq 0$
11	$+ (X\ \boldsymbol{m}\ Y)$	$X^{+}= Y^{-}$
12	$- (X\ \boldsymbol{m}\ Y)$	$X^{+}–Y^{-} \geq 1 \mid Y^{-}–X^{+} \geq 1$
13	$+ (X\ \boldsymbol{o}\ Y)$	$Y^{-}–X^{-} \geq 1, X^{+}–Y^{-} \geq 1, Y^{+}–X^{+} \geq 1$
14	$- (X\ \boldsymbol{o}\ Y)$	$X^{-}–Y^{-} \geq 0 \mid Y^{-}–X^{+} \geq 0 \mid X^{+}–Y^{+} \geq 0$
15	$+ (X\ \boldsymbol{f}\ Y)$	$X^{-}–Y^{-} \geq 1,\ X^{+}= Y^{+}$
16	$- (X\ \boldsymbol{f}\ Y)$	$Y^{-}–X^{-} \geq 0 \mid X^{+}–Y^{+} \geq 1 \mid Y^{+}–X^{+} \geq 1$
17	$+ (X\ \boldsymbol{s}\ Y)$	$X^{-}= Y^{-}, Y^{+}–X^{+} \geq 1$
18	$- (X\ \boldsymbol{s}\ Y)$	$X^{-}–Y^{-} \geq 1 \mid Y^{+}–X^{+} \geq 1 \mid X^{+}–Y^{+} \geq 0$
19	$+ (X\ \boldsymbol{d}\ Y)$	$X^{-}–Y^{-} \geq 1, Y^{+}–X^{+} \geq 1$
20	$- (X\ \boldsymbol{d}\ Y)$	$Y^{-}–X^{-} \geq 0 \mid X^{+}–Y^{+} \geq 0$
21	$+ (X\ \boldsymbol{e}\ Y)$	$X^{-}= Y^{-}, X^{+}= Y^{+}$
22	$- (X\ \boldsymbol{e}\ Y)$	$Y^{-}–X^{-} \geq 1 \mid Y^{-}–X^{-} \geq 1 \mid Y^{+}–X^{+} \geq 1 \mid Y^{+}–X^{+} \geq 1$
23	$+ (X\ \theta^{*} Y)$	$+ (Y\ \theta\ X)$
24	$- (X\ \theta^{*} Y)$	$- (Y\ \theta\ X)$
25	$+ X – Y \geq a$	$X – Y \geq a$
26	$– X – Y \geq a$	$Y – X \geq 1– a$
27	$+ X – Y \leq a$	$X – Y \geq – a$
28	$– X – Y \leq a$	$Y – X \geq 1+ a$

It is clear that this logical consequence is valid if and only if the set of formulas belonging to the initial branch is impossible.

At the first step of the output, rule number 3 from table 3 was applied to the formula + (p ∧ (A s C)). The fact that this application was performed at step 1 according to rule 3 from table 2 is shown by using the label ‘[1, T2, 3]’, standing to the right of this formula. The result of this application is the formulas + p and + (A s C), which with the left label ‘1:’ were added one by one to the initial branch (this label indicates that the formulas were attached in step 1)

In step 6, rule 7 from Table 2 was applied to the formula + (p → (A b C) ∧ (A b B)). This rule is disjunctive in the sense that it gives an alternative of two formulas-p and + ((A d C) ∧ (A b B)). The result of applying the rule is the addition of a "fork" of these two formulas.

