

# Connecting Granular and Topological Relations through Description Logics

Elio Hbeich<sup>1,2</sup>, Ana Roxin<sup>2</sup>, and Nicolas Bus<sup>1</sup>

<sup>1</sup> Université de Bourgogne Franche-Comté– LIB EA7534, Dijon 21000, France

<sup>2</sup> Information System and Applications Division, CSTB, Sophia Antipolis 06560, France

\*email:[elio.hbeich@cstb.fr](mailto:elio.hbeich@cstb.fr), [ana-maria.roxin@u-bourgogne.fr](mailto:ana-maria.roxin@u-bourgogne.fr),  
[nicolas.bus@cstb.fr](mailto:nicolas.bus@cstb.fr)

**Abstract.** Granularity deals with organizing in greater or lesser detail data, information, and knowledge that resides at a granular level. This organization is carried out according to certain criteria, which thereby provide a context view or dimension also called granular perspective. Topological relations express spatial associations among geospatial features (points, polylines, and polygons); they represent a horizontal spatial analysis. The two domains allow scientists to conceive different perspectives of the world. In this article, we aim to combine the two representations through Description Logics (DL) rules to relate granular (vertical representation) and geospatial topological (horizontal representation) relations. The following consequences are thus noted: (1) geospatial features become granules, (2) geospatial features are grouped into different levels of granularity and different granules, and finally, (3) granular construction and decomposition operations are integrated into the spatial domain.

**Keywords:** Geospatial Data, GeoSPARQL, Description Logic, Topological Relations, Granular Computing, Granular Relations.

## 1 Introduction

Scientists are continually endeavoring to structure their perception of the environment and the world, i.e., geospatial and building data. With the recent advances in Artificial Intelligence (AI), there is a growing need to analyze and reason over such data in the context of numerous use cases, i.e., disaster management, compliance checking (Bus et al., 2018). Granular Computing (GrC) has been recognized as a promising approach for representing human reasoning and problem solving through the levels of granularity (Keet, 2008). It considers for modelling a specific domain of knowledge or a worldview. While not fully implemented in ontology languages (such as OWL), GrC relationships structure knowledge into multi-level hierarchies by identifying parts of such knowledge, their relations, and their connections to the whole. In the present article, we seek to identify logical relations between GrC principles and existing topological relations as defined for existing geographical datasets. The overall goal is to use such logical rules to help automatically build perspectives or granular levels for knowledge in a specific area. Like the Level of Detail (LoD) concept used in CityGML

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(Gröger et al., 2012), such logical rules would facilitate different computing perspectives of the geospatial data available for a considered area. We aim to combine the two representations using Description Logics (DL) rules to relate granular and geospatial topological relations. We consider geospatial features as granules, grouped into different levels of granularity and different granules. The article is organized as follows: Section 2 introduces geospatial topological relations; section 3 presents the granular computing perspective, notions, and relationships; section 4 introduces related work; section 5 presents DL rules that relate topological and granular relationships, section 6 discusses our use case and future work, and finally we conclude in section 7.

## 2 Geospatial Topological Relations

Geospatial data contains geospatial features with geographic aspects, e.g., objects, locations. Geospatial features are connected through two types of geospatial relations: geometrical and topological (Zlatanova, 2015). This article focuses on topological connections, as they provide a general structure linking geospatial features. GeoSPARQL topological relations are used to link data among different geospatial resources. (Perry & Herring, 2012) GeoSPARQL is an Open Geospatial Consortium (OGC) standard used to represent and query geospatial data on the Web. Furthermore, it provides a vocabulary for asserting topological relations between geospatial features such as simple feature, RCC8 (Region Connection Calculus), and Egenhofer (see table below).

**Table 1.** GeoSPARQL topological relations (GeoSPARQL Ontology, 2012)(Available online at [http://schemas.opengis.net/geosparql/1.0/geosparql\\_vocab\\_all.rdf#](http://schemas.opengis.net/geosparql/1.0/geosparql_vocab_all.rdf#) ).

Simple Feature	Egenhofer	RCC8
Equals <b>sfEquals</b>	Equals <b>ehEquals</b>	Equals <b>rcc8eq</b>
Disjoint <b>sfDisjoints</b>	Disjoint <b>ehDisjoint</b>	Disconnected <b>rcc8dc</b>
Intersects <b>sfIntersects</b>	Meets <b>ehMeet</b>	Externally connected <b>rcc8ec</b>
Touches <b>sfTouches</b>	Overlaps <b>ehOverlap</b>	Partially overlaps <b>rcc8po</b>
Within <b>sfWithin</b>	Covers <b>ehCovers</b>	Tangential proper part inverse <b>rcc8tpi</b>
Contains <b>sfContains</b>	Covered by <b>ehCoveredBy</b>	Tangential proper part <b>rcc8tpp</b>
Overlaps <b>sfOverlaps</b>	Inside <b>ehInside</b>	Non-tangential proper part <b>rcc8ntpp</b>
Crosses <b>sfCrosses</b>	Contains <b>ehContains</b>	Non-tangential proper part inverse <b>rcc8ntppi</b>

The vocabulary described above illustrates connectivity, adjacency, and enclosure relations among geospatial features. For example, **sfTouches** describes whether two geospatial features are next to each other.