Table 3. Output rules for sentences with metric constraints

Number	**Antecedent**	**Consequences**
1	$+ (\lvert X\rvert \le a)$	$X^{-} - X^{+} \ge - a$
2	$- (\lvert X\rvert \le a)$	$X^{+} - X^{-} \ge 1+a$
3	$+ (\lvert X\rvert \ge a)$	$X^{+} - X^{-} \ge a$
4	$- (\lvert X\rvert \ge a)$	$X^{-} - X^{+} \ge 1 - a$
5	$+ (\lvert X\rvert = a)$	$X^{+} - X^{-} \ge a,\ X^{-} - X^{+} \ge - a$
8	$- (\lvert X\rvert = a)$	$X^{-} - X^{+} \ge 1 - a\ \mid A^{+} - A^{+} \ge 1+ a$
9	$+ X\ \theta(\sigma)\ Y$	$+ X\ \theta\ Y,\ + \theta \bullet \sigma / A, B$
10	$- A\ \theta(\tau)\ Y$	$- A\ \theta\ B\ \mid - \theta \bullet \sigma$
11	$+ \lambda;\sigma$	$+ \lambda, + \sigma$
12	$- \lambda;\sigma$	$- \lambda\ \mid - \sigma$

In general, if we consider the application of a conjunctive (or disjunctive) rule to a formula φ made up of several subformules φ_i, then the result of its application is the addition to each branch of the formulas φ_i passing through φ, written one after the other (or "forks" of the formulas φ_i).

At step 6, two branches were formed. We see that the first (left) branch contains a contrarian pair (+p,–p), which means that the set of formulas of this branch is impossible. The second (right) branch is also impossible. To prove this, we write out from this branch all inequalities of the form X-Y ≥r, adding to this list the inequalities $A^{-}–C^{-} \ge 0$ and $C^{-}– A^{-} \ge 0$ (which replace the equality $A^{-} = C^{-}$), as well as the inequalities $A^{+}–A^{-} \ge 1$, $B^{+}–B^{-} \ge 1$, $C^{+}– C^{-} \ge$ (which are valid by definition):

B = $\{A^{+}– A^{-} \ge a,\ B^{+}– B^{-} \ge b,\ C^{-}– C^{+} \ge – a – b – 1,\ C^{+}–A^{+} \ge 1,$
$B^{-}– C^{-} \ge 1,\ C^{+}– B^{+} \ge 1,\ B^{-}– A^{+} \ge 1,\ A^{-}– C^{-} \ge 0,$
$C^{-}– A^{-} \ge 0,\ A^{+}– A^{-} \ge 1,\ B^{+}– B^{-} \ge 1,\ C^{+}– C^{-} \ge 1\}.$

This list is interpreted as a conjunction, therefore, the inequalities $A^{+}–A^{-} \ge 1$, $B^{+}–B^{-} \ge 1$ can be excluded from it, since (given that a ≥ 1 and b ≥ 1) The inequality $A^{+}– A^{-} \ge a$ absorbs the inequality $A^{+}–A^{-} \ge 1$, and $B^{+}– B^{-} \ge a$ absorbs $B^{+}– B^{-} \ge 1$.

Then we construct the following directed graph G (B) with marked arcs (Figure 3). Its vertices are A^{+}, A^{-}, B^{+}, B^{-}, C^{-}, C^{+}, and the marked arcs are triples (X, r, Y) such that the inequality X-Y ≥ r is included in the set B. The equality $A^{-} = C^{-}$ corresponds to the edge connecting A^{-} and C^{-}. Note that an edge as an unordered pair $[A^{-},C^{-}]$ corresponds to two oppositely directed arcs as triples $(A^{-},0,C^{-})$ and $(C^{-},0,A^{-})$.

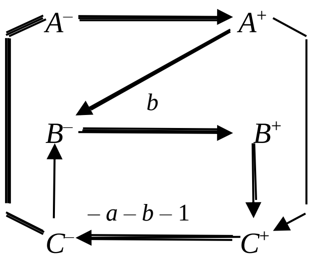

Figure 3.

Graph G (B) for the set of inequalities from Example 3

Graph G (B) has a cycle, shown in Figure 3 with bold lines

$$A^{+} (b)\ B^{-} (1)\ B^{+}(1)\ C^{+}(– a – b – 1)\ C^{-} (0)\ A^{-} (a)\ A^{+}.. \qquad (4)$$

The length of this cycle is 1, since b + 1 + 1 + (– a – b -1) + a = 1.

The cycle (4) corresponds to a sequence of inequalities

$$B^{-}– A^{+} \ge 1, B^{+}– B^{-} \ge b, C^{+}– B^{+} \ge 1, C^{-}–C^{+} \ge – a – b – 1, A^{-}– C^{-} \ge 0,$$

folding that get

$$(B^{-}– A^{+}) + (B^{+}– B^{-}) + (C^{+}– B^{+}) + (C^{-}– C^{+}) + (A^{-}– C^{-}) \ge$$
$$b + 1 + 1 + (– a – b –1) + a.$$

It can be seen that the left part of this inequality is equal to 0, and the right part is equal to 1. Thus, we have a contradiction of $0 \geq 1$. Hence, the set of inequalities lying on the second branch of the output tree is impossible.

Let us consider a general situation when an arbitrary (finite) set E of the indicated formulas is given, and we must find out whether the set E is impossible.

To do this, we build a tree Tr(E) using the set E, applying the rules from Table.2 and Table.3. (Note that rules 9-12 from Table 3 are not used in IL-1 logic.) Let $Br_1, Br_2, \ldots, Br_m$ be all branches of the tree Tr (E). From each branch of the Br_i tree Tr (E), we write out all inequalities of the form $X - Y \geq r$. Let $\mathbf{B}_i$ be the set obtained from the list of these inequalities by removing those of them that are absorbed by other inequalities that were included in the list. A branch of Br_i is called closed if the set of $\mathbf{B}_i$ is impossible. A tree Tr(E) is called closed if all its branches are closed.

In the future, the set $\mathbf{S}_1$ consisting of the rules of Table 2 and 3 (excluding rules 9-12) will be called the system of inference rules for IL-1 logic. The following theorem is valid.

Theorem 1. The system S_1 of inference rules for IL-1 logic is consistent and complete. This means that

- all rules from S1 are consistent;
- The set E of the indicated formulas of the IL-1 logic is impossible if and only if the tree Tr (E) is closed.

A rule of inference that preserves truth is called consistent, i.e. in any interpretation, if the antecedents are true, then the consequences are also true.

Theorem 1 is proved according to the standard scheme with the use of the concept of Hintikkov sets.

On the other hand, it is easy to prove that all the inference rules from Tables 2 and 3 are consistent. A rule of inference is called consistent if, under any interpretation, the consequences of the rule are true when its antecedents are true.

There is a simple and fast algorithm for finding out the impracticability (incompatibility) of a set of inequalities of the form $X - Y \geq r$, the idea of which is described above in Example 3. Using the set B, we construct a directed graph G (B). In this graph, we are looking for a positive cycle, i.e. a cycle whose length is a positive number. If there is no such cycle, then the set is impossible, otherwise it is feasible (jointly).

So, the algorithm that finds out the impracticability of the set E of the formulas of the IL-1 logic has the following stages:

1. Applying the output rules of system S_1 to the set E, we build an output tree Tr (E).
2. We make a list $\{Br_1, Br_2, \ldots, Br_m\}$ from those branches of the tree Tr(E) that do not end with the symbol 'X'.
3. From each branch of Br_i, we write out all inequalities of the form $X - Y \geq r$. Let B_i be the resulting set of inequalities.
4. Using the set B_i, we construct a graph G (B_i).
5. We apply an algorithm for finding a positive cycle to each graph G(B_i).
6. The message YES is issued if all graphs G (Bi) contain positive cycles; otherwise, the message NO is issued.