## 3 Granular Computing

(Yao, 2007) explains that GrC is studied from three perspectives philosophical, methodological, and computing. In this article, we are interested in the philosophical perspective as it explores the compositing of parts, their relations, and their connections

to the whole. GrC's philosophical perspective exploits structures in terms of granules, levels, and hierarchies based on multilevel representations. Consequently, a granule can be considered part of another granule or may include a family of granules, creating a hierarchical structure. (J. T. Yao et al., 2013) details three basic notions of GrC: (1) Granule: defined as a small particle among numerous particles forming a large unit. It represents classes, objects, data, elements, or any sort of real or virtual information. The partition of a granule into smaller ones results in subgranules. (2) Granulation: presents construction or decompose operations. The construction process forms high-level granules from lower-level subgranules; the decomposition process splits high-level granules into lower-level subgranules. (3) Granular relationships: used as a foundation to gather lower-level granules into higher-level ones (interrelationship) or to split high-level granules into low-level granules (intrarelationship). Note that high-level granules represent abstract concepts, and lower-level granules represent specific concepts. In the presented work, we consider the DL formalisms for modelling granules and the granular relations between them. The notation  $\text{granular-relation}(x, y)$  thus represents the granular relation between granules  $x$  and  $y$ . We also consider granules  $x$  and  $y$  as concepts (or concept instances) in DL. Based on these assumptions, the table below presents granular relationships as defined in (J. T. Yao et al., 2013) :

**Table 2. Granular relationships mathematical definition and DL notation**

Relation	Definition	DL notation
Refine	$x_i \in X, y_j \in Y$ $\forall x_i \in X \Rightarrow x_i \subset \exists y_j \in Y$	$\text{refine}(x, y)$
Coarse	$x_i \in X, y_j \in Y$ $\forall y_j \in Y \Rightarrow y_j \supset \exists x_i \in X$	$\text{coarse}(x, y)$
Partial fine	$x_i \in X, y_j \in Y$ $\exists x_i \in X \Rightarrow x_i \not\subset \forall y_j \in Y$	$\text{prefine}(x, y)$
Partial coarse	$x_i \in X, y_j \in Y$ $\exists y_j \in Y \Rightarrow y_j \not\supset \forall x_i \in X$	$\text{pcoarse}(x, y)$
Partition	<i>if <math>U</math> a finite partition <math>\pi = \{X_i   1 \leq i \leq m\}</math>, then</i> $X_i \neq \emptyset, \forall i \neq j, X_i \cap X_j = \emptyset, \bigcup_1^m X_i = U$	$\text{partition}(x, y)$
Covering	<i>if <math>U</math> a finite partition <math>\pi = \{X_i   1 \leq i \leq m\}</math>, then</i> $X_i \neq \emptyset, \forall i \neq j, \bigcup_1^m X_i = U$	$\text{covering}(x, y)$
Similar	$\text{Sim}(x, y) = \frac{1}{m \times n} \sum_{i=1}^m \sum_{j=1}^n \text{Sim}(x_i, y_j)$	$\text{similar}(x, y)$

In GrC domain, the term subgranule is used to differentiate granules that exist on two different levels, where the high-level contains granules and the low-level contains subgranules. In other words,  $\text{refine}(g1, g2)$  indicates that  $g1$  and  $g2$  are distinct granules located on two different granular levels, and that  $g1$  is located on a higher-level than  $g2$ . As a result,  $g1$  is considered a granule, while  $g2$  is considered a subgranule. To incorporate GrC notions (granules, levels, and hierarchies) into OWL, we need to address certain differences between them. While OWL differentiates concepts and instances, GrC represents them both using granules. While OWL uses subclass relation

to refer to hierarchy, this relation does not represent the complexities of the granular world (high-level and lower-level granule). At the same time, OWL uses different types of relations to represent connection between concepts, instances, etc. GrC only uses granular relationships. Therefore, the below DL rules allow building granular levels that do not exist in OWL.