3.2. IL-2 logic

The IL-2 logic extends the IL-1 logic by including interval durations in the Allen relations. Let's consider what is shown in the Table.3 (left and top) the ratio b with intervals $I = X^+ - X^-$, $J = Y^- - X^-$, $K = Y^+ - X^+$, $L = Y^- - X^-$, $M = Y^+ - X^+$, $N = Y^+ - X^-$. Using these intervals, you can enter such temporal relations, such as:

$$\boldsymbol{b}(I = 2),\ \boldsymbol{b}(2 \leq I \leq 5; J \geq 3),\ \boldsymbol{b}(K \neq 4),$$

interpreted in the following way:

"$X\ \boldsymbol{b}(I = 2)\ Y$" = 1 □ "$X$" $\boldsymbol{b}$ "Y" $\wedge (X^+ - X^- = 2)$,

"$X\ \boldsymbol{b}(2 \leq I \leq 5; J \geq 3)\ Y$" = 1 □ "$X$" $\boldsymbol{b}$ "Y" $\wedge (X^+ - X^- \geq 2) \wedge$ $(X^+ - X^- \leq 5) \wedge (Y^- - X^- \geq 3)$,

"$X\ \boldsymbol{b}(K \neq 4)\ Y$" = 1 □ "$X$" $\boldsymbol{b}$ "Y" $\wedge [(Y^+ - X^+ < 4) \vee (Y^+ - X^+ > 4)]$.

The information contained in Table 4 about the relationships of the intervals I, J, K, L, M and N with the Allen relations can be displayed using the ‘ • ’ operation written in Table.5.

The IL-2 logic uses a system of output rules S_2, which contains rules 23-28 from Table 2 and rules from Table 2 in addition to the rules of the S_2 system.5.

In the logic of IL-2, as well as in the logic of IL-1, it is possible to solve typical problems for T-metagraphs:

A. Let O be an ontology (see above) representing knowledge about the simulated application using IL-2 logic. Find out if onotology is About impossible;

B. Let φ be a sentence in IL-2 logic. Find out whether it follows from the ontology O;

B. Let χ be a request to the ontology O. Calculate the answer to the request χ addressed to the ontology O.

Table 4.

Intervals for Allen relations

<table>
<tr>
<td>X b Y
X^- ===== X^+ Y^- ======Y^+
|------I-----|----J----|---- K -----|
$I = X^+ – X^-$, $J = Y^- – X^-$, $K = Y^+ – X^+$,
$L = Y^- – X^-$, $M = Y^+ – X^+$, N
$=Y^+ – X^-$</td>
<td>X m Y
X^- =======X^+
Y^- ======Y^+
|------I------|---- K -----|
$I = X^+ – X^-$, $J = 0$, $K = Y^+ – X^+$,
$N = Y^+ – X^-$</td>
</tr>
<tr>
<td>X d Y
X^- ==== X^+
Y^- ================Y^+
|-----I----|----J----|----K ----|
$I = X^- – Y^-$, $J = X^+ – X^-$, $K = Y^+ –$
X^+,
$L = X^+ – Y^-$, $M = Y^+ – X^-$, $N = Y^+ –$
X^+</td>
<td>X s Y
X^- =====X^+
Y^-=============Y^+
|------I----|----K-----|
$I = X^+ – X^-$, $J = 0$, $K = Y^+ – X^+$,
$N = Y^+ – X^-$</td>
</tr>
<tr>
<td>X o F
X^- ============X^+
Y^-===========Y^+
|-----I-----|-----J----|----K ---|
$I = Y^- – X^-$, $J = X^+ – Y^-$, $K = Y^+ –$
X^+,
$L = X^+ – X^-$, $M = Y^+ – X^+$, $N = Y^+ –$
X^-</td>
<td>X f F
X^- ======X^+
Y^-=============Y^+
|-----I-----|-----J-------|
$I = X^- – Y^-$, $J = Y^+ – X^-$, $K = 0$,
$N = F^+ – F^-$</td>
</tr>
</table>

Table 5.

"Multiplication table" for the '•'operation

	I	J	K	L	M	N
B	$X^+ – X^-$	$Y^- – X^+$	$Y^+ – X^+$	$Y^- – X^-$	$Y^+ – X$	$Y^+ – X^+$
M	$X^+ – X^-$	0	$Y^+ – X^+$	0	0	$Y^+ – X^-$
S	$X^+ – Y^-$	$X^+ – X^-$	$Y^+ – X^+$	$X^+ – Y^-$	$Y^+ – X^-$	$Y^+ – X^+$
F	0	$X^+ – X^-$	$Y^+ – X^+$	0	0	$Y^+ – Y^-$
O	$Y^- – X^-$	$X^+ – E^-$	$Y^+ – X^-$	$X^+ – X^-$	$Y^+ – X^+$	$Y^+ – X^-$
D	$X^- – Y^-$	$X^+ – X^-$	$Y^+ – X^+$	$X^+ – Y^-$	$Y^+ – X^-$	$Y^+ – X^+$
E	$X^+ – X^-$	0	0	0	0	0

Let's consider an example of solving problem B.

Example 4. Let there be vertices in some T-metagraph for actions a, b, c with time intervals A, B, C. Let the lengths of these intervals be 4, 6 and 5, respectively. Let there also be conditions p and q for which the following statements are true:

(1) if the condition p is true, then action a is performed during the execution of action b, and a ends 2-3 units of time before the end of b;

(2) if the condition q is true, then the work with ends together with the

action b.

Consider the question: "Does action a overlap with action c in time, assuming that both conditions p and q are met? If YES, specify the best estimate for the overlap time."

Knowledge of (1) and (2) in the MAL language can be written as an ontology

$$\boldsymbol{O} = \{|A| = 4,\ |B| = 6,\ 4\ |C| = 5,\ p \to A\ \boldsymbol{d}(2 \le K \le 3)\ B,\ q \to C\ \boldsymbol{f}\ B\}.$$

The above question can be formally represented by the following query to the ontology O:

$$?\ (\max x, \min y):\ \ p \wedge q \to A\ \boldsymbol{o}(x \le J \le y)\ C.$$

The answer to this query is the largest value of x and the smallest value of y, such that the ontology O2 logically follows $p \wedge q \to A\ o(x \le J \le y)\ C$.

Figure 4 shows the output tree for the set

$$+\ O2 \cup \{-\ p \wedge q \to A\ o(x \le J \le y)\ C\ \}.$$

This tree has 7 branches, the first two of which are closed due to the fact that they contain contrasting pairs (+ p,– p) and (+ q,– q). Let's write out all the inequalities and equalities from the other branches:

$\mathbf{B}_3 = \{A^+ – A^- \ge 4,\ A^- – A^+ \ge – 4,\ B^+ – B^- \ge 8,\ B^- – B^+ \ge – 8,$
$C^+ – C^- \ge 5,\ C^- – C^+ \ge – 5,\ A^- – B^- \ge 1,\ B^+ – A^+ \ge 1,\ B^+ – A^+ \ge 2,$
$A^+ – B^+ \ge – 3,\ \ C^- – B^- \ge 1,\ \ C^+ = B^+,\ \ A^- – C^- \ge 0\};$

$\mathbf{B}_4 = \{A^+ – A^- \ge 4,\ A^- – A^+ \ge – 4,\ B^+ – B^- \ge 8,\ B^- – B^+ \ge – 8,\ C^+ – C^- \ge 5,$
$C^- – C^+ \ge – 5,\ \ A^- – B^- \ge 1,\ B^+ – A^+ \ge 1,\ B^+ – A^+ \ge 2,$
$A^+ – B^+ \ge – 3,\ C^- – B^- \ge 1,\ \ C^+ = B^+,\ C^- – A^+ \ge 0\};$