## 4 Related Work

Inspired by geospatial topological relations, the authors in (Dube & Egenhofer, 2009) created coarse topological relations to analyze data in different contexts. For example, inside and coveredBy are grouped in a single relation called IN to indicate that one region is a proper subset of the other. In addition, the authors mapped both topological relations to address the zonal representation of the relations neighborhoods between spatial entities. (Fent et al., 2005) proposes an extension of GeoGraph called Granular GeoGraph that supports spatial and semantic granularity by adding granular notion (aggregation and generalization) to geospatial conceptual models. These notions allow modification in geometry description and topological relation between spatial objects. The authors in (Khamespanah et al., 2016) propose a reliable model for earthquake vulnerability assessment to manage the uncertainty associated with the experts' opinions. To achieve their objective geospatial data were integrated using Dempster-Shafer theory, and granular-tree was applied to extract rules with minimum incompatibility from the information table provided by the experts.

## 5 Connecting Topological and Granular Relations through Logical Rules

Topological relations describe the interactions among geospatial features horizontally; they consider that all geospatial features are situated on the same level or layer. GrC builds knowledge in a hierarchical structure by exploring the composition/decomposition of parts, their relations, and connections. Nevertheless, both domains represent information from orthogonal perspectives. We have noted the potential connection between granular and topological relations. To apply the philosophical perspective of GrC to the spatial domain, we develop a set of DL rules to build granular levels from geospatial data and their topological relationships, thus bringing a multilevel interpretation to the spatial domain. Before connecting granular and topological relations, we noticed the following (equivalent:  $\equiv$ , not:  $\neg$ , Union:  $\cup$ , Intersection  $\cap$ ):

- $sfEquals \equiv ehEquals \equiv rcc8ec$
- $sfDisjoints \equiv ehDisjoint \equiv rcc8dc$
- $sfIntersects \equiv \neg ehDisjoint \equiv \neg rcc8dc$
- $sfTouches \equiv ehMeet \equiv \neg rcc8dc$
- $sfWithin \equiv (ehInside \cup ehCoveredBy) \equiv (rcc8ntpp \cup rcc8tppi)$
- $sfContains \equiv (ehContains \cup ehCovers) \equiv (rcc8ntppi \cup rcc8tpp)$
- $sfOverlaps \equiv ehOverlap \equiv rcc8po$

The above equivalences imply that if one topological relation is associated with a granular relationship, its equivalent vocabulary is also linked. For example, if *sfContains* is connected to refine relation and  $\text{sfContains} \equiv (\text{ehCoveredBy} \cup \text{ehInside})$ , then  $(\text{ehCoveredBy} \cup \text{ehInside})$  is connected to the refine relation. However, GrC (granules and granular relationships) has not been implemented as an ontology, such as GeoSPARQL [7]. Hence, the tables below represent DL rules that relate topological and granular relationships.

**Table 3.** DL rules relating granular and Simple Feature topological relations

DL rules	
1	$\text{sfContains}(x, y) \rightarrow \text{refine}(x, y)$
2	$\text{sfWithin}(y, x) \rightarrow \text{coarse}(y, x)$
3	$\text{sfCrosses}(x, y) \rightarrow \text{prefine}(x, y) \cap \text{pcoarse}(y, x)$
4	$\text{sfOverlaps}(x, y) \rightarrow \text{prefine}(x, y) \cap \text{pcoarse}(y, x)$
5	$\text{sfEquals}(x, y) \rightarrow \text{similar}(x, y)$
6	$\text{sfContains}(x, y) \cap \text{sfContains}(x, z) \cap \text{sfTouches}(y, z) \rightarrow \text{partition}(x, y) \cap \text{partition}(x, z)$
7	$\text{sfContains}(x, y) \cap \text{sfContains}(x, z) \cap \text{sfDisjoint}(y, z) \rightarrow \text{partition}(x, y) \cap \text{partition}(x, z)$
8	$\text{sfContains}(x, y) \cap \text{sfContains}(x, z) \cap \text{sfOverlaps}(y, z) \rightarrow \text{covering}(x, y) \cap \text{covering}(x, z)$
9	$\text{sfContains}(x, y) \cap \text{sfContains}(x, z) \cap \text{sfCrosses}(y, z) \rightarrow \text{covering}(x, y) \cap \text{covering}(x, z)$

**Table 4.** DL rules relating granular and Egenhofer topological relations

DL rules	
10	$\text{ehContains}(x, y) \cup \text{ehCovers}(x, y) \rightarrow \text{refine}(x, y)$
11	$\text{ehInside}(y, x) \cup \text{ehCoveredBy}(y, x) \rightarrow \text{coarse}(x, y)$
12	$\text{ehOverlaps}(x, y) \rightarrow \text{prefine}(x, y) \cap \text{pcoarse}(y, x)$
13	$\text{ehEquals}(x, y) \rightarrow \text{similar}(x, y)$
14	$(\text{ehContains}(x, y) \cup \text{ehCovers}(x, y)) \cap ((\text{ehContains}(x, z) \cup (\text{ehCovers}(x, z)) \cap \text{ehMeet}(y, z) \rightarrow \text{partition}(x, y) \cap \text{partition}(x, z)$
15	$(\text{ehContains}(x, y) \cup \text{ehCovers}(x, y)) \cap ((\text{ehContains}(x, z) \cup (\text{ehCovers}(x, z)) \cap \text{ehDisjoint}(y, z) \rightarrow \text{partition}(x, y) \cap \text{partition}(x, z)$
16	$(\text{ehContains}(x, y) \cup \text{ehCovers}(x, y)) \cap ((\text{ehContains}(x, z) \cup (\text{ehCovers}(x, z)) \cap \text{ehOverlaps}(y, z) \rightarrow \text{covering}(x, y) \cap \text{covering}(x, z)$
17	$(\text{ehContains}(x, y) \cup \text{ehCovers}(x, y)) \cap ((\text{ehContains}(x, z) \cup (\text{ehCovers}(x, z)) \cap \neg \text{ehDisjoint} \rightarrow \text{covering}(x, y) \cap \text{covering}(x, z)$