$\mathbf{B}_5 = \{A^+ – A^- \ge 4,\ A^- – A^+ \ge – 4,\ B^+ – B^- \ge 8,\ B^- – B^+ \ge – 8,\ C^+ – C^- \ge 5,$
$C^- – C^+ \ge – 5,\ A^- – B^- \ge 1,\ B^+ – A^+ \ge 1,\ B^+ – A^+ \ge 2,$
$A^+ – B^+ \ge – 3,\ C^- – B^- \ge 1,\ C^+ = B^+,\ \ A^+ – C^+ \ge 0\};$

$\mathbf{B}_6 = \{A^+ – A^- \ge 4,\ A^- – A^+ \ge – 4,\ B^+ – B^- \ge 8,\ B^- – B^+ \ge – 8,\ C^+ – C^- \ge 5,$
$C^- – C^+ \ge – 5,\ \ A^- – B^- \ge 1,\ B^+ – A^+ \ge 1,\ B^+ – A^+ \ge 2,$
$A^+ – B^+ \ge – 3,\ C^- – B^- \ge 1,\ \ C^+ = B^+,\ A^- – A^+ \ge 1 – x\};$

$+\ |A| = 4$ [1]
$+\ |B| = 6$ [2]
$+\ |C| = 5$ [3]
$+\ p \to A\ \boldsymbol{d}(2 \le\ K \le 3)\ B$ [6]
$+\ q \to C\ \boldsymbol{f}\ B$ [11]
$–\ p \wedge q \to A\ \boldsymbol{o}(x \le J \le y)\ C$ [4]

$\mathbf{B}_7 = \{A^+ – A^- \ge 4,\ A^- – A^+ \ge – 4,\ B^+ – B^- \ge 8,\ \ B^- – B^+ \ge – 8,$ 3:
$C^+ – C^- \ge 5,\ \ C^- – C^+ \ge – 5,\ \ A^- – B^- \ge 1,\ \ B^+ – A^+ \ge 1,$
$B^+ – A^+ \ge 2,\ A^+ – B^+ \ge – 3,\ C^- – B^- \ge 1,\ \ C^+ = B^+,$
$A^+ – A^- \ge 1 + y.$

1: $A^+ – A^- \ge 4$
1: $A^- – A^+ \ge – 4$
2: $B^+ – B^- \ge 6$
2: $B^- – B^+ \ge – 6$
3: $C^+ – C^- \ge 5$
3: $C^- – C^+ \ge – 5$
4: $+\ p \wedge q$ [5]
4: $–\ A\ \boldsymbol{o}(x \le J \le y)\ C$ [14]
5: $+\ p$ [7]
5: $+\ q$ [12]

6: $–\ p$ [7]
7: X
Br1

6: $+\ A\ \boldsymbol{d}(2 \le K \le 3)\ B$ [8]
8: $+\ A\ \boldsymbol{d}\ B$ [9]
8: $+\ \boldsymbol{d} \bullet (2 \le K \le 3)\ /\ A, B =\ (B^+ – A^+ \ge 1)$ [10]
9: $A^- – B^- \ge 1$
9: $B^+ – A^+ \ge 1$
10: $B^+ – A^+ \ge 2$
10: $A^+ – B^+ \ge – 3$

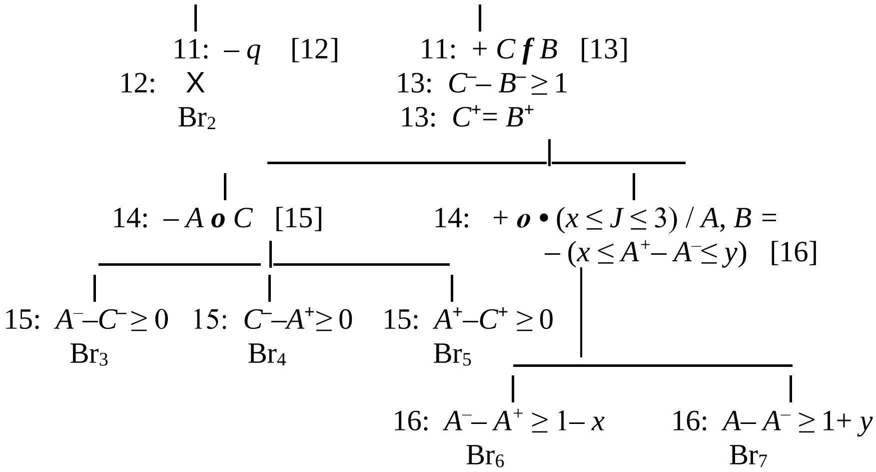

Figure 4. Output tree for the ontology and query from Example 4

If you build graphs G(B_3), D(B_4) and G(B_5), it is possible to find in them a positive cycles:

$C_3 = A^+ (2)\ B^+(-6)\ B^- (1)\ A^- (4)\ A^+$, length $C_3 = 2+(-6)+1+4 = 1$;

$C_4 = A^+ (0)\ C^- (5)\ C^+(0)\ B^+(-3)\ A^+$, length $C_4 = 0+5+0+(-3) = 2$;

$C_5 = A^+ (2)\ B^+(0)\ C^+ (0)\ A^+$, length $C_5 = 2+0+0 = 2$.

Therefore, the branches Br_3, Br_4 and Br_5 are closed. But the branches Br_6 and Br_7 can be closed only for certain values of the variables x and y. To find these values, we construct graphs G (B_6) and G (B_7) (Figure 5). We see that in these graphs there are the following cycles containing 1– x and 1+ y:

$$C_1 = A^+(1-x)\ A^-(5)\ A^+(0)\ B^+(-3)\ A^+ \text{ and}$$

$$C_2 = A^+(2)\ B^+(0)\ A^+(-5)\ A^-\ (1+y)\ A^+,$$

which respectively have a length of 1– x + 5 + 0 + (– 3) = 3– x 2 + (– 5)+(1+ y) = y– 2. The cycle C_1 is positive if and only if when 3– x ≥ 1, i.e. when x ≤ 2; cycle C_2 is positive if and only if when the u – 2 ≥ 1, i.e., when y ≥ 3. Hence, we get that the answer to the query is the sentence p ∧ q →A o (2 ≤ J ≤ 3) C.

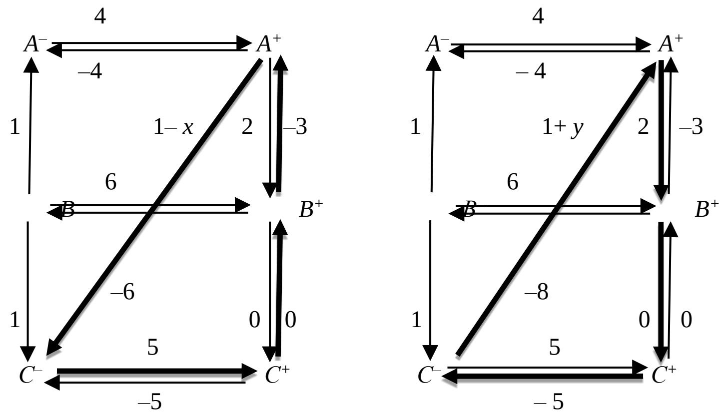

Figure 5. Graphs G (B6) and G (B7) from Example 4

4. CONCLUSION

The paper introduces the definition of a temporal metagraph. Since the function τ that maps time intervals to its vertices is partial, this metagraph contains vertex variables. A general T-metagraph defined in this way is a scheme from which specific T-metagraphs are obtained. For the formal expression of restrictions on the values of temporal variables, the interval logics IL-1 and IL-2 are constructed. Examples show how knowledge in the form of constraints for a T-metagraph can be represented in these logics. A deduction method based on analytical tables for interval ontologies is proposed. A theorem on the consistency and completeness of the system of inference rules for the logic of constraints in T-metagraphs is presented.

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