**Table 5.** DL rules relating granular and RCC8 topological relations

DL rules	
18	$\text{rcc8ntppi}(x, y) \cup \text{rcc8tpp}(x, y) \rightarrow \text{refine}(x, y)$
19	$\text{rcc8ntpp}(y, x) \cup \text{rcc8tppi}(y, x) \rightarrow \text{coarse}(x, y)$

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- 20  $rcc8po(x, y) \rightarrow refine(x, y) \cap pcoarse(y, x)$   
 21  $rcc8eq(x, y) \rightarrow similar(x, y)$   
 22  $(rcc8ntppi(x, y) \cup rcc8tpp(x, y)) \cap ((rcc8ntppi(x, z) \cup (rcc8tpp(x, z)) \cap rcc8dc(x, z))$   
      $\rightarrow partition(x, y) \cap partition(x, z)$   
 23  $(rcc8ntppi(x, y) \cup rcc8tpp(x, y)) \cap ((rcc8ntppi(x, z) \cup (rcc8tpp(x, z)) \cap rcc8po(y, z))$   
      $\rightarrow covering(x, y) \cap covering(x, z)$   
 24  $(rcc8ntppi(x, y) \cup rcc8tpp(x, y)) \cap ((rcc8ntppi(x, z) \cup (rcc8tpp(x, z)) \cap \neg rcc8dc(y, z))$   
      $\rightarrow covering(x, y) \cap covering(x, z)$
- 

In addition, we have noted that refine is the inverse of coarse and that refine is the inverse relation of pcoarse. Even when granular relationships connect two levels of granularity, one can infer granular relationships to accommodate multiple scales. For example,  $refine(x, y) \cap refine(y, z) \rightarrow refine(x, z)$ .

## 6 Use Case

Our scope of work focuses on creating a multiscale semantic checker that verifies the compliance of construction at urban and building levels. After creating our knowledge base that integrates and connects urban and building concepts, and divided French urban regulations into several scales: building, district, city, and region (Hbeich et al., 2019). We will apply the GrC notion and relationships on our knowledge base to produce a multiscale structure, enabling us to connect the urban regulation to the appropriate level of the knowledge base. The reason behind the implementation of GrC notion and relations refers to the limitations of OWL language to represent the complexity of the GrC philosophical perspective. For example, OWL uses subclass to highlight hierarchy, e.g., A subclassOf B. This relation implies that (1) all the instances of A are instances of B, (2) A inherent all relations and restriction from B, and finally (3) A inherent all properties of B. While granular relation such as A refine B indicates (1) A is at a lower level than B, (2) A and B could represent a concept or instance, (3) A and B are different concepts or instances, and finally (4) both granules don't inherit any relations or restriction from one another. In this article, we have investigated the relations between topological and granular relationships. Our future work will apply the same methodology (philosophical perspective) to the Building Information Model (BIM), more specifically to IFC relations, in order to create a hierarchical structure for building models. By connecting the two hierarchical structures (geospatial and building), we will then be able to generate a multiscale knowledge base ranging from City to building elements.

## 7 Conclusion and Future Work

As mentioned above, topology structures information horizontally, whereas GrC creates hierarchies of information. Our work combines the two representations using

DL rules to relate granular and geospatial topological relations. Thus, we consider geospatial features as granules, grouped into different levels of granularity and into different granules. In this way, geospatial data is presented as granular multiscale hierarchies. Our future work will specify an OWL vocabulary for granular relations and further apply Linked Data principles along with the DL rules elaborated here. The goal is to create an ontology similar to GeoSPARQL for GrC and use it to structure existing geospatial datasets. In doing so, the rules presented above can be adapted either into SHACL rules (Shapes Constraint Language (SHACL), s. d.) or SPARQL queries (SPARQL Query Language for RDF, 2019), thus structuring geospatial knowledge (pertaining to the considered datasets) into multiscale, granular knowledge. This would enable multiscale compliance checking rules, for example considering a building's environment when checking specific regulations.

